Bounding Errors Due to Switching Delays in Incrementally Stable Switched Systems

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Outline

- The approximate bisimulation workflow
- Our use: delays in switched systems
  - Simper setting ➔ applicability
- Technical contributions
- Two-step synthesis,
  via state-space discretization
- Case study
Approximate Bisimulation

\[ T_i = (Q_i, L, \overset{i}{\rightarrow}, O, H_i) \] for \( i = 1, 2 \)

\( O \) equipped with a metric \( d \)

**Def.**

\( R_\varepsilon \subseteq Q_1 \times Q_2 \) is an \( \varepsilon \)-approximate bisimulation relation between \( T_1 \) and \( T_2 \) if for all \( (q_1, q_2) \in R_\varepsilon \),

- \( d(H_1(q_1), H_2(q_2)) \leq \varepsilon \)
- \( \forall q_1, \overset{1}{\rightarrow}^{l} q'_1, \exists q_2, \overset{2}{\rightarrow}^{l} q'_2, \) s.t. \( (q'_1, q'_2) \in R_\varepsilon \)
- \( \forall q_2, \overset{2}{\rightarrow}^{l} q'_2, \exists q_1, \overset{1}{\rightarrow}^{l} q'_1, \) s.t. \( (q'_1, q'_2) \in R_\varepsilon \)
Approximate Bisimulation

[Def.]
$T_i = (Q_i, L, \overset{i}{\rightarrow}, O, H_i)$ for $i = 1, 2$

$(O$ equipped with a metric $d)$

$R_\varepsilon \subseteq Q_1 \times Q_2$ is an \(\varepsilon\)-approximate bisimulation relation between $T_1$ and $T_2$ if for all $(q_1, q_2) \in R_\varepsilon$,

- $d(H_1(q_1), H_2(q_2)) \leq \varepsilon$
- $\forall q_1 \xrightarrow{1} q'_{1}, \exists q_2 \xrightarrow{1} q'_{2},$ s.t. $(q'_{1}, q'_{2}) \in R_\varepsilon$
- $\forall q_2 \xrightarrow{2} q'_{2}, \exists q_1 \xrightarrow{1} q'_{1},$ s.t. $(q'_{1}, q'_{2}) \in R_\varepsilon$

Once in a relationship, always henceforth
Approximate Bisimulation

\[ T_i = (Q_i, L, \xrightarrow{i}, O, H_i) \text{ for } i = 1, 2 \]
\( (O \text{ equipped with a metric } d) \)

**Def.**
\( R_\varepsilon \subseteq Q_1 \times Q_2 \) is an \( \varepsilon \)-approximate bisimulation relation between \( T_1 \) and \( T_2 \) if for all \( (q_1, q_2) \in R_\varepsilon \),

- \( d(H_1(q_1), H_2(q_2)) \leq \varepsilon \)
- \( \forall q_1 \xrightarrow{1} q_1', \exists q_2 \xrightarrow{2} q_2', \text{ s.t. } (q_1', q_2') \in R_\varepsilon \)
- \( \forall q_2 \xrightarrow{2} q_2', \exists q_1 \xrightarrow{1} q_1', \text{ s.t. } (q_1', q_2') \in R_\varepsilon \)

Once in a relationship, always henceforth
Incremental Stability (δ-GUAS)

Incrementally Globally Uniformly Asymptotically Stable

**Def.** A dynamics is said to be $\delta$-GAS if $\exists \mathcal{KL}$ function $\beta$ s.t.

$$\| x(x, t) - x(y, t) \| \leq \beta(\| x - y \|, t)$$

where $x(x, -)$ is the trajectory starting at $x$. 
**Incremental Stability (δ-GUAS)**

**Def.** A dynamics is said to be δ-*GAS* if there exists a KL function \( \beta \) such that

\[
\| x(x, t) - x(y, t) \| \leq \beta(\|x - y\|, t)
\]

where \( x(x, -) \) is the trajectory starting at \( x \).

