Bounding Errors Due to Switching Delays in Incrementally Stable Switched Systems

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SOKENDAI

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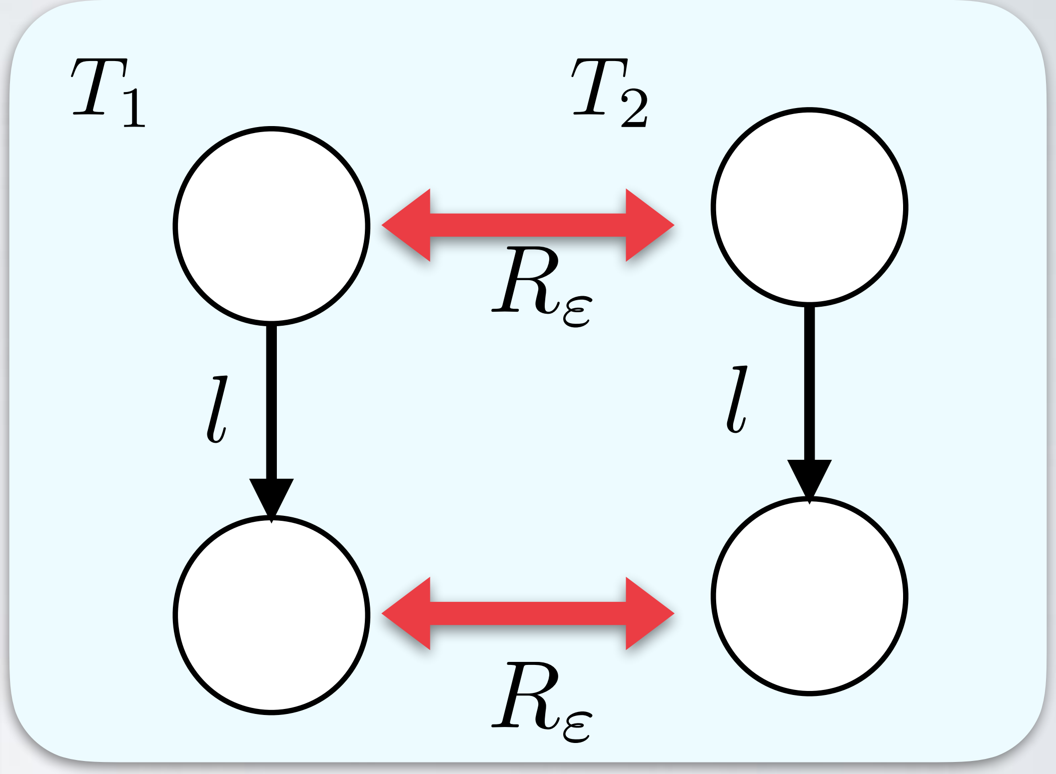
Outline

- The approximate bisimulation workflow
- Our use: delays in switched systems
  - Simper setting ➜ applicability
- Technical contributions
- Two-step synthesis,
  via state-space discretization
- Case study

Approximate Bisimulation

[Girard & Pappas, IEEE Transactions on Automatic Control'07]

$$T_i = (Q_i, L, \underset{i}{\longrightarrow}, O, H_i) \text{ for } i = 1, 2$$
$$(O \text{ equipped with a metric } d)$$



<u>Def.</u>

$R_\varepsilon \subseteq Q_1 \times Q_2$ is an <u>ε–approximate bisimulation</u> relation between T_1 and T_2 if for all $(q_1, q_2) \in R_\varepsilon$,

- $d(H_1(q_1), H_2(q_2)) \leq \epsilon$
- $\forall q_1 \overset{l}{\underset{1}{\rightarrow}} q_1', \exists q_2 \overset{l}{\underset{2}{\rightarrow}} q_2',$ s.t. $(q_1', q_2') \in R_\varepsilon$
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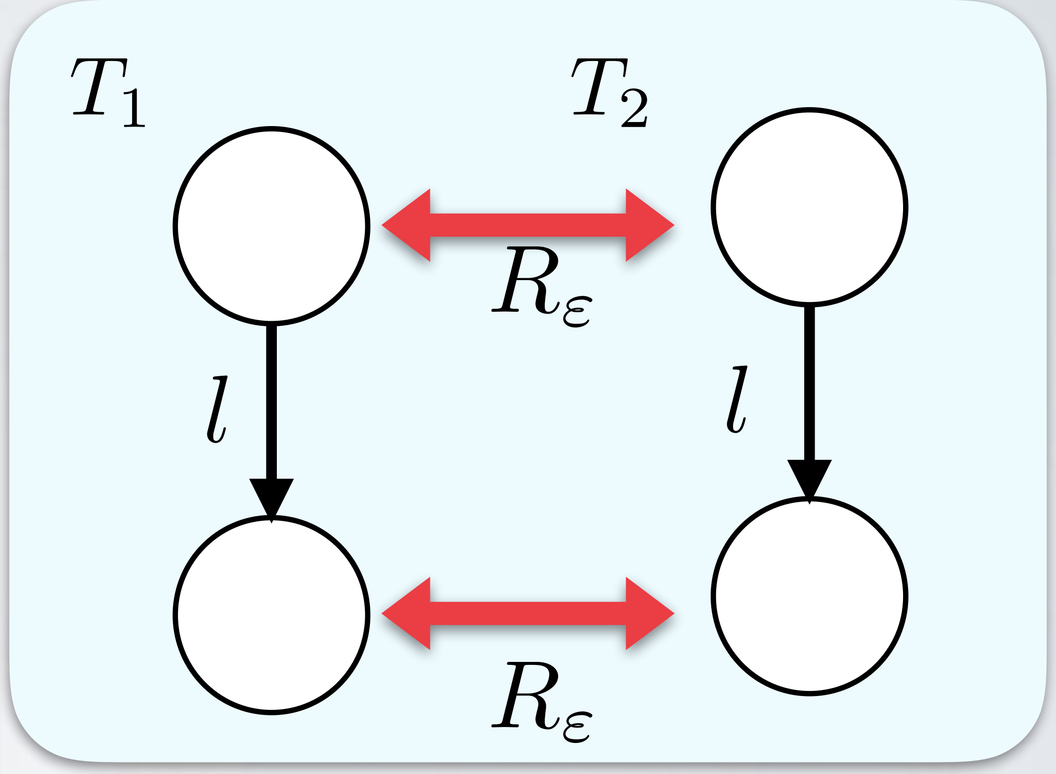

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Once in a relationship, always henceforth

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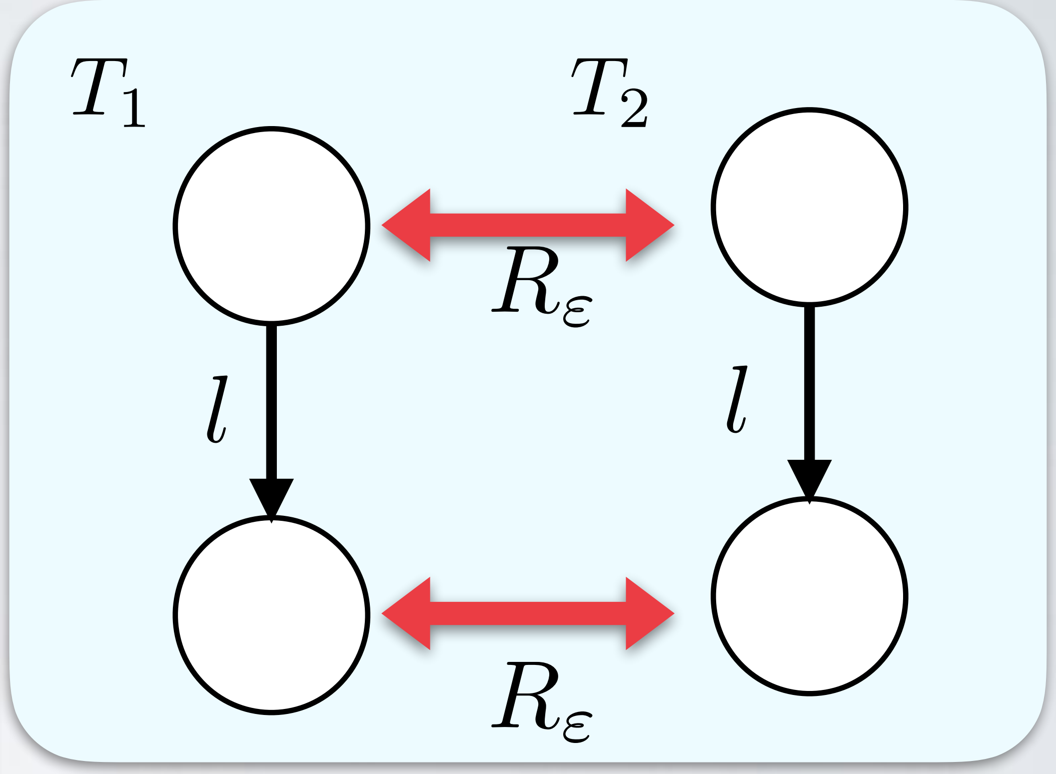

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Once in a relationship, always henceforth

Incremental Stability (δ-GUAS)

Incrementally **G**lobally **U**niformly **A**symptotically **S**table

Def. A dynamics is said to be δ-$\boldsymbol{GAS}$ if $\exists$ $\mathcal{KL}$ function β s.t.

$$\left\| \mathbf{x}(x,t) - \mathbf{x}(y,t) \right\| \;\leq\; \beta\big(\, \|x-y\|,\, t\,\big)$$

where $\mathbf{x}(x,-)$ is the trajectory starting at x.

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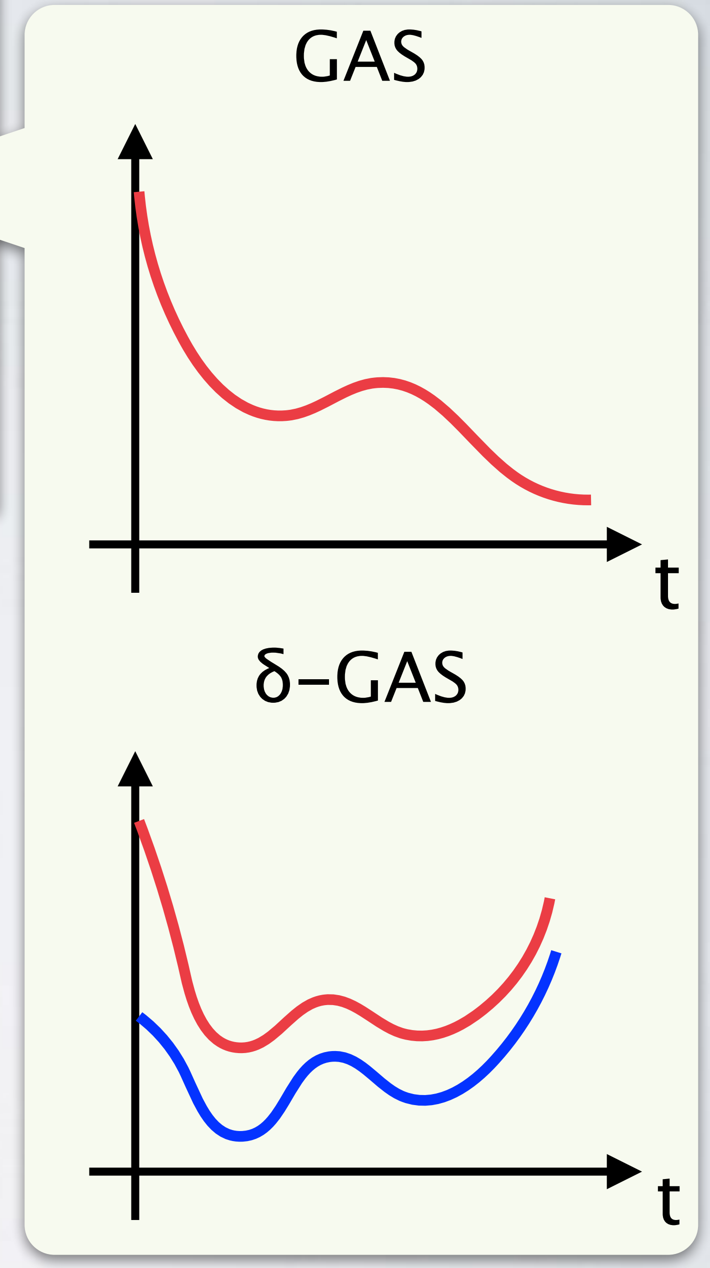

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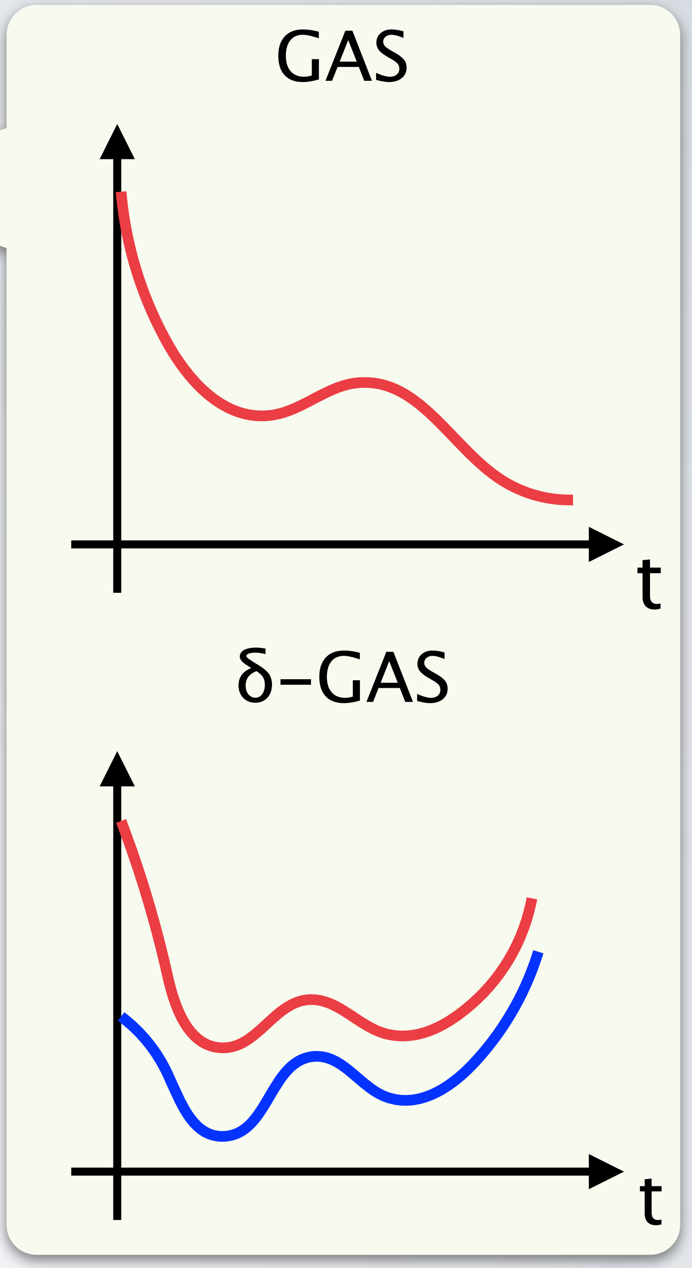

