Mathematical Semantics of Computer Systems, *MSCS* (4810-1168)
Handout for Lecture 11 (2016/12/19)

Ichiro Hasuo, Dept. Computer Science, Univ. Tokyo
http://www-mmm.is.s.u-tokyo.ac.jp/∼ichiro

Video recording of the lectures is available at: http://www-mmm.is.s.u-tokyo.ac.jp/videos/mscs2016

1 Cartesian Closed Categories as Models of Typed λ-Calculus

We continue and follow slides by Samson Abramsky (Oxford) found at www.math.helsinki.fi/logic/sellc-2010/course/LectureIII.pdf. See [2] for further details. There is a big body of literature on the λ-calculus, including [1, 3, 4].

- On conversion in λ-calculus
- Categorical Semantics, as a typed λ-calculus and Cartesian closed categories as examples

Definition. Type judgment. Type derivation tree.

NB: we use the term calculus *a la Church* (where bound variables have explicit types).

Lemma 1. *Each derivable type judgment has a unique derivation tree.*

Definition. Cartesian closed category: a category with finite products and exponentials.

Definition. Interpretation $[\![\_]\!]$ of typed λ-calculus. Interpreting types, type derivation trees, type judgments, and terms.

Definition. Substitution lemma: interpretation of $s[t/x]$ is given by composition of arrows.

Definition. Conversion rules, including congruence rules.

Theorem. *Soundness of categorical semantics: if $s =_{\beta\eta} t$, then $[\![s]\!] = [\![t]\!]$.*

If we have time:

- The Curry-Howard correspondence; terms as proofs; conversion as proof normalization

2 Lectures Remaining

Coalgebras and State-Based Transition Systems. We will follow [Jacobs 2012, Chapters 1-3]

- 2017.1.16 (Mon)
- 2017.1.23 (Mon)
- 2017.1.30 (Mon)

References

[1] H.P. Barendregt. *The Lambda Calculus. Its Syntax and Semantics.* North-Holland, Amsterdam, 2nd rev. edn., 1984.

[2] J. Lambek and P.J. Scott. *Introduction to higher order Categorical Logic.* No. 7 in Cambridge Studies in Advanced Mathematics. Cambridge Univ. Press, 1986.

[3] M.H. Sørensen and P. Urzyczyn. *Lectures on the Curry-Howard Isomorphism*, vol. 149 of *Studies in Logic and the Foundations of Mathematics.* Elsevier Science Inc., New York, NY, USA, 2006.

[4] G. Winskel. *The Formal Semantics of Programming Languages.* MIT Press, 1993.