**Incrementally Globally Uniformly Asymptotically Stable**

![GAS Diagram](image)

![δ-GAS Diagram](image)
Incremental Stability (δ-GUAS)

Incrementally Globally Uniformly Asymptotically Stable

**Def.** A dynamics is said to be δ-\textit{GAS} if

\[ \exists \mathcal{KL} \text{ function } \beta \text{ s.t.} \]

\[ \| x(x, t) - x(y, t) \| \leq \beta(\|x - y\|, t) \]

where \( x(x, -) \) is the trajectory starting at \( x \).
**Incremental Stability (δ-GUAS)**

**Def.** A dynamics is said to be δ-GAS if there exist $K_C$ functions $\alpha, \overline{\alpha}$ and $\kappa > 0$ such that

$$\alpha(||x - y||) \leq V(x, y) \leq \overline{\alpha}(||x - y||)$$

$$\frac{\partial V}{\partial x}(x, y)f(x) + \frac{\partial V}{\partial y}(x, y)f(y) \leq -\kappa V(x, y)$$

- increasing in $||x - y||$
  - $0$ if $||x - y|| = 0$
  - $\to \infty$ if $||x - y|| \to \infty$
- decreasing in $t$
  - $\to 0$ if $t \to \infty$

**Def.** For a dynamics $\dot{x} = f(x)$ in $\mathbb{R}^n$, a smooth function $V: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^+$ is a common δ-GAS Lyapunov function if there exist $\mathcal{K}_\infty$ functions $\alpha, \overline{\alpha}$ and $\kappa > 0$ such that

$$\alpha(||x - y||) \leq V(x, y) \leq \overline{\alpha}(||x - y||)$$

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Globally Uniformly Asymptotically Stable
**Incremental Stability (δ-GUAS)**

**Incrementally Globally Uniformly Asymptotically Stable**

**Def.** A dynamics is said to be $\delta$-GAS if there exist $\mathcal{KL}$ function $\beta$ s.t.

$$||x(x,t) - x(y,t)|| \leq \beta(||x - y||, t)$$

where $x(x, -)$ is the trajectory starting at $x$.

**Def.** For a dynamics $\dot{x} = f(x)$ in $\mathbb{R}^n$, a smooth function $V: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^+$ is a common $\delta$-GAS Lyapunov function if there exist $\mathcal{K}_\infty$ functions $\alpha$, $\overline{\alpha}$ and $\kappa > 0$ such that

$$\alpha(||x - y||) \leq V(x, y) \leq \overline{\alpha}(||x - y||)$$

$$\frac{\partial V}{\partial x}(x, y)f(x) + \frac{\partial V}{\partial y}(x, y)f(y) \leq -\kappa V(x, y)$$

$V(x, y)$ is more or less $||x - y||$...

that decreases along the dynamics

$\kappa \to 0$ if $t \to \infty$
**Fundamental Stability (δ-GUAS)**

**Fundamentally Globally Uniformly Asymptotically Stable**

**Def.** A dynamics is said to be **δ-GAS** if there exist \( \mathcal{K} \mathcal{L} \) functions \( \beta \) s.t.

\[
\| x(x, t) - x(y, t) \| \leq \beta(\| x - y \|, t)
\]

where \( x(x, -) \) is the trajectory starting at \( x \).