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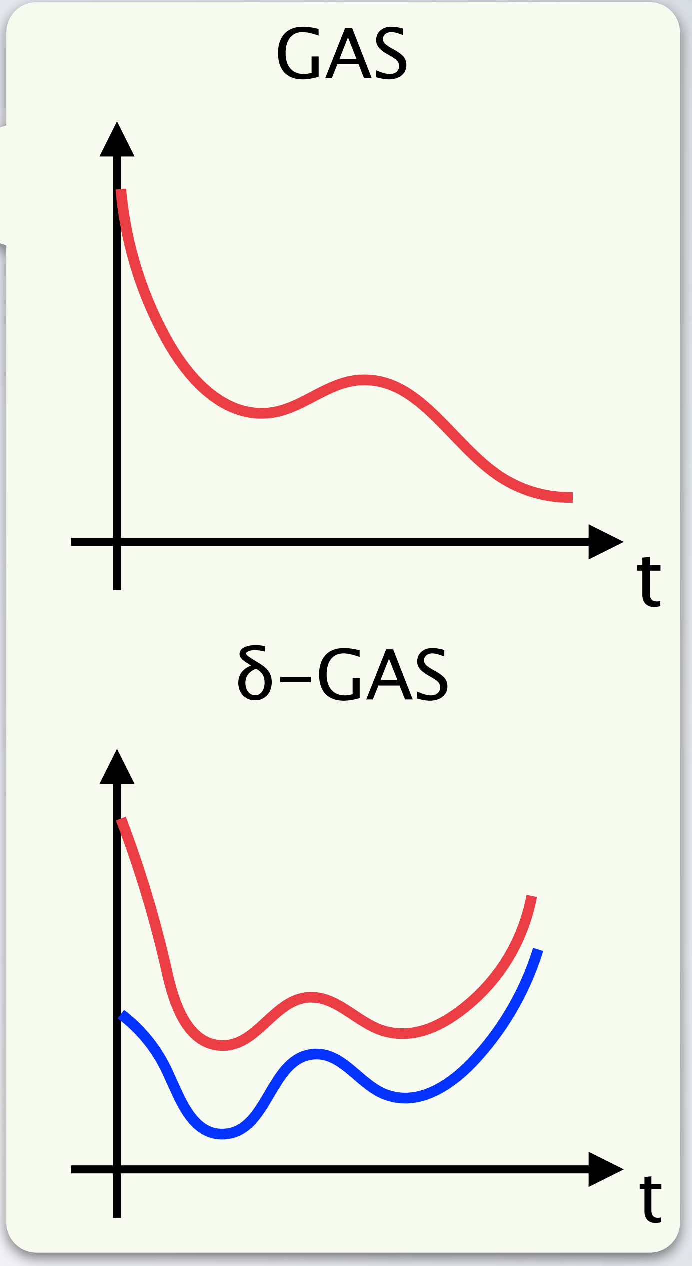

Def. For a dynamics $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ in $\mathbb{R}^{\boldsymbol{n}}$, a smooth function $\boldsymbol{V} : \mathbb{R}^{\boldsymbol{n}} \times \mathbb{R}^{\boldsymbol{n}} \to \mathbb{R}^{+}$ is a *common $\boldsymbol{\delta}$-GAS Lyapunov function* if there exist $\mathcal{K}_\infty$ functions $\underline{\boldsymbol{\alpha}}$, $\overline{\boldsymbol{\alpha}}$ and $\boldsymbol{\kappa} > \mathbf{0}$ such that

$$\underline{\boldsymbol{\alpha}}(\|\boldsymbol{x}-\boldsymbol{y}\|) \;\le\; \boldsymbol{V}(\boldsymbol{x},\boldsymbol{y}) \;\le\; \overline{\boldsymbol{\alpha}}(\|\boldsymbol{x}-\boldsymbol{y}\|)$$

$$\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{x}}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{f}(\boldsymbol{x}) + \frac{\partial \boldsymbol{V}}{\partial \boldsymbol{y}}(\boldsymbol{x},\boldsymbol{y})\boldsymbol{f}(\boldsymbol{y}) \;\le\; -\boldsymbol{\kappa}\boldsymbol{V}(\boldsymbol{x},\boldsymbol{y})$$

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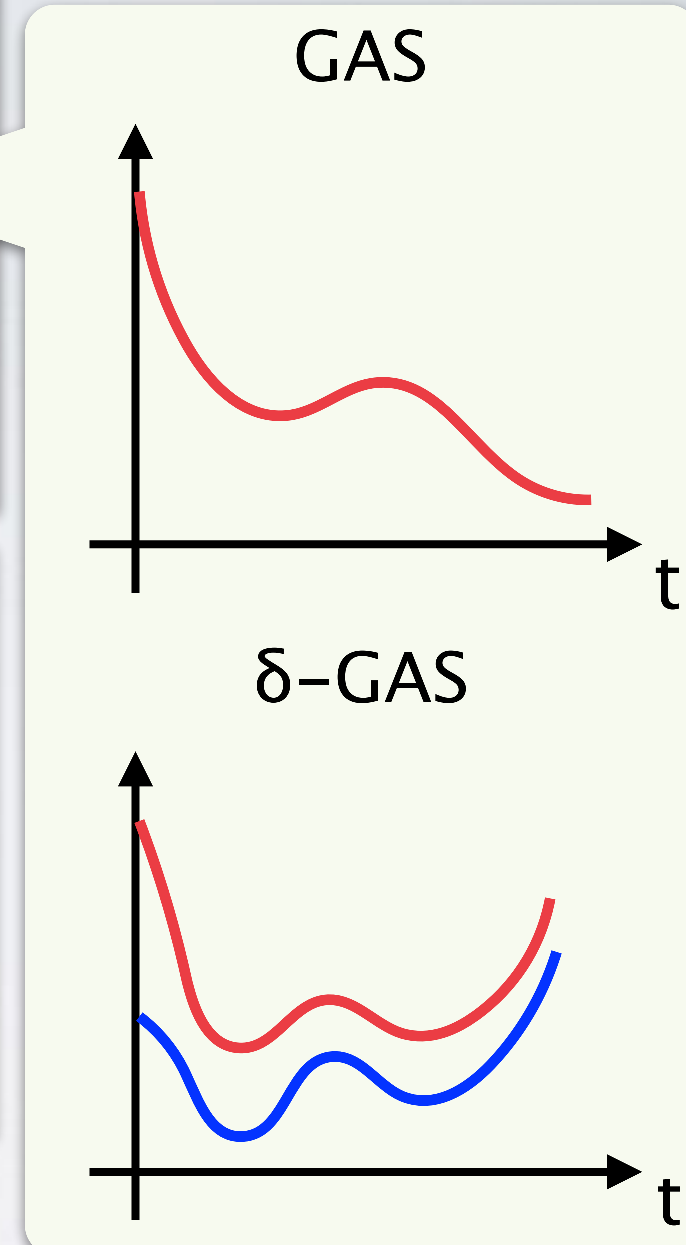

$V(x,y)$ is more or less $\|\, x - y \,\|$...

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$$\underline{\alpha}(\|x - y\|) \;\leq\; V(x,y) \;\leq\; \overline{\alpha}(\|x - y\|)$$

$$\frac{\partial V}{\partial x}(x,y) f(x) + \frac{\partial V}{\partial y}(x,y) f(y) \;\leq\; -\kappa V(x,y)$$

that decreases along the dynamics

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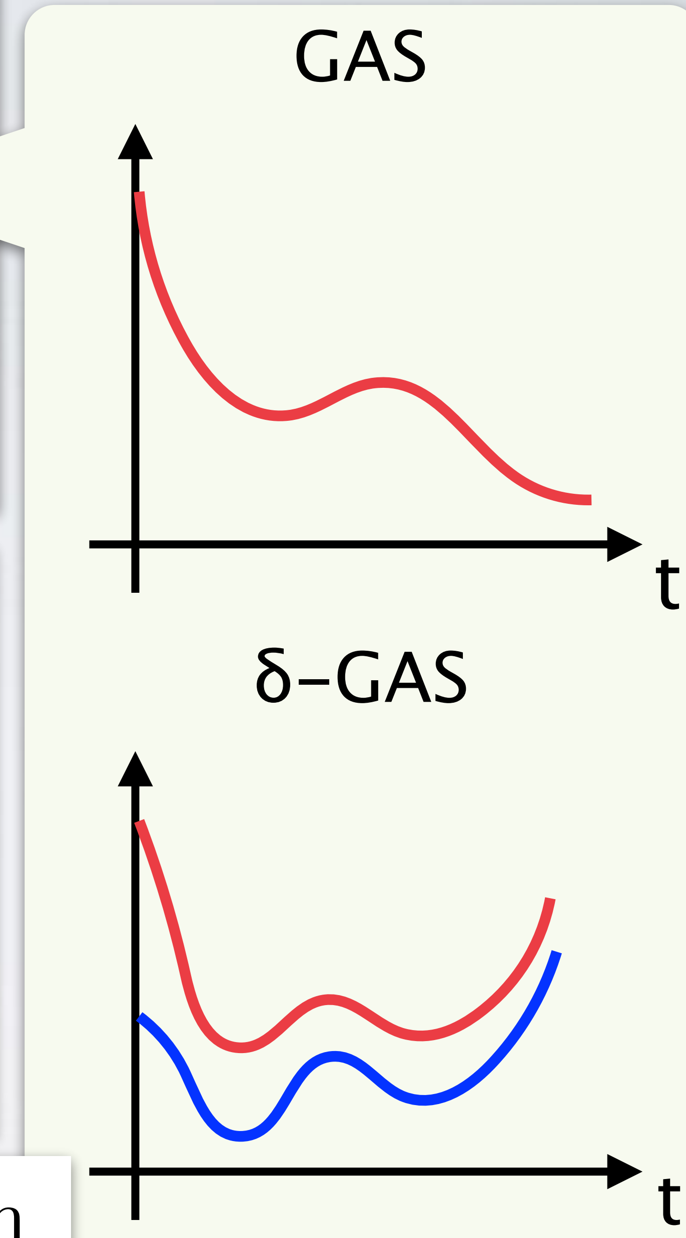

Thm. There is a $\boldsymbol{\delta}$-GAS Lyapunov function $\Longrightarrow$ incrementally stable ($\boldsymbol{\delta}$-GAS).

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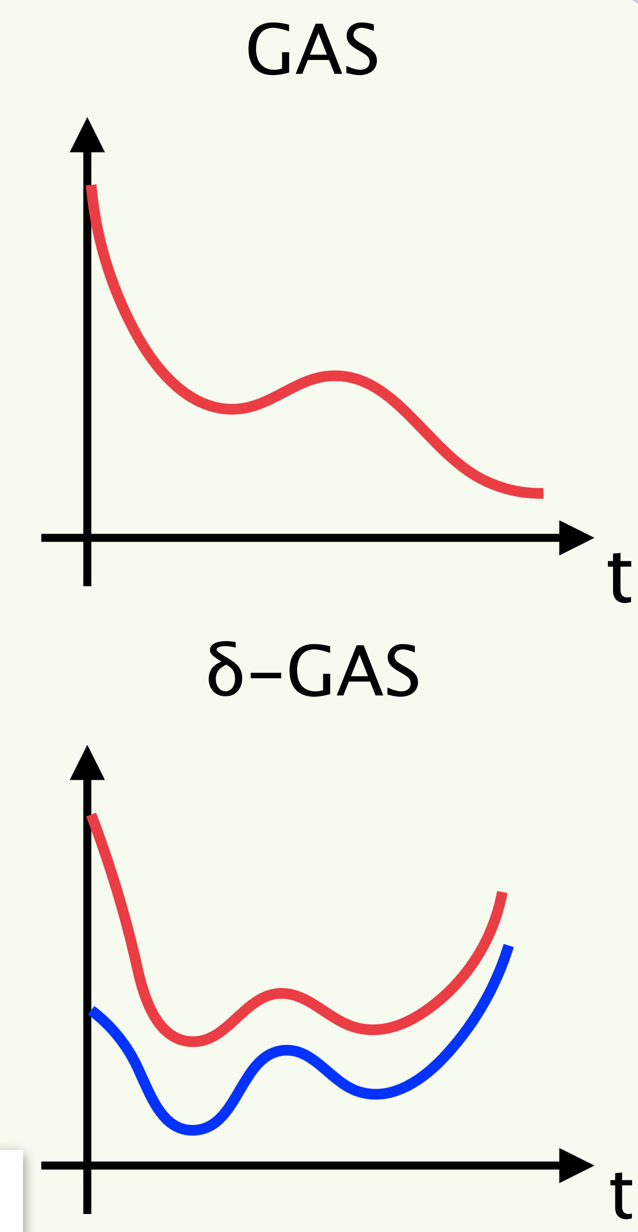

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Thm. There is a δ-GAS Lyapunov function $\Longrightarrow$ incrementally stable (δ-GAS).

Switched extension: δ-GUAS, common Lyapunov func.

