

# Mathematical Semantics of Computer Systems, *MSCS* (4810-1168) Handout for Lecture 2 (2016/10/3)

Ichiro Hasuo, Dept. Computer Science, Univ. Tokyo  
<http://www-mmm.is.s.u-tokyo.ac.jp/~ichiro>

Video recording of the lectures is available at:

<http://www-mmm.is.s.u-tokyo.ac.jp/~ichiro/mscs2016>

(ask me for username, password)

## 1 Today's Goal

Come to understand and prove the following statement.

**Proposition 1.** *Let  $X, Y \in \mathbb{C}$ . A product of  $X$  and  $Y$ , if it exists, is unique up-to a canonical isomorphism.*

## 2 Today's Agenda

### 2.1 Categories, Functors

**Definition.** Category, object, arrow (= morphism).

**Example.** Sets; a preorder as a category; a monoid as a category; the category **Mon** of monoids and monoid homomorphisms.

**Remark 1.** To familiarize yourself with the following abstract categorical notions, always try to think of their instances when the category in question is one of the previous examples.

**Remark 2.** An “abstract” definition of category by a deductive system, in [Lambek & Scott]

**Definition.** Proper class, small set, small category, locally small category

**Definition.** Functor

**Example.** Monotone functions as functors (for preorders considered as categories); monoid homomorphisms as functors (for monoids).

**Example.** Graphs as functors. Monoid/group actions as functors.

### 2.2 Reasoning with Arrows

**Definition.** Epi, mono. Split epi, split mono.

**Definition.** Product, coproduct.

## 3 Exercises

1. Describe Russell's paradox.
2. Prove “Today's Goal.”