**Def.** For a dynamics \( \dot{x} = f(x) \) in \( \mathbb{R}^n \), a smooth function \( V : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^+ \) is a common **δ-GAS Lyapunov function** if there exist \( \mathcal{K}_\infty \) functions \( \alpha, \overline{\alpha} \) and \( \kappa > 0 \) such that

\[
\alpha(\| x - y \|) \leq V(x, y) \leq \overline{\alpha}(\| x - y \|)
\]

\[
\frac{\partial V}{\partial x}(x, y)f(x) + \frac{\partial V}{\partial y}(x, y)f(y) \leq -\kappa V(x, y)
\]

**Thm.** There is a **δ-GAS Lyapunov function \( \Longrightarrow \) incrementally stable (δ-GAS).**

---

- increasing in \( \| x - y \| \)
  - 0 if \( \| x - y \| = 0 \)
  - \( \rightarrow \infty \) if \( \| x - y \| \rightarrow \infty \)
- decreasing in \( t \)
  - \( \rightarrow 0 \) if \( t \rightarrow \infty \)

\( V(x, y) \) is more or less \( \| x - y \| \... \)

that decreases along the dynamics
\textbf{Incremental Stability (δ-GUAS)}

\textbf{Globally Unconditionally Uniformly Asymptotically Stable (GUAS)}

\textbf{Def.} A dynamics is said to be \textbf{δ-GAS} if there exist a \textbf{common Lyapunov function} \( \beta \) s.t.

\[
\| x(x,t) - x(y,t) \| \leq \beta(\| x - y \|, t)
\]

where \( x(x,-) \) is the trajectory starting at \( x \).

\textbf{Def.} For a dynamics \( \dot{x} = f(x) \) in \( \mathbb{R}^n \), a smooth function \( V : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^+ \) is a \textit{common δ-GAS Lyapunov function} if there exist \( K_\infty \) functions \( \alpha, \overline{\alpha} \) and \( \kappa > 0 \) such that

\[
\alpha(\| x - y \|) \leq V(x, y) \leq \overline{\alpha}(\| x - y \|)
\]

\[
\frac{\partial V}{\partial x}(x, y)f(x) + \frac{\partial V}{\partial y}(x, y)f(y) \leq -\kappa V(x, y)
\]

\textbf{Thm.} There is a \( \delta \)-GAS Lyapunov function \( \implies \) incrementally stable (\( \delta \)-GAS).

\textbf{Switched extension: δ-GUAS, common Lyapunov func.} [Girard, Pola, Tabuada, IEEE TAC '10]

\textbf{Hasuo (NII, JP) 4}
Discrete Abstraction via Incremental Stability

[Girard, Pola, Tabuada, IEEE TAC '10]

For a switched dynamics $\dot{x} = f_p(x)$ ...

Common $\delta$-GAS Lyapunov func. $V$

 witnesses

Incremental stability ($\delta$-GUAS)
Discrete Abstraction via Incremental Stability

For a switched dynamics $\dot{x} = f_p(x)$ ...

Common
$\delta$-GAS
Lyapunov func.
$V$

witnesses

Inductive stability
($\delta$-GUAS)

Induces

Statespace abstraction

(Fig. from [GirardPT'10])

$||x - q'|| \leq \eta$

$x(\tau_s, q, p)$

$q'$

Approximate bisimulation

Thm. Let $V$ be a common $\delta$-GAS Lyapunov func. lower-bounded by $\alpha$. Then

$(x, y) \in R \iff V(x, y) \leq \alpha(\varepsilon)$

yields an $\varepsilon$-approximate bisimulation.
Discrete Abstraction via Incremental Stability

[Thm.] Let $V$ be a common $\delta$-GAS Lyapunov func. lower-bounded by $\alpha$. Then

$$(x, y) \in R \iff V(x, y) \leq \alpha(\varepsilon)$$

yields an $\varepsilon$-approximate bisimulation.

For a switched dynamics $\dot{x} = f_p(x)$ ...

Common $\delta$-GAS Lyapunov func. $V$

witnesses

induces

Approximate bisimulation

Incremental stability ($\delta$-GUAS)

Discrete verification, synthesis, supervisory control, ...

Statespace abstraction

(Fig. from [GirardPT'10])

Figure 1.
Outline

- The approximate bisimulation workflow
- Our use: delays in switched systems
  - Simper setting ➔ applicability
- Technical contributions
- Two-step synthesis, via state-space discretization
- Case study
Time Delays in Control

- Networked control, IoT, …
Time Delays in Control

- Networked control, IoT, …

- Cloud control
Switched System \( \dot{x} = f_p(x) \)

Def.