[Girard, Pola, Tabuada, IEEE TAC '10]

Discrete Abstraction via Incremental Stability

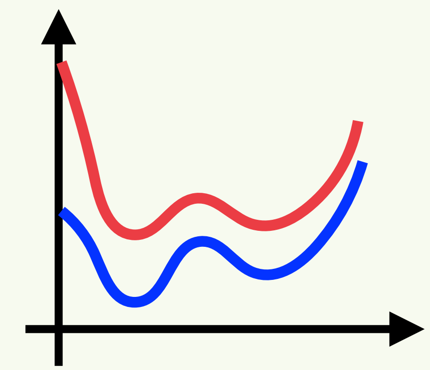
[Girard, Pola, Tabuada, IEEE TAC '10]

For a switched dynamics $\dot{x} = f_p(x)$ …

Common δ-GAS Lyapunov func. V

witnesses →

Incremental stability (δ-GUAS)



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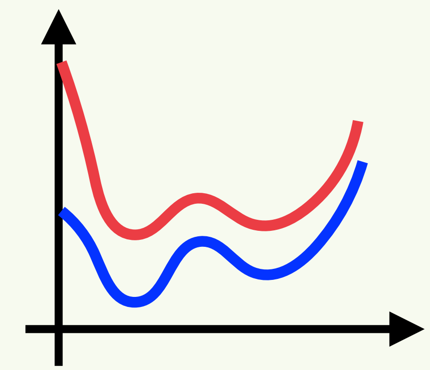


induces

Approximate bisimulation

Thm. Let $\boldsymbol{V}$ be a common $\boldsymbol{\delta}$-GAS Lyapunov func. lower-bounded by $\underline{\boldsymbol{\alpha}}$. Then

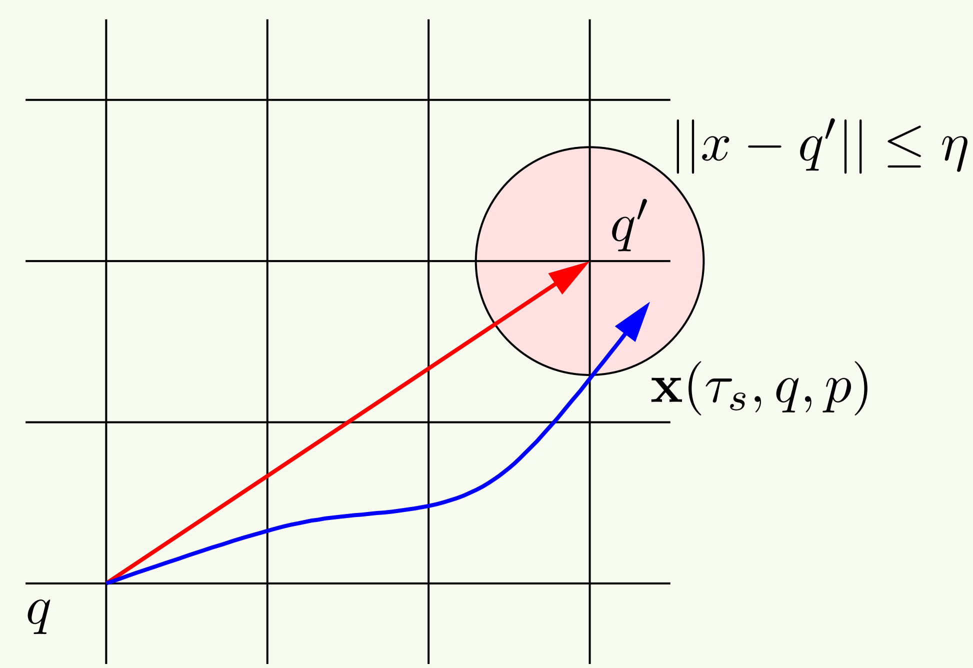
$$(\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{R} \quad \overset{\text{def}}{\Longleftrightarrow} \quad \boldsymbol{V}(\boldsymbol{x}, \boldsymbol{y}) \le \underline{\boldsymbol{\alpha}}(\boldsymbol{\varepsilon})$$

yields an $\boldsymbol{\varepsilon}$-approximate bisimulation.

correctness guarantee

Statespace abstraction

(Fig. from [GirardPT'10])



Discrete Abstraction via Incremental Stability

[Girard, Pola, Tabuada, IEEE TAC '10]

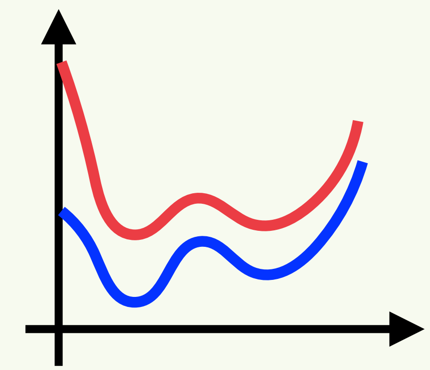
Discrete verification, synthesis, supervisory control, …

For a switched dynamics $\dot{x} = f_p(x)$ …

Common δ-GAS Lyapunov func. V

witnesses →

Incremental stability (δ-GUAS)



↓ induces

Approximate bisimulation

Thm. Let V be a common δ-GAS Lyapunov func. lower-bounded by $\underline{\alpha}$. Then

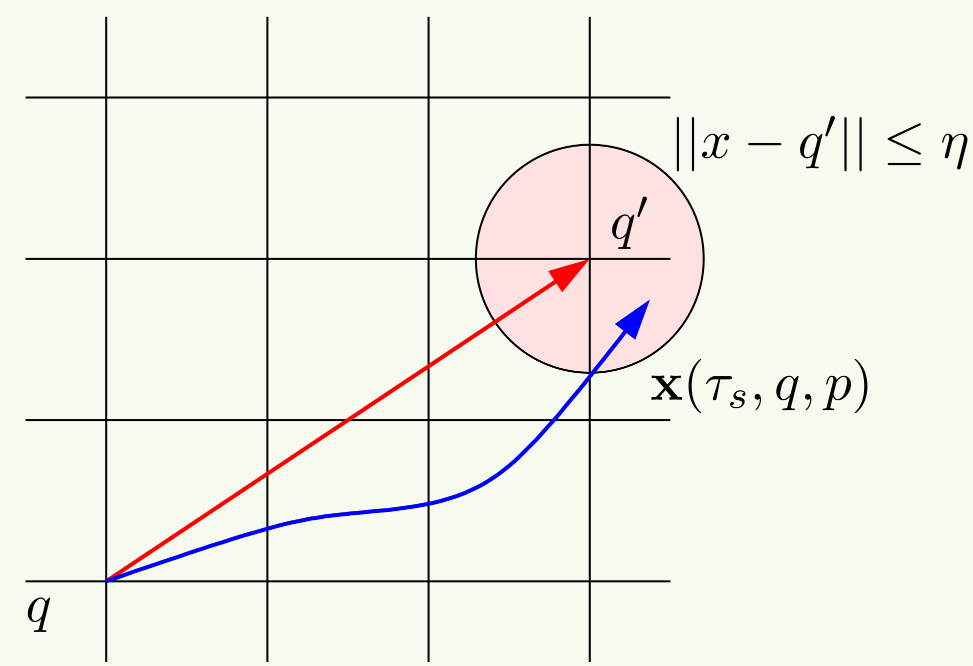
$$(x, y) \in R \quad \overset{\text{def}}{\iff} \quad V(x, y) \leq \underline{\alpha}(\varepsilon)$$

yields an ε-approximate bisimulation.

→ correctness guarantee

Statespace abstraction

(Fig. from [GirardPT'10])



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Time Delays in Control

- Networked control, IoT, …

Time Delays in Control

- Networked control, IoT, …

- Cloud control

Switched System $\dot{x} = f_p(x)$

Def.

A switched system is a quadruple $(\mathbb{R}^n, P, \mathcal{P}, F)$

- $\mathbb{R}^n$: state space
- $P = \{1, \cdots, m\}$: finite set of modes
- $\mathcal{P} \subseteq \mathbb{R}^+ \to P$: the set of switching signals (piecewise constant functions that are continuous from the right and non-Zeno)
- $F = \{f_1, \cdots, f_m\}$: a set of functions ($f_p : \mathbb{R}^n \to \mathbb{R}^n$ for each mode p)

Notation

p : mode

$\mathbf{p}$: switching signal

Trajectory of Switched Systems

Given a switching signal $\mathbf{p}$,

- $x(t) \in \mathbb{R}^n$ behaves according to $\dot{x}(t) = f_p(x(t))$ while $\mathbf{p}(t) = p$.
- $x(t) \in \mathbb{R}^n$ is continuous even at switching

Periodic Switched Systems

cf. [Al Khatib et al., HSCC'16]

Periodic: k-th switching occurs at

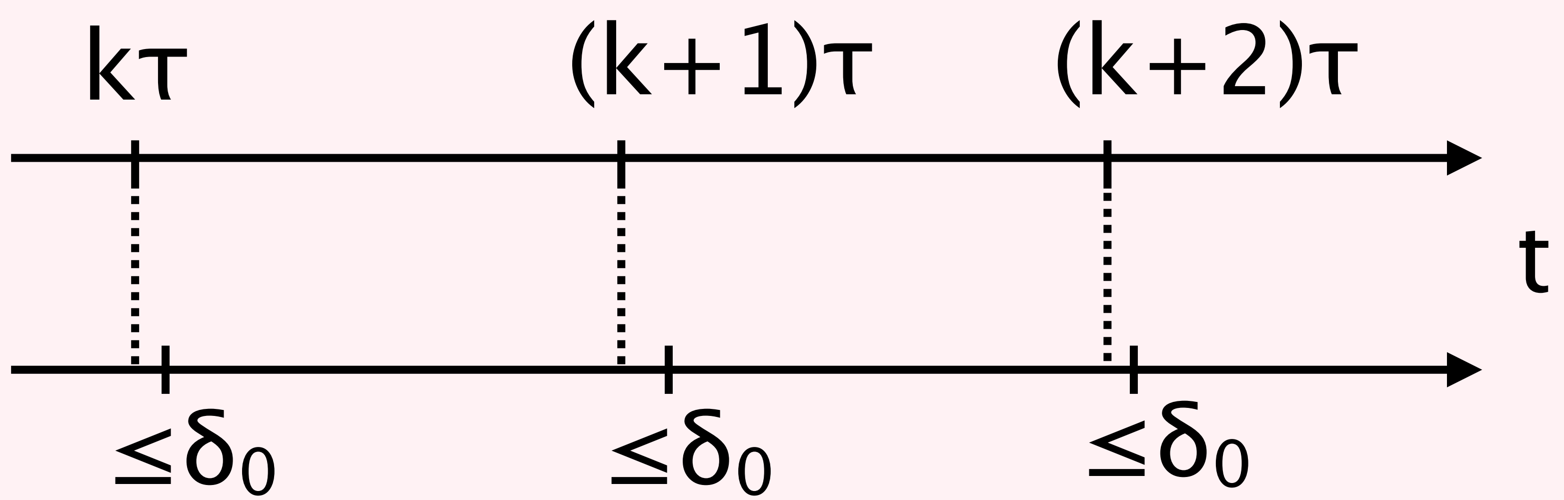
$$t = k\tau \quad (k \in \mathbb{N})$$

Nearly periodic: k-th switching occurs at

$$t \in [k\tau, k\tau + \delta_0] \quad (k \in \mathbb{N})$$

Our assumption:

