

Categorical Algebra and Coalgebra in Computer Science

概要

- 7/3 (水), 7/4 (木), 7/5 (金), 各日 14:45-18:00
- [名古屋大学 大学院多元数理科学研究科](#)

講義の目的・内容

Category theory is an abstract mathematical language that is used in many different branches of mathematics. It has also found its successful applications in computer science—in fact, in many different ways. The classic example is in the semantics of functional programming, where types are objects and programs are arrows. In this course we focus on another eminent use of categories in computer science, namely categorical *algebra and coalgebra*.

The bottom-line here is: a coalgebra is a categorical abstraction of *dynamics*, i.e. a state-based system like an automaton; and an algebra (especially an initial one) is an abstraction of *syntax*, i.e. the set of well-formed programs. Plotkin's *structural operational semantics*—connecting syntax and dynamics—also allows an elegant categorical modeling via a distributive law.

After exhibiting these basics of the (co)algebraic modeling in computer science, we proceed to a more advanced categorical structure of *presheaf categories*. We introduce the necessary categorical machineries—(co)end, Kan extension, the Yoneda lemma, etc.—as well as demonstrate their applications in name-passing calculi like pi-calculus.

No preliminary knowledge in category theory is assumed. The course materials will be announced at the course website.

Course Structure

1. Categorical Algebra and Coalgebra
 - Introduction to category theory I: the category of sets and functions
 - System as coalgebra
 - Final coalgebra
 - Syntax via algebra
 - Initial algebra
2. Categorical Structural Operational Semantics
 - Distributive Law
 - (Monad, comonad)
3. (Co)algebra in a presheaf category
 - Introduction to category theory II: working with presheaves
 - Name-passing calculi

教科書および参考書

教科書を講義の初回到配布する。

その他参考書は、

- Bart Jacobs, Introduction to Coalgebra: Towards Mathematics of States and Observations (Version 2.0), 2012. Available on the author's website.
- Steve Awodey, Category Theory, 2nd Edition (2010). Oxford University Press.
- Bartek Klin, Bialgebras for structural operational semantics: An introduction. Theor. Comput. Sci. 412(38), 2011.
- Marcelo Fiore, Rough Notes on Presheaves, 2001. Available on the author's website.
- 圏論の歩き方 (数学セミナー連載, 単行本準備中). 蓮尾担当回の原稿を配布.

評価

レポートによる。教育研究支援室横レポート提出ボックスか、または講師のメールアドレス宛提出すること。