A switched system is a quadruple \((\mathbb{R}^n, P, \mathcal{P}, F)\)

- \(\mathbb{R}^n\): state space
- \(P = \{1, \cdots, m\}\): finite set of modes
- \(\mathcal{P} \subseteq \mathbb{R}^+ \rightarrow P\): the set of switching signals (piecewise constant functions that are continuous from the right and non-Zeno)
- \(F = \{f_1, \cdots, f_m\}\): a set of functions \((f_p : \mathbb{R}^n \rightarrow \mathbb{R}^n\) for each mode \(p\))

Notation

- \(p\): mode
- \(P\): switching signal
Trajectory of Switched Systems

Given a switching signal $p$,

- $x(t) \in \mathbb{R}^n$ behaves according to $\dot{x}(t) = f_p(x(t))$

  while $p(t) = p$ .

- $x(t) \in \mathbb{R}^n$ is continuous even at switching
Periodic Switched Systems

cf. [Al Khatib et al., HSCC’16]

Periodic: k-th switching occurs at

\[ t = k\tau \quad (k \in \mathbb{N}) \]

Nearly periodic: k-th switching occurs at

\[ t \in [k\tau, k\tau + \delta_0] \quad (k \in \mathbb{N}) \]

Our assumption:

Switching delays do not accumulate

\[ k\tau \quad (k+1)\tau \quad (k+2)\tau \]

\[ \leq \delta_0 \quad \leq \delta_0 \quad \leq \delta_0 \]
Periodic Switched Systems

Periodic: $k$–th switching occurs at

$$t = k\tau \quad (k \in \mathbb{N})$$

Nearly periodic: $k$–th switching occurs at

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Our assumption:

Switching delays do not accumulate

$$k\tau \quad \leq \delta_0 \quad (k+1)\tau \quad \leq \delta_0 \quad (k+2)\tau \quad \leq \delta_0$$

Controller

Plant

Sensor

Actuator

Hasuo (NII, JP)
Switched System with Actuation Delays

\[ \dot{x} = f_1(x) \]

\[ \dot{x} = f_2(x) \]

Ideal, without delay

Actual, with delay

\( \tau \)

\( 2\tau \)

\( 3\tau \)

\( 4\tau \)
\[ \dot{x} = f_1(x) \]
\[ \dot{x} = f_2(x) \]

Ideal, without delay

Actual, with delay
Thm. If the switched system has a common $\delta$-GAS Lyapunov function $V$, then the gap ($L^\infty$ dist., $\downarrow$) is bounded by

$$\alpha^{-1} \left( \frac{\nu \delta_0}{1 - e^{-\kappa(\tau - \delta_0)}} \right)$$

Here $\alpha$, $\kappa$, $\nu$ are param’s of the dynamics and $V$. 

$$\dot{x} = f_1(x)$$
$$\dot{x} = f_2(x)$$
Thm. If the switched system has a common $\delta$-GAS Lyapunov function $V$, then the gap ($L_\infty$ dist., $\downarrow\uparrow$) is bounded by

$$\alpha^{-1} \left( \frac{\nu \delta_0}{1 - e^{-\kappa(\tau-\delta_0)}} \right)$$

Here $\alpha$, $\kappa$, and $\nu$ are parameters of the dynamics and $V$. No accumulation of $\downarrow\uparrow$
Approximate Bisimulation

[Def.]