Switching delays do not accumulate



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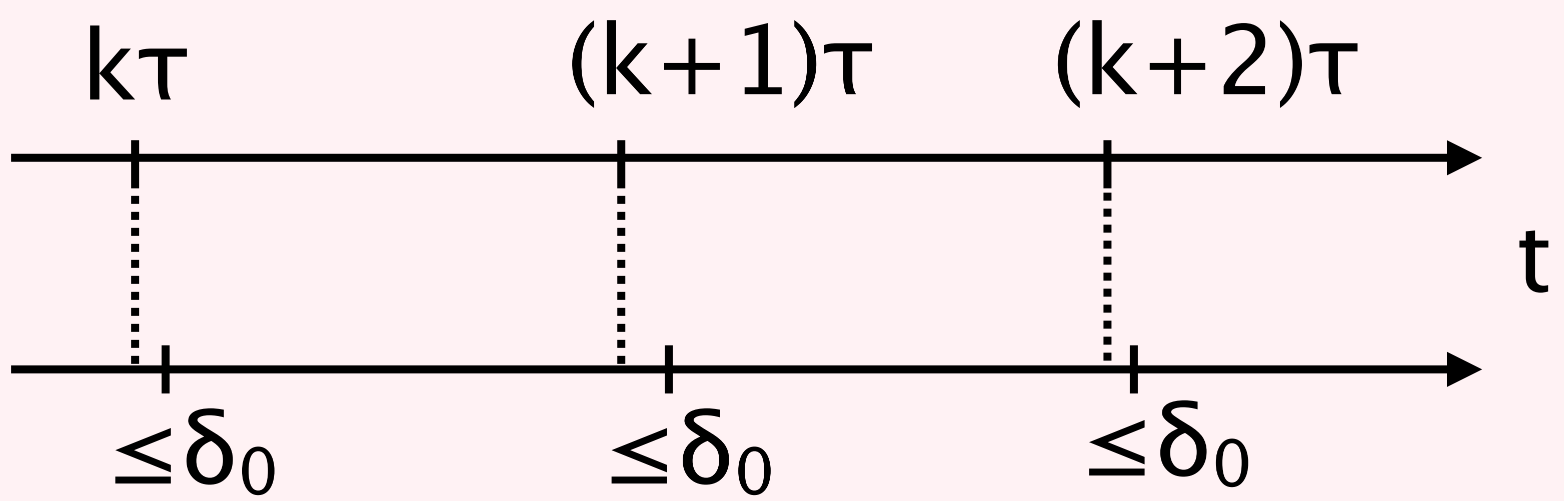
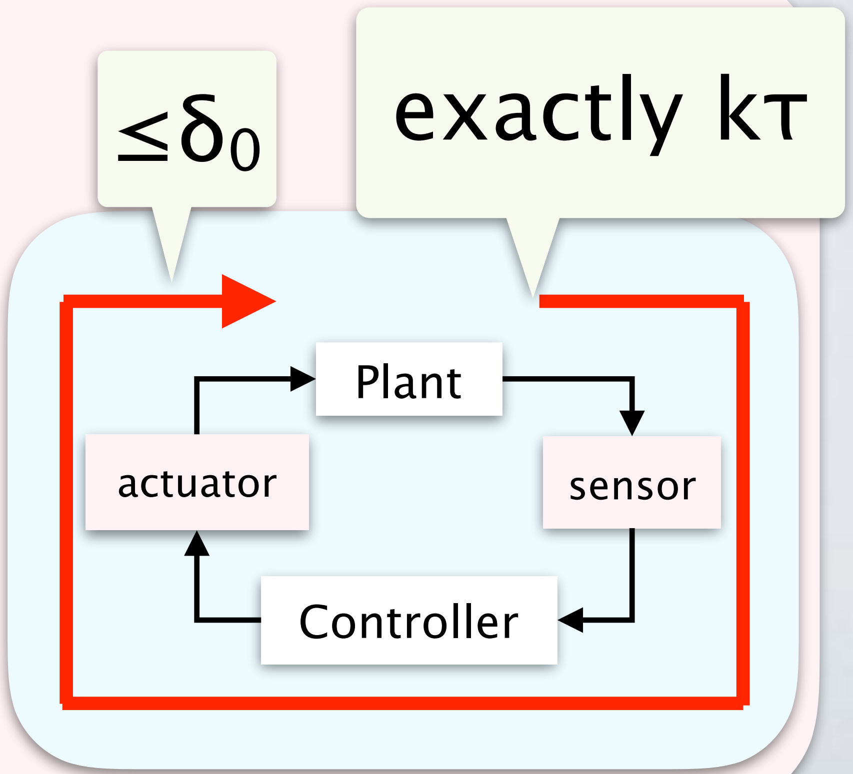
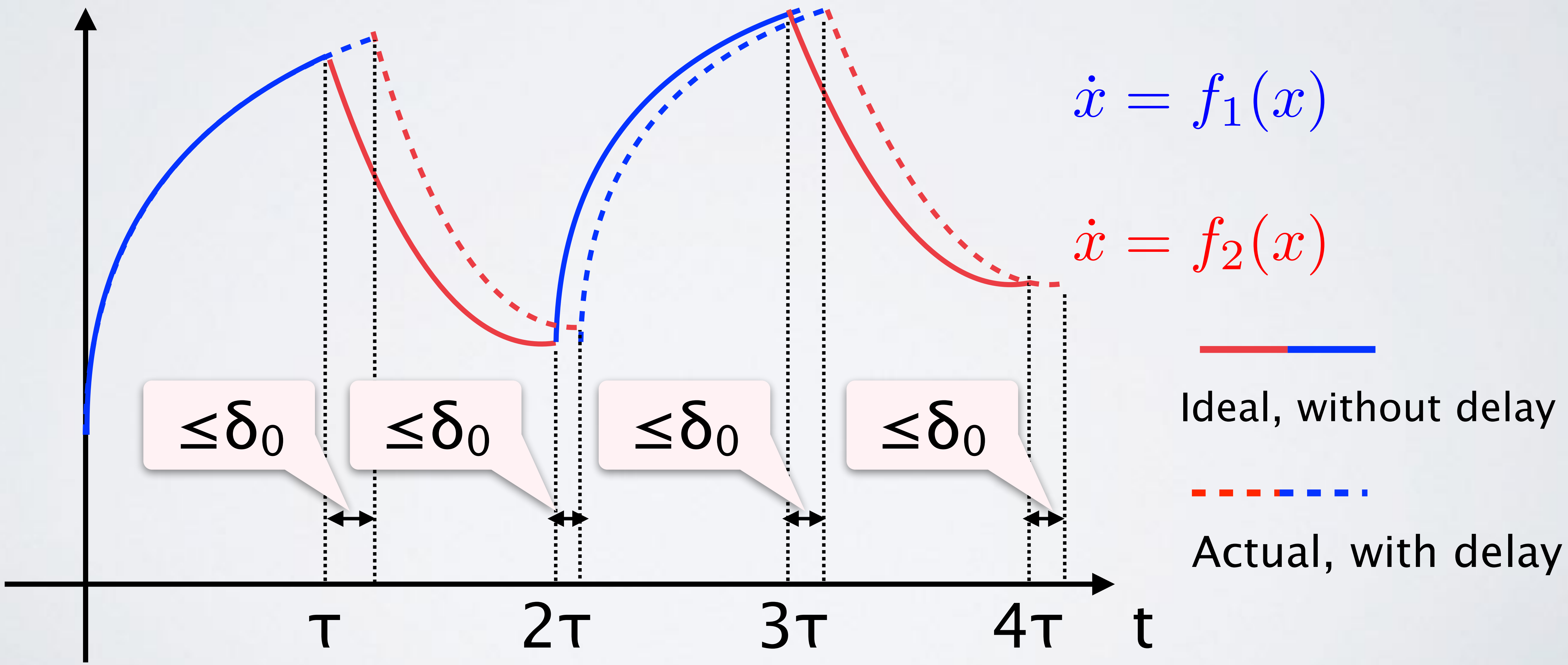
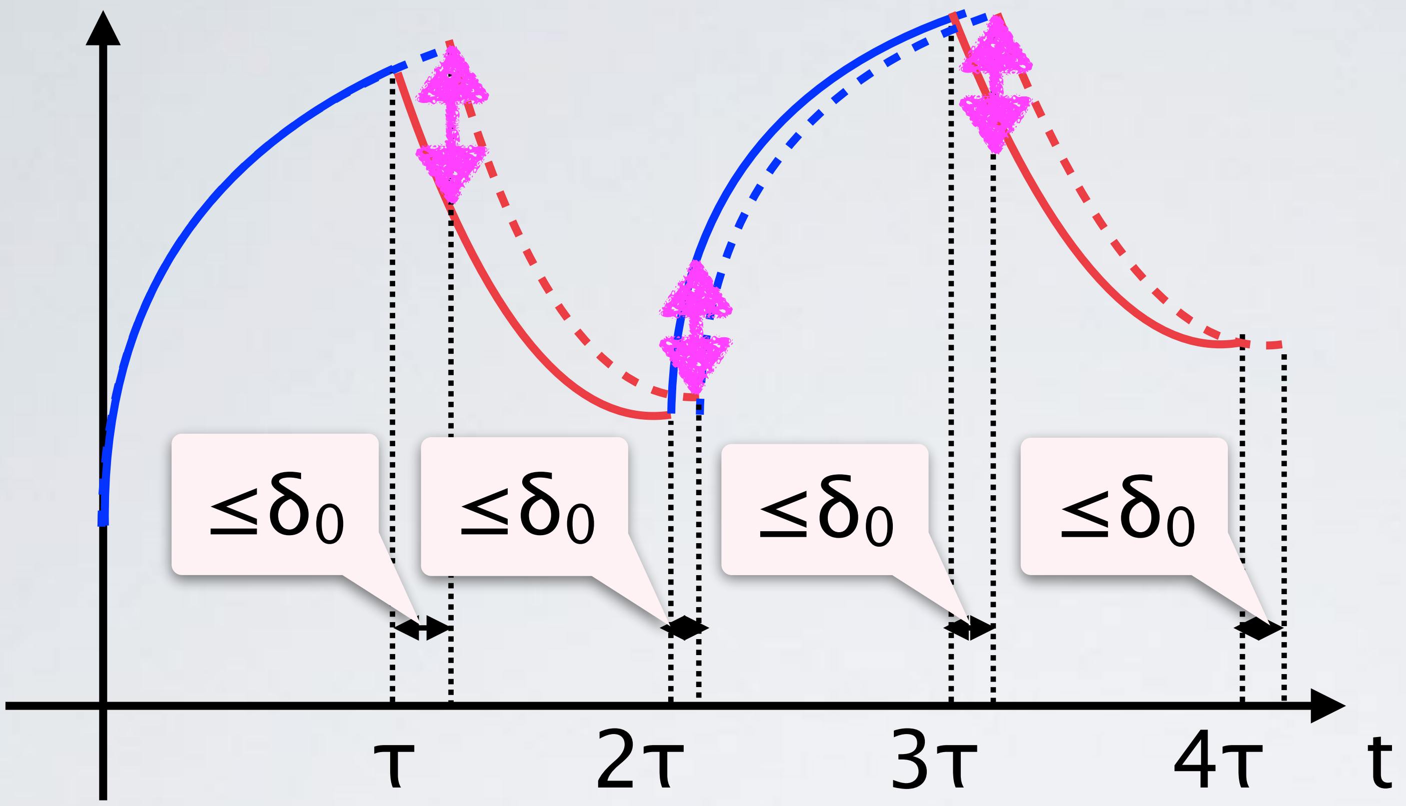




Switched System with Actuation Delays



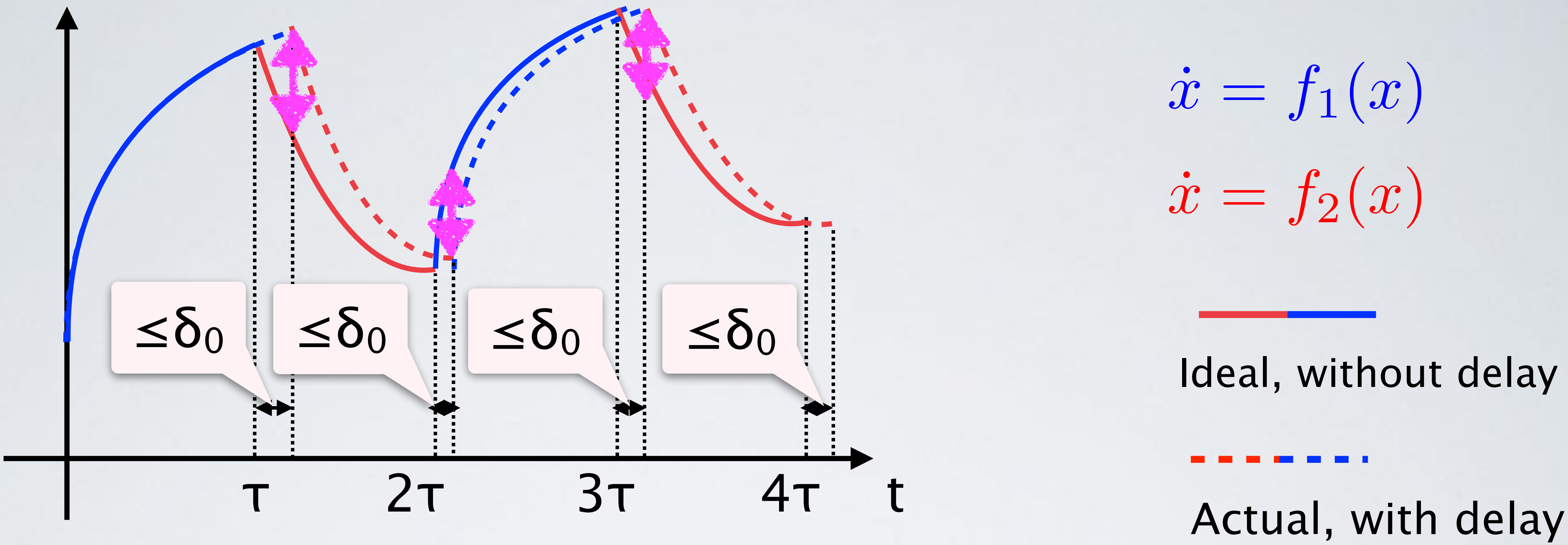


$$\dot{x} = f_1(x)$$

$$\dot{x} = f_2(x)$$

Ideal, without delay

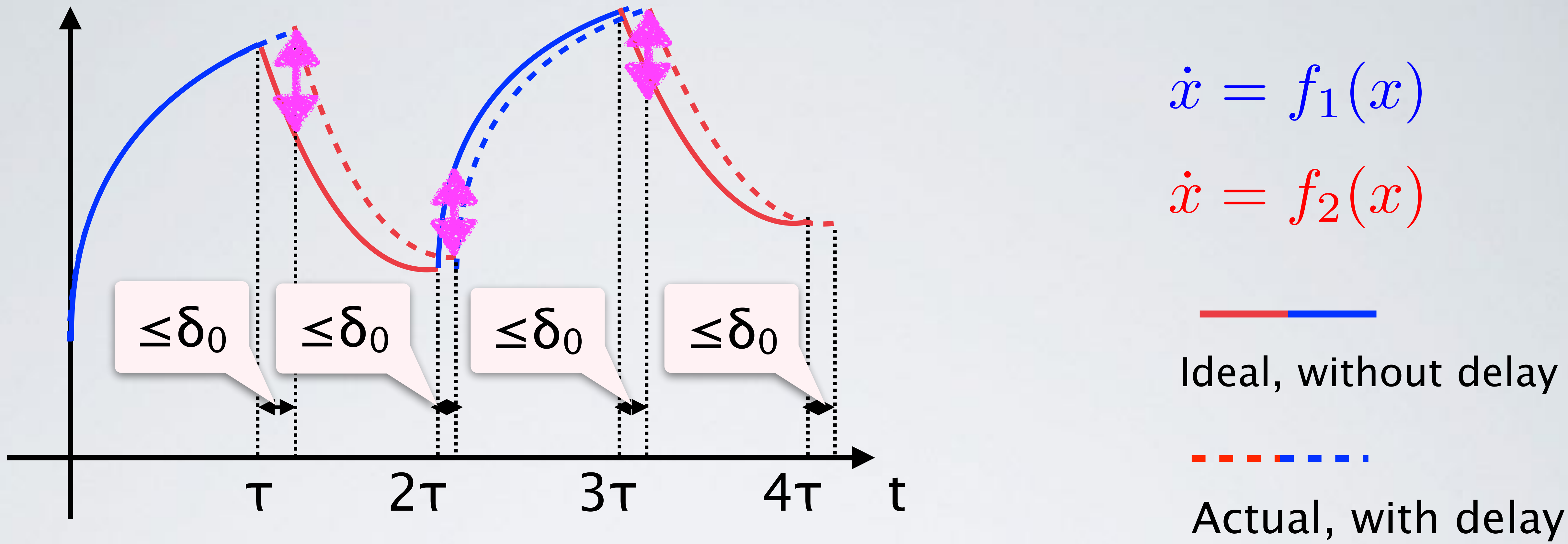
Actual, with delay



Thm. If the switched system has a common δ-GAS Lyapunov function V, then the gap (L∞ dist., ↕) is bounded by

$$\underline{\alpha}^{-1}\left(\frac{\nu\delta_0}{1-e^{-\kappa(\tau-\delta_0)}}\right)$$

Here $\underline{\alpha}$, $\varkappa$, ν are param's of the dynamics and V.



