

Mathematical Semantics of Computer Systems, *MSCS* (4810-1168) Handout for Lecture 2 (2014/10/20)

Ichiro Hasuo, Dept. Computer Science, Univ. Tokyo

<http://www-mmm.is.s.u-tokyo.ac.jp/~ichiro>

Video recording of the lectures is available at:

<http://www-mmm.is.s.u-tokyo.ac.jp/~ichiro/video/mscs2014>

(ask me for username, password)

1 Today's Goal

Come to understand and prove the following statement.

Proposition. *Let $X, Y \in \mathbb{C}$. A product of X and Y , if it exists, is unique up-to a canonical isomorphism.*

2 Today's Agenda

2.1 Categories, Functors

Definition. Category, object, arrow (= morphism).

Example. **Sets**; a preorder as a category; a monoid as a category; the category **Mon** of monoids and monoid homomorphisms.

Remark. To familiarize yourself with the following abstract categorical notions, always try to think of their instances when the category in question is one of the previous examples.

Remark. An “abstract” definition of category by a deductive system, in [Lambek & Scott]

Definition. Proper class, small set, small category, locally small category

Definition. Functor

Example. Monotone functions as functors (for preorders considered as categories); monoid homomorphisms as functors (for monoids).

Example. Graphs as functors. Monoid/group actions as functors.

2.2 Reasoning with Arrows

Definition. Epi, mono. Split epi, split mono.

Definition. Product, coproduct.

3 Exercises

1. Describe Russell's paradox.
2. Prove “Today's Goal.”