\( R_\varepsilon \subseteq Q_1 \times Q_2 \) is an \( \varepsilon \)-approximate bisimulation relation between \( T_1 \) and \( T_2 \) if for all \( (q_1, q_2) \in R_\varepsilon \),

- \( d(H_1(q_1), H_2(q_2)) \leq \varepsilon \)
- \( \forall q_1 \xrightarrow{l_1} q_1', \exists q_2 \xrightarrow{l_2} q_2', \text{s.t. } (q_1', q_2') \in R_\varepsilon \)
- \( \forall q_2 \xrightarrow{l_2} q_2', \exists q_1 \xrightarrow{l_1} q_1', \text{s.t. } (q_1', q_2') \in R_\varepsilon \)
Approximate Bisimulation

[Def.]

\( R_{\varepsilon} \subseteq Q_1 \times Q_2 \) is an \( \varepsilon \)-approximate bisimulation relation between \( T_1 \) and \( T_2 \) if for all \( (q_1, q_2) \in R_{\varepsilon} \),

- \( d(H_1(q_1), H_2(q_2)) \leq \varepsilon \)
- \( \forall q_1 \xrightarrow{l}{1} q'_1, \exists q_2 \xrightarrow{l}{2} q'_2, \text{ s.t. } (q'_1, q'_2) \in R_{\varepsilon} \)
- \( \forall q_2 \xrightarrow{l}{2} q'_2, \exists q_1 \xrightarrow{l}{1} q'_1, \text{ s.t. } (q'_1, q'_2) \in R_{\varepsilon} \)

Our proof is by approximate bisimulation…

**Key:** what are states \( q_1, q_2 \)?

\( T_i = (Q_i, L, \xrightarrow{i}{}, O, H_i) \) for \( i = 1, 2 \)

\( (O \text{ equipped with a metric } d) \)
Transition Systems for Switched Systems

delayed system

delay-free model

\[ T(\Sigma_{\tau,\delta_0}) \quad T(\Sigma_{\tau}) \]

\[ \xrightarrow{R_\varepsilon} \]

\[ p \]

\[ \xleftarrow{R_\varepsilon} \]

\[ p \]
Transition Systems for Switched Systems

Delayed system

Delay-free model

\[ T(\Sigma_{\tau,\delta_0}) \leftrightarrow T(\Sigma_{\tau}) \]

\[ \begin{align*} L &= P \\
O &= \mathbb{R}^n \times \mathbb{R}^+ \times P \\
H &\colon \text{canonical embedding} \end{align*} \]

\[ (x', k\tau, p) \]

\[ (x(\tau, x', p), (k + 1)\tau, p') \]
Transition Systems for Switched Systems

 delayed system

delay–free model

\[ T(\Sigma_{\tau,\delta_0}) \quad T(\Sigma_{\tau}) \]

\[ \begin{array}{c}
\text{States are switching points}
\end{array} \]

\[ L = P \]

\[ O = \mathbb{R}^n \times \mathbb{R}^+ \times P \]

\[ H : \text{canonical embedding} \]

\[ (x', k\tau, p) \]

\[ (x(\tau, x', p), (k + 1)\tau, p') \]
Transition Systems for Switched Systems

**delayed system**

\[(x, t \in [k\tau, k\tau + \delta_0], p)\]

**delay-free model**

\[(x, (t - t, x, p), t' \in [(k + 1)\tau, (k + 1)\tau + \delta_0], (p'))\]

\[T(\Sigma_{\tau}, \delta_0) \quad T(\Sigma_{\tau})\]

\[R_\epsilon \quad R_\epsilon\]

\[L = P\]

\[O = \mathbb{R}^n \times \mathbb{R}^+ \times P\]

\[H : \text{canonical embedding}\]

\[(x', k\tau, p)\]

\[(x(t - t), x', p)\]

\[(x(\tau, x'), p), (k + 1)\tau, p')\]

States are switching points
Transition Systems for Switched Systems

(delayed system)

\((x, t \in [k\tau, k\tau + \delta_0], p)\)

(delay-free model)

\(T(\Sigma_{\tau, \delta_0}) \xrightarrow{R_\epsilon} T(\Sigma_{\tau})\)