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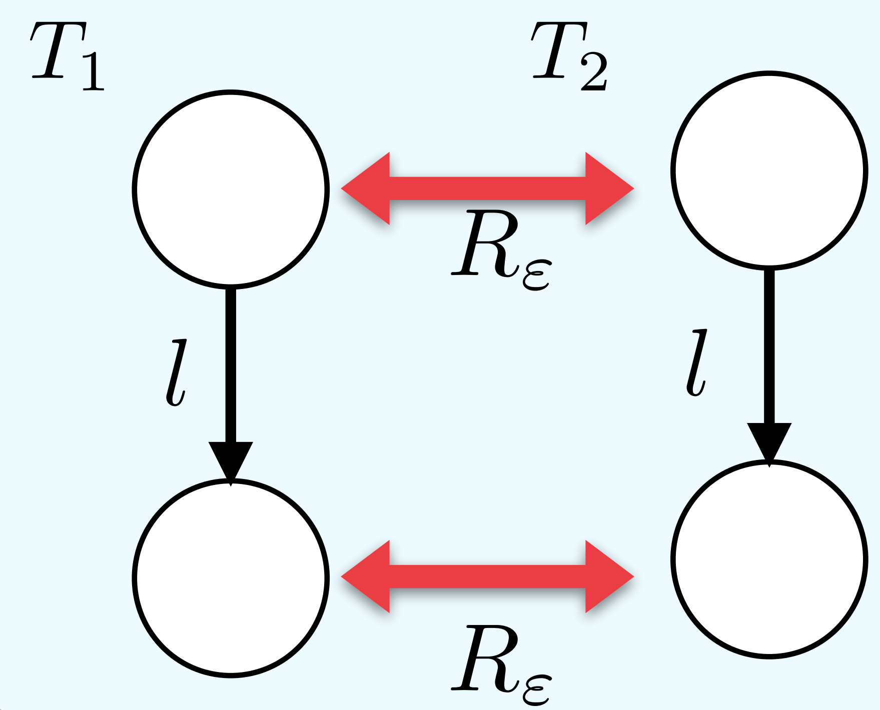

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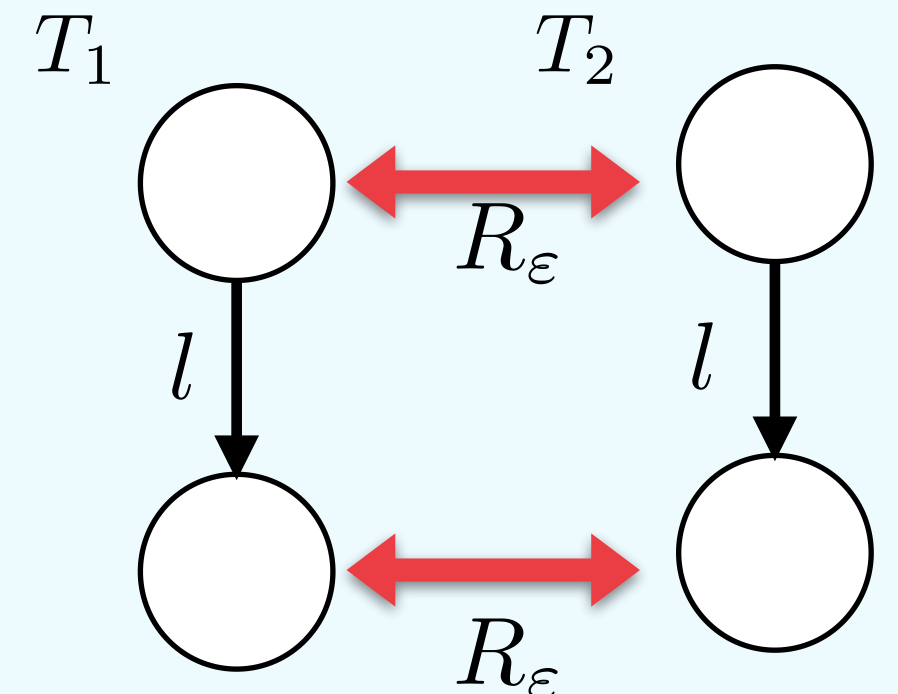

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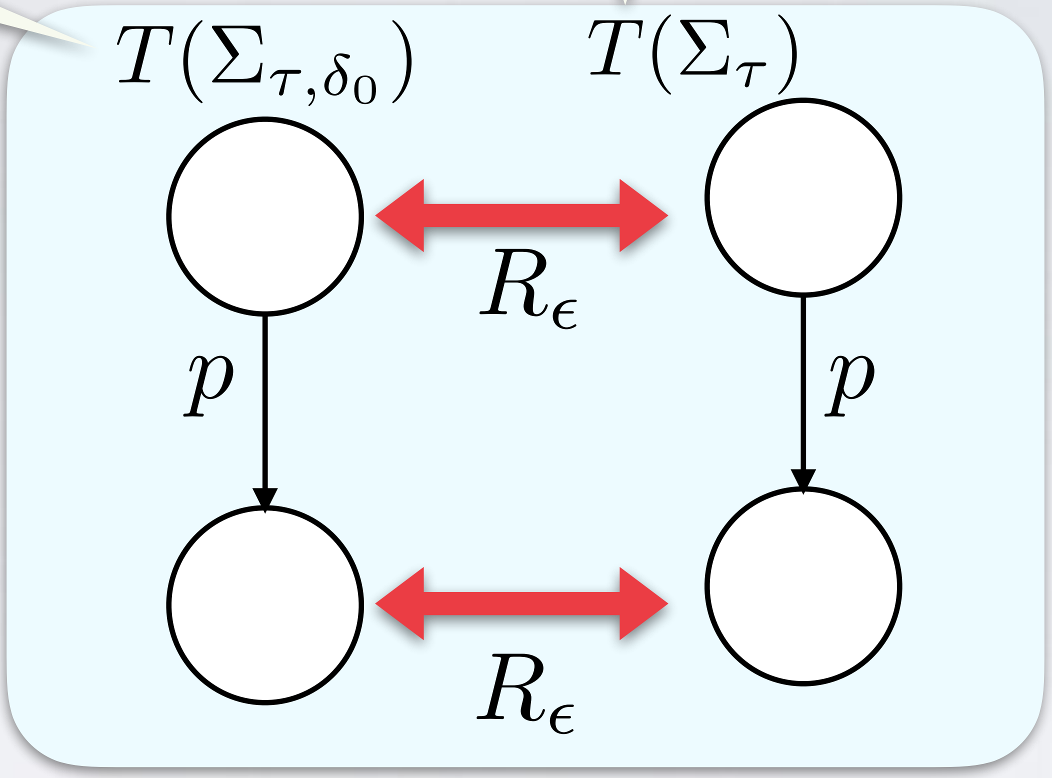
Our proof is by approximate bisimulation…

Key: what are states q1, q2?

Transition Systems for Switched Systems

delayed system

delay-free model



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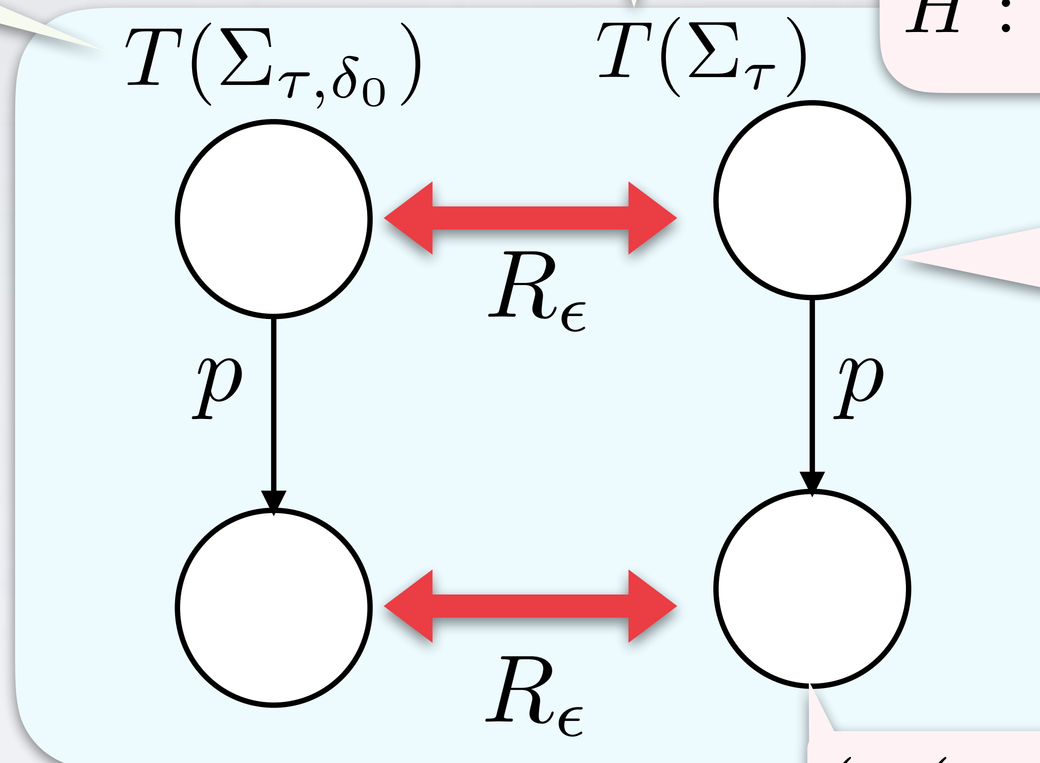
delayed system

delay-free model

$L = P$
$O = \mathbb{R}^n \times \mathbb{R}^+ \times P$
H : canonical embedding



$(x', k\tau, p)$

$(\mathbf{x}(\tau, x', p), (k+1)\tau, p')$

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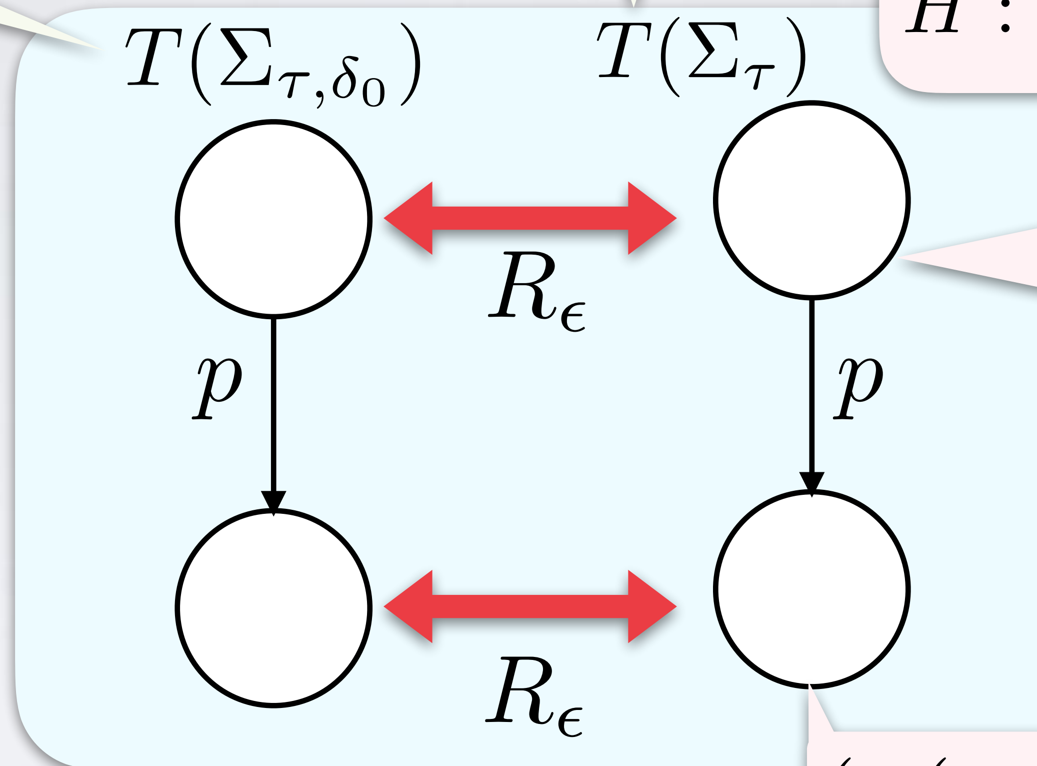
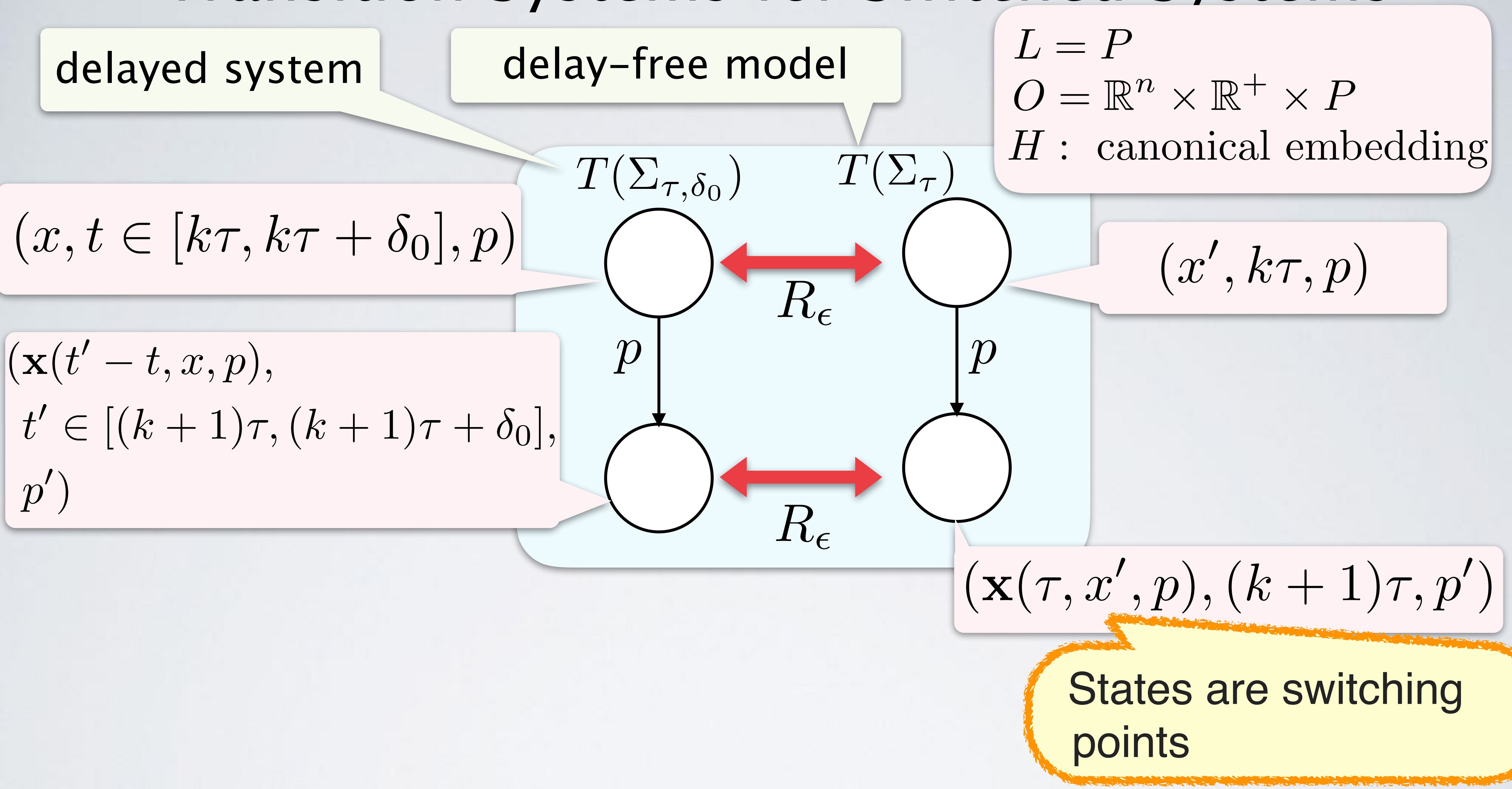


