

Mathematical Structures in Formal Methods, *MSFM*

Handout for Lecture 4 (2018/5/24)

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1 Today's Lecture

Closure properties of Buchi automata. Impossibility of determinization. Complementation via Rabin automata. We use [Vardi, Section 2].

Report Assignment

1. Give a formal proof for the first half of the proof of Proposition 9 (that it suffices to prove the claim for Rabin conditions that consist of a single pair).
2. Given a language $L' \subseteq A^*$, we define an ω -language $\overrightarrow{L'} \subseteq A^\omega$ by

$$\overrightarrow{L'} := \{w \in A^\omega \mid w \text{'s infinitely many prefixes are in } L'\} .$$

(Note that L' is a set of *finite* words; $\overrightarrow{L'}$ is a set of *infinite* words.)

- (a) Show that if $L \subseteq A^\omega$ is recognized by some *deterministic* Büchi automaton, there exists a regular language $L' \subseteq A^*$ such that $L = \overrightarrow{L'}$. (Note that you must show both $L \subseteq \overrightarrow{L'}$ and $L \supseteq \overrightarrow{L'}$)
- (b) Use (a) to show that $(0+1)^*1^\omega$ is not recognized by any deterministic Büchi automaton.
- (c) Conclude that deterministic Büchi automata have an expressive power that is strictly less than (general) Büchi automata.
- (d) Prove the converse of (a).