\(L = P\)

\(O = \mathbb{R}^n \times \mathbb{R}^+ \times P\)

\(H :\) canonical embedding

\((x', k\tau, p)\)

\((x(t' - t, x, p), t' \in [(k + 1)\tau, (k + 1)\tau + \delta_0], p')\)

... that can delay

\((x(\tau, x', p), (k + 1)\tau, p')\)

States are switching points
Transition Systems for Switched Systems

**delayed system**

\[(x, t \in [k\tau, k\tau + \delta_0], p)\]

**delay-free model**

\[T(\Sigma_\tau, \delta_0) \xrightarrow{R_\epsilon} T(\Sigma_\tau)\]

\[L = P\]

\[O = \mathbb{R}^n \times \mathbb{R}^+ \times P\]

\[H : \text{canonical embedding}\]

\[(x', k\tau, p)\]

\[(x(t' - t, x, p), t' \in [(k + 1)\tau, (k + 1)\tau + \delta_0], p')\]

... that can delay

\[x, x'\]

States are switching points

\[x(t, x', p), (k + 1)\tau, p')\]
Transition Systems for Switched Systems

Delayed system

\[(x, t \in [k\tau, k\tau + \delta_0], p)\]

Delay-free model

\[T(\Sigma_{\tau,\delta_0}) \xrightarrow{\epsilon} T(\Sigma_{\tau})\]

\(L = P\)

\(O = \mathbb{R}^n \times \mathbb{R}^+ \times P\)

\(H : \text{ canonical embedding}\)

\[\begin{align*}
(x(t' - t, x, p), \\
t' \in [(k + 1)\tau, (k + 1)\tau + \delta_0], \\
p')
\end{align*}\]

\[\begin{align*}
(x(\tau, x', p), (k + 1)\tau, p') \quad \text{States are switching points}
\end{align*}\]

\[p \quad \text{delayed system} \quad p\]

\[x', x \quad \text{not} \]

\[t', t\]

\[\cdots \text{that can delay}\]
Transition Systems for Switched Systems

delayed system

$\left( x, t \in [k\tau, k\tau + \delta_0], p \right)$

delay–free model

$T(\Sigma_{\tau, \delta_0}) \xrightarrow{R_{\epsilon}} T(\Sigma_{\tau})$

$L = P$

$O = \mathbb{R}^n \times \mathbb{R}^+ \times P$

$H : \text{canonical embedding}$

$(x', k\tau, p)$

$(x(\tau, x', p), (k + 1)\tau, p')$

States are switching points

…that can delay

$x$
Transition Systems for Switched Systems

delayed system

\[(x, t \in [k\tau, k\tau + \delta_0], p)\]

delay-free model

\[(x', k\tau, p)\]

\[L = P\]
\[O = \mathbb{R}^n \times \mathbb{R}^+ \times P\]
\[H : \text{canonical embedding}\]

\[T(\Sigma_{\tau}, \delta_0) \rightarrow T(\Sigma_{\tau})\]

\[R_\epsilon\]

\[d((x, t, p), (x', t', p')) :=\]
\[
\begin{cases}
\|x - x(t - t', x', p)\| & \text{if } p = p', t' = k\tau \text{ and } t \in [t', t' + \delta_0] \text{ for some } k \in \mathbb{N} \\
\infty & \text{otherwise.}
\end{cases}
\]

States are switching points

\[\text{not}\]

\[\text{but}\]
Example I: Boost DC-DC Converter

\[ \dot{x}(t) = A_p x(t) + b \quad \text{for } p \in \{ \text{ON, OFF} \} , \] where

\[ A_{ON} = \begin{bmatrix} -\frac{r_l}{x_l} & 0 \\ 0 & -\frac{1}{x_c(r_o+r_c)} \end{bmatrix} , \quad b = \begin{bmatrix} \frac{v_s}{x_l} \\ 0 \end{bmatrix} \quad \text{and} \]

\[ A_{OFF} = \begin{bmatrix} -\frac{r_l r_o + r_l r_c + r_o r_c}{x_l(r_o+r_c)} & -\frac{r_l r_o + r_l r_c + r_o r_c}{x_l(r_o+r_c)} \\ \frac{r_o}{x_c(r_o+r_c)} & -\frac{1}{x_c(r_o+r_c)} \end{bmatrix} . \]
Example I: Boost DC-DC Converter