$(x', k\tau, p)$

$(\mathbf{x}(\tau, x', p), (k+1)\tau, p')$

States are switching points

Transition Systems for Switched Systems



Transition Systems for Switched Systems

delayed system

delay-free model

$L = P$
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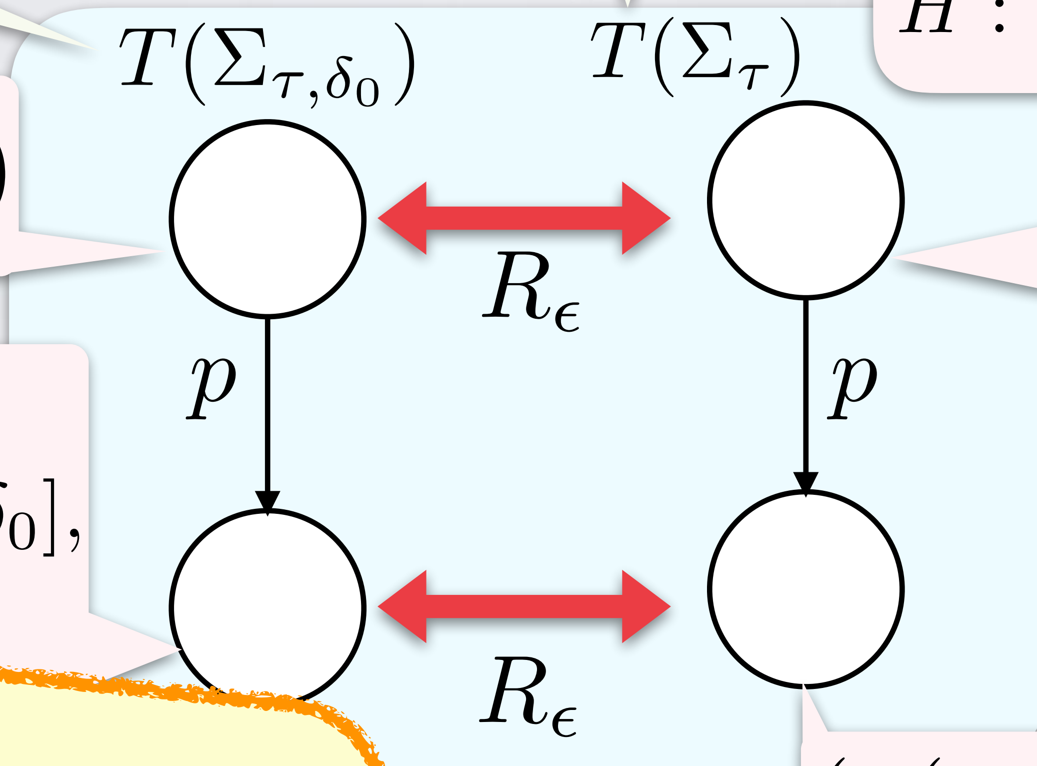

$(x, t \in [k\tau, k\tau + \delta_0], p)$

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$(\mathbf{x}(t' - t, x, p),$
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… that can delay

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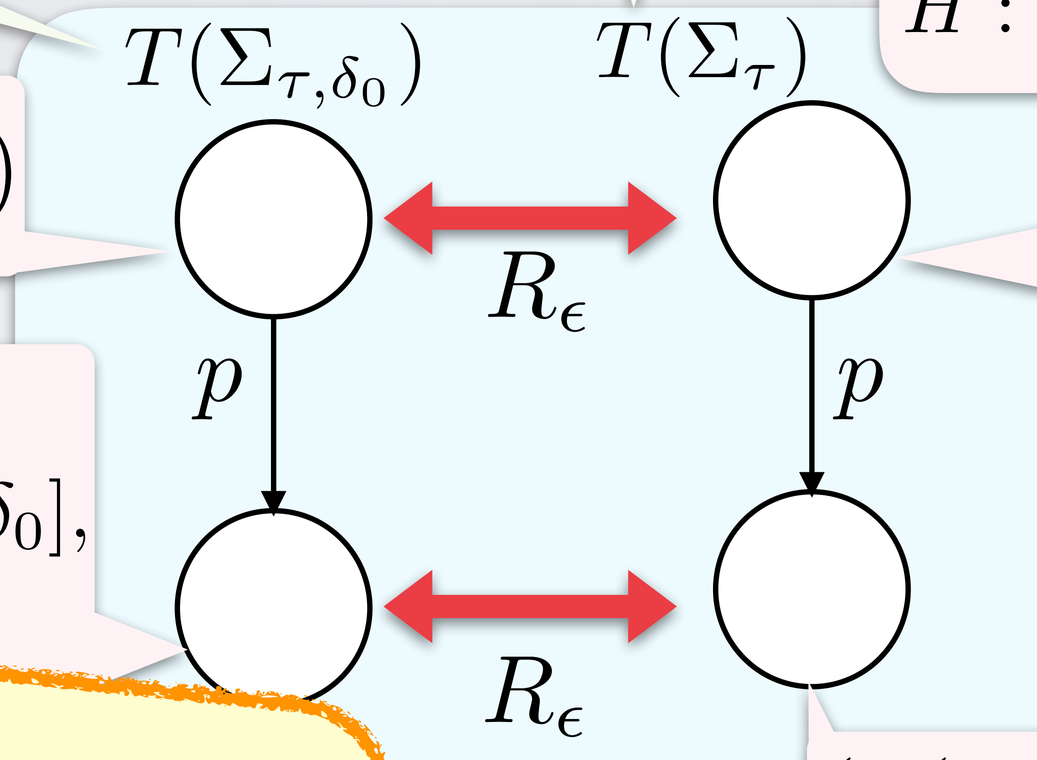

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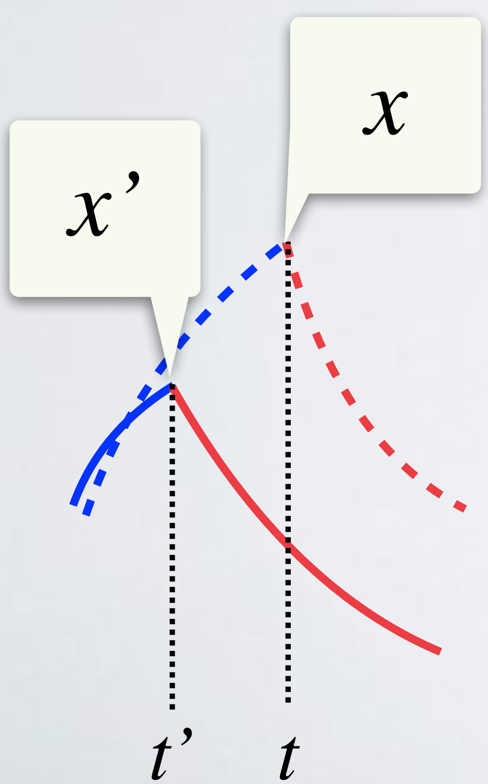

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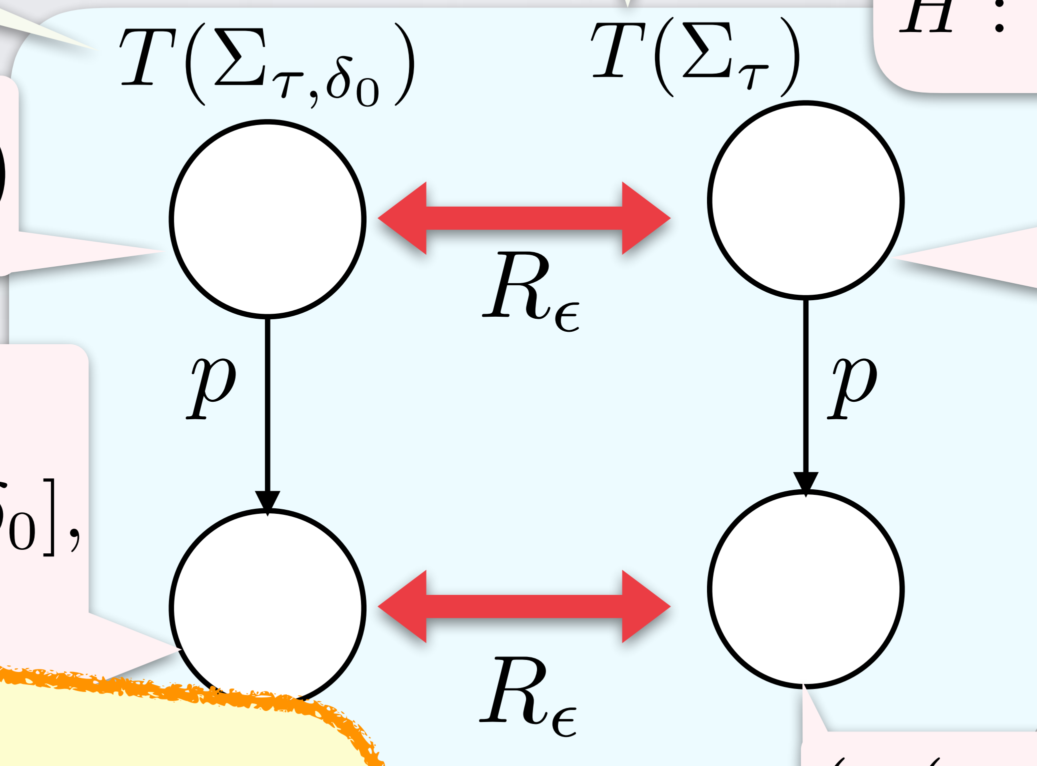

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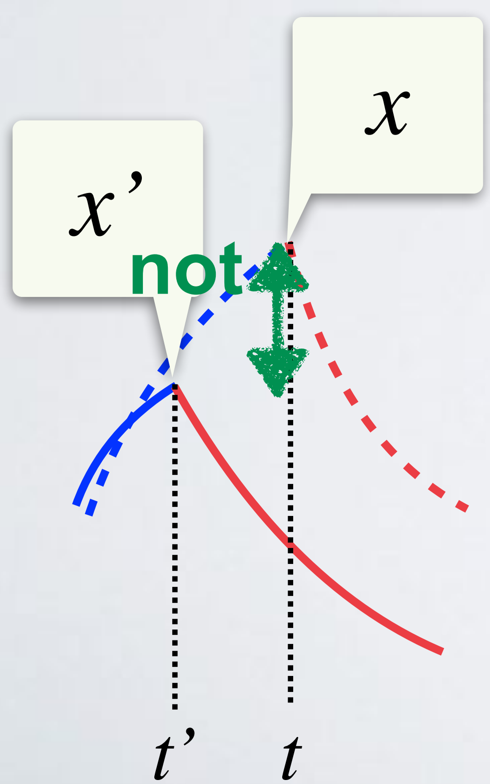

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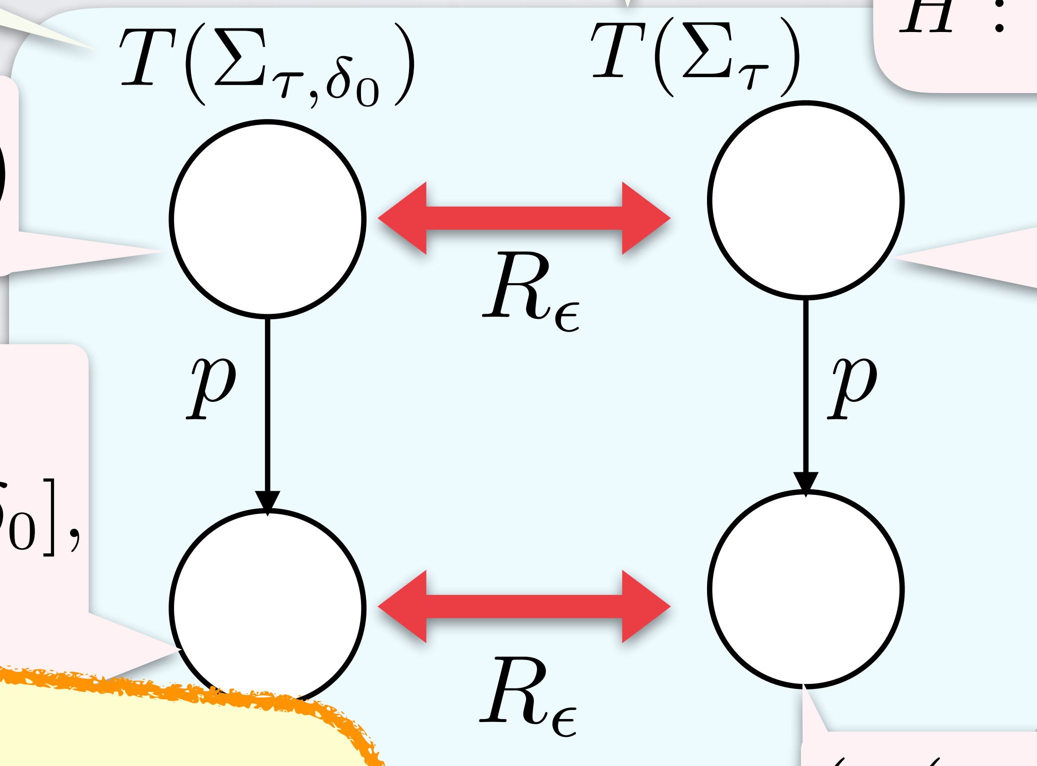

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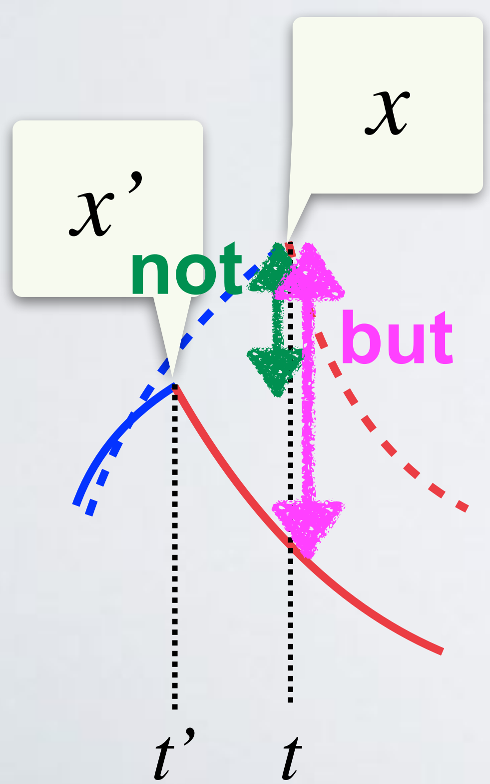

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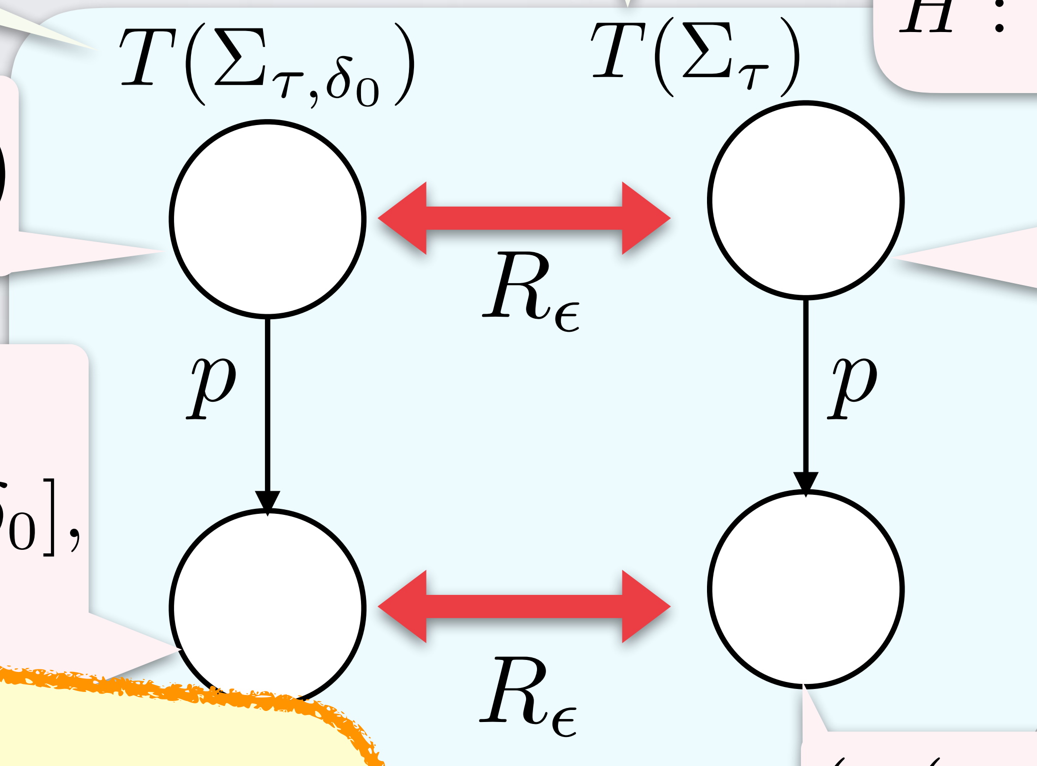

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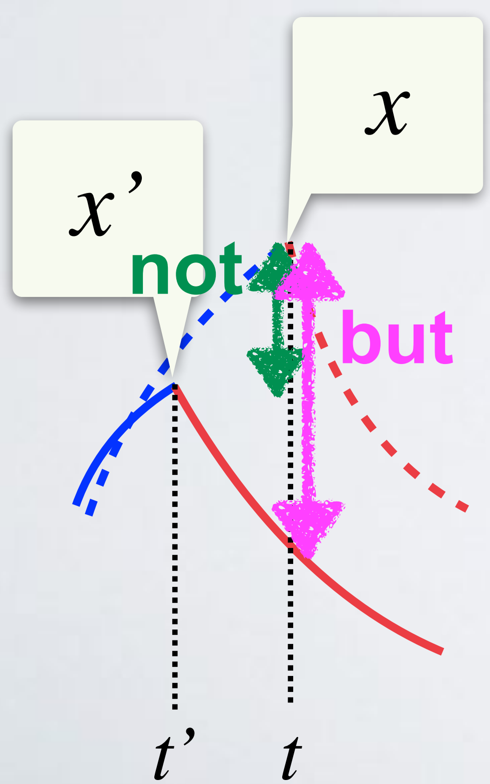
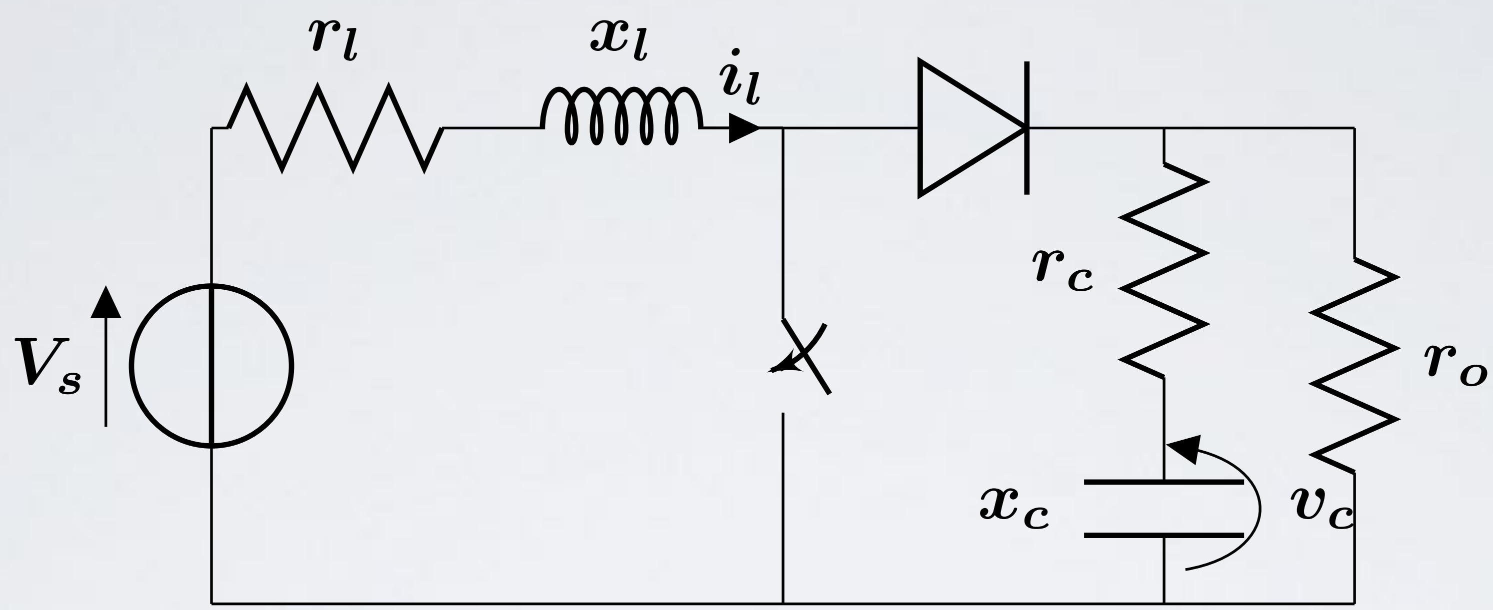


$$d((x,t,p),(x',t',p')) := \begin{cases} \|x - \mathbf{x}(t - t', x', p)\| & \text{if } p = p', t' = k\tau \text{ and } \\ & \quad t \in [t', t' + \delta_0] \text{ for some } k \in \mathbb{N} \\ \infty & \text{otherwise.} \end{cases}$$

Example I: Boost DC-DC Converter

[BeccutiPM, CDC'05] [GirardPT, IEEE Tranc. Autom. Ctrl. '10]



$$\dot{x}(t) = A_p x(t) + b \quad \text{for } p \in \{ON, OFF\} \ , \ \text{where}$$

$$A_{ON} = \begin{bmatrix} -\frac{r_l}{x_l} & 0 \\ 0 & -\frac{1}{x_c(r_o+r_c)} \end{bmatrix}, \quad b = \begin{bmatrix} \frac{v_s}{x_l} \\ 0 \end{bmatrix} \text{ and}$$

$$A_{OFF} = \begin{bmatrix} -\frac{r_l r_o + r_l r_c + r_o r_c}{x_l(r_o+r_c)} & -\frac{r_l r_o + r_l r_c + r_o r_c}{x_l(r_o+r_c)} \\ \frac{r_o}{x_c(r_o+r_c)} & -\frac{1}{x_c(r_o+r_c)} \end{bmatrix} .$$

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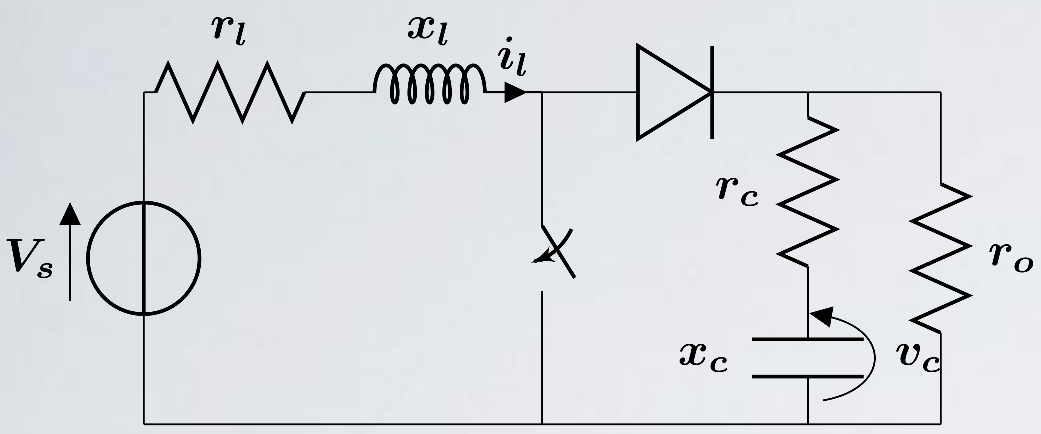

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- After rescaling, a common Lyapunov func. is found

[GirardPT'10]

$$V(x,y) = \sqrt{(x-y)^T M (x-y)} \text{ with } M = \begin{bmatrix} 1.0224 & 0.0084 \\ 0.0084 & 1.0031 \end{bmatrix}$$

with parameters $\underline{\alpha}(s) = s, \overline{\alpha}(s) = 1.0127s$ and $\kappa = 0.014$.

- Now with switching interval $\tau = 0.5$ and the maximum delay $\delta_0 = \frac{\tau}{1000}$ we use our main theorem to derive the error bound $\varepsilon_1 = 0.0294176$

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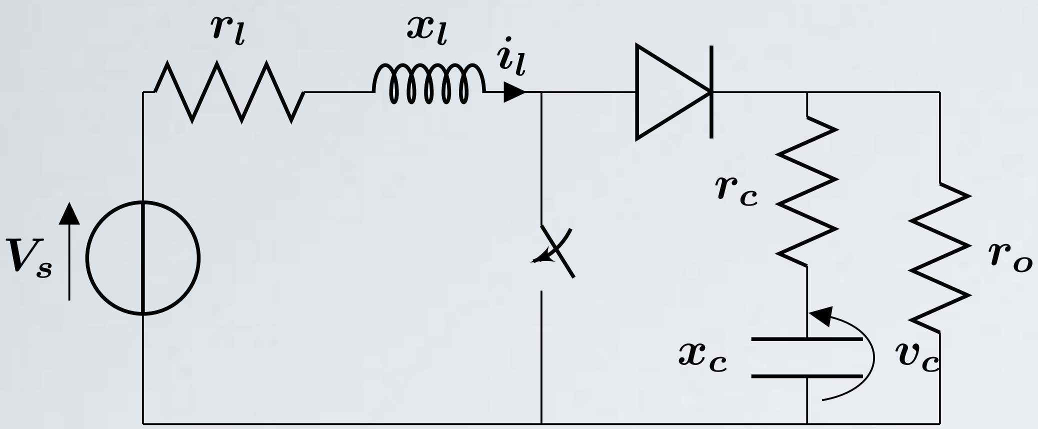