We use our main theorem to derive the error bound

\[ V(x, y) = \sqrt{(x - y)^T M (x - y)} \]

with parameters \( \alpha(s) = s, \overline{\alpha}(s) = 1.0127s \) and \( \kappa = 0.014 \).

Now with switching interval \( \tau = 0.5 \) and the maximum delay \( \delta_0 = \frac{\tau}{1000} \)
we use our main theorem to derive the error bound \( \varepsilon_1 = 0.0294176 \).
Example I: Boost DC-DC Converter

After rescaling, a common Lyapunov func. is found

\[ V(x, y) = \sqrt{(x - y)^T M (x - y)} \]

with parameters \( \alpha(s) = s, \bar{\alpha}(s) = 1.0127s \) and \( \kappa = 0.014 \).

Now with switching interval \( \tau = 0.5 \) and the maximum delay \( \delta_0 = \frac{\tau}{1000} \)
we use our main theorem to derive the error bound \( \varepsilon_1 = 0.0294176 \).
Two-Step Control Synthesis for Periodically Switched Systems with Delays

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<td>discrete synthesis $\Rightarrow$ A switching signal for $T^\text{symb}_{\tau}$</td>
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<td>A switching signal for $\Sigma_{\tau},\delta_0$ with precision $\varepsilon_1 + \varepsilon_2$</td>
<td>A switching signal for $\Sigma_{\tau}$ with precision $\varepsilon_2$</td>
<td></td>
</tr>
</tbody>
</table>

- One Lyapunov, two approx. bisim
- In the boost DC-DC ex: $\varepsilon_1 = 0.0294176$ $\varepsilon_2$ : trade-off with grid size
- Another example in the arxiv ver. (nonlinear water tank)
Two-Step Control Synthesis for Periodically Switched Systems with Delays

Symbolic abstraction $T^{\text{symb}}_\tau$
A discrete-space transition system built as an abstraction of $\Sigma_\tau$

\[ \sim_{\varepsilon_2} \text{ approximately bisimilar} \]

Discrete synthesis for $T^{\text{symb}}_\tau$
A switching signal

One Lyapunov,
two approx. bisim

In the boost DC-DC ex:
$\varepsilon_1 = 0.0294176$
$\varepsilon_2 : \text{ trade-off}$
with grid size

Another example
in the arxiv ver.
(nonlinear water tank)

Lyapunov function
Related Work

- Approximate bisimulation for state-space discretization. Synthesis, switched systems, …

- Delays as adversarial disturbance → alternating approximate bisimulation
  - Pola, G., Pepe, P., and Benedetto, M.D.D. Alternating approximately bisimilar symbolic models for nonlinear control systems with unknown time-varying delays. CDC 2010

- Synthesis by solving games

- “Delay-tolerating” specification
  - Liu, J. and Ozay, N. Finite abstractions with robustness margins for temporal logic-based control synthesis. HSCC 2016

- Discrete delays $\tau$, $2\tau$, $3\tau$, … by zero-order hold → discrete games

- $(\tau, \epsilon)$-closeness, Skorokhod distance
Conclusions

- The approximate bisimulation workflow
- Our use: delays in switched systems
- Simper setting ➔ applicability
- Technical contributions. Key: states and distance
- Two-step synthesis, via state-space discretization
Conclusions

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logic + automata + categories + machine learning + software engineering
➜ CPS, automated driving

Thank you for your attention!
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