$\dot{x}(t) = A_p x(t$

$A_{ON} = \Big[-\frac{r_l}{x_l} \quad 0$

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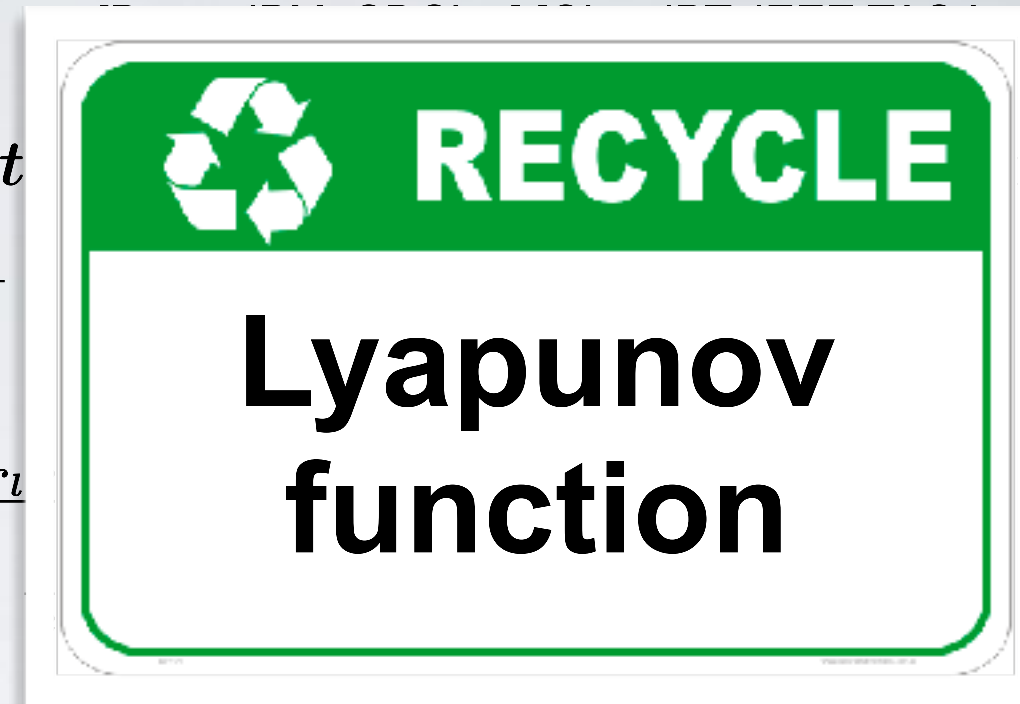

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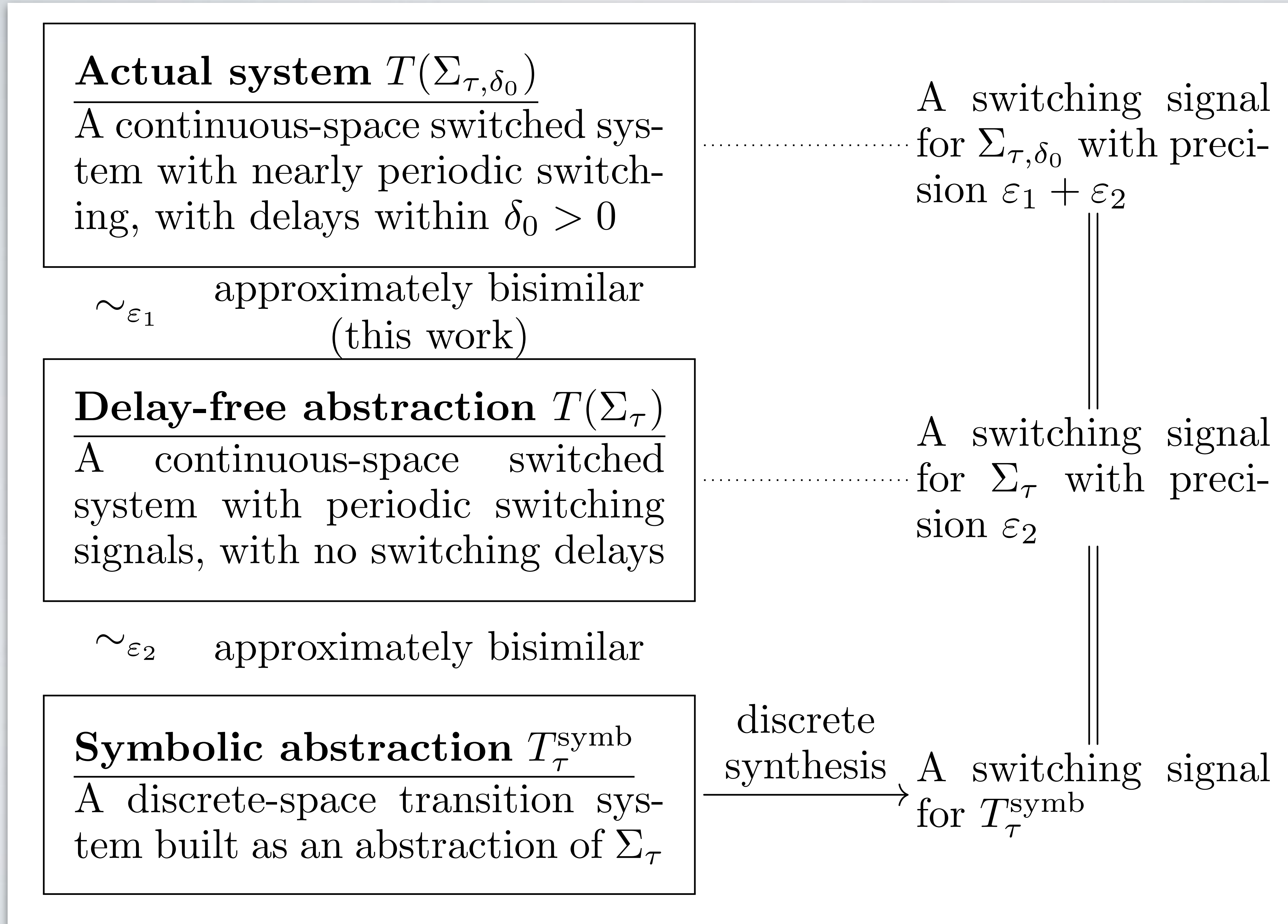
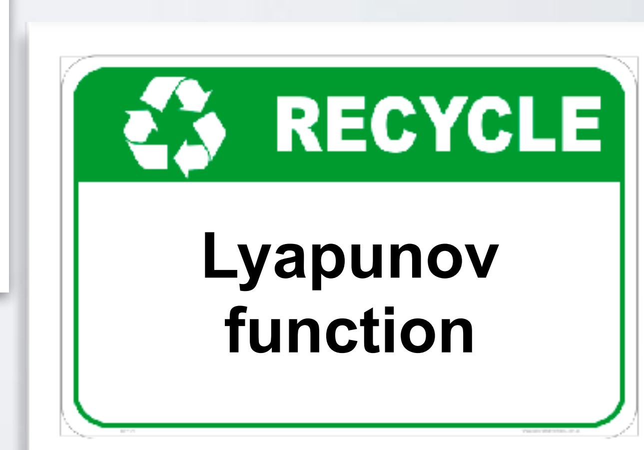
Two-Step Control Synthesis

for Periodically Switched Systems with Delays



- One Lyapunov, two approx. bisim
- In the boost DC-DC ex: $\varepsilon_1 = 0.0294176$ ε2 : trade-off with grid size
- Another example in the arxiv ver. (nonlinear water tank)



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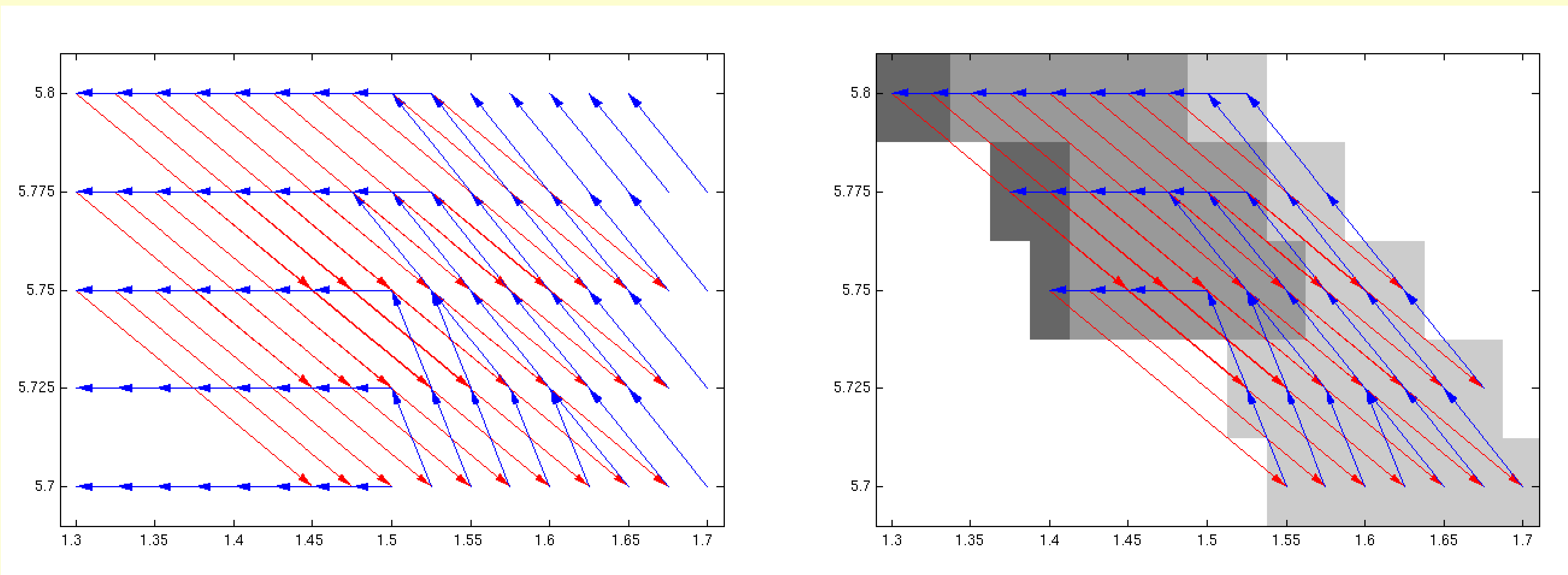

[GirardPT'10]

$\sim_{\varepsilon_2}$ approximately bisimilar

Symbolic abstraction T_τ^{symb}
A discrete-space transition system built as an abstraction of Σ_τ

discrete synthesis → A switching signal for T_τ^{symb}

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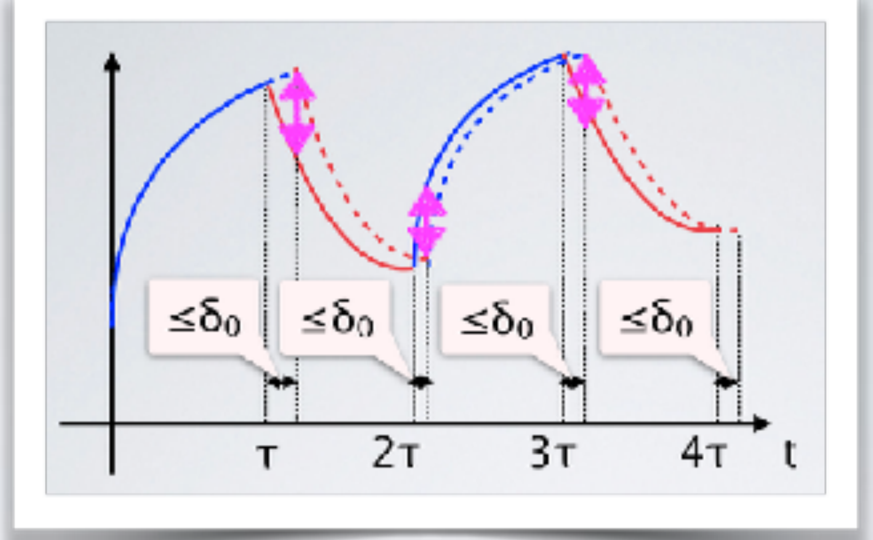
Related Work

- Approximate bisimulation for state-space discretization. Synthesis, switched systems, …
  - Overview: Girard, A. and Pappas, G.J. (2011). Approximate bisimulation: A bridge between computer science and control theory. Eur. J. Control , 17(5-6), 568-578.
- Delays as adversarial disturbance ➜ **alternating** approximate bisimulation
  - Pola, G., Pepe, P., and Benedetto, M.D.D. Alternating approximately bisimilar symbolic models for nonlinear control systems with unknown time-varying delays. CDC 2010
  - Synthesis by solving games
- “Delay-tolerating” specification
  - Liu, J. and Ozay, N. Finite abstractions with robustness margins for temporal logic-based control synthesis. HSCC 2016
- Discrete delays τ, 2τ, 3τ, … by **zero-order hold** ➜ discrete games
  - Zamani, M., Jr, M.M., Khaled, M., and Abate, A. (2017). Symbolic abstractions of networked control systems. IEEE Transactions on Control of Network Systems
- (τ,ε)-closeness, Skorokhod distance
  - Abbas, H. and Fainekos, G.E. (2015). Towards composition of conformant systems. CoRR , abs/1511.05273.

Conclusions



- The approximate bisimulation workflow
- Our use: delays in switched systems
  - Simper setting ➜ applicability
- Technical contributions. Key: states and distance
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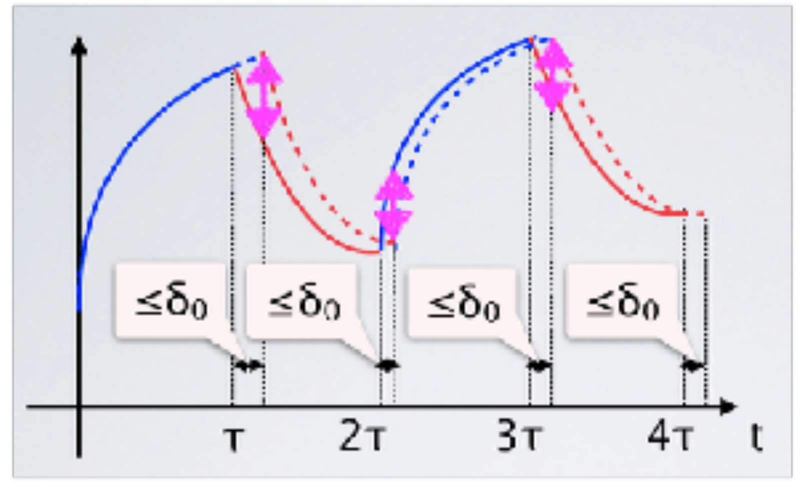

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