terminality implies non-signalling

(and other abstract nonsense about causality)

Primary reference:

— (2014) *Terminality implies non-signalling*. arXiv:1405.3681

Secondary references:

— & R. Lal (2012) *Time-asymmetry of probabilities versus relativistic causal structure: an arrow of time*. Physical Review Letters **108**, 200403. arXiv:1108.1988

– causal stuff –

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- J. Pearl (2000) Causality: Models, Reasoning and Inference. Cambridge University Press.

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- G. Chiribella, G. M. D'Ariano & P. Perinotti (2010) Probabilistic theories with purification. Physical Review A, **81**, 062348. arXiv:0908.1583

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BUT, this is not the case anymore for non-signalling!

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$$R^{t} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} :: \begin{cases} 00, 10 & \mapsto & 00, 01, 11 \\ 01, 11 & \mapsto & 01, 10, 11 \end{cases}$$

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When Bob inputs **0**, his (i.e. Bob's) output is exactly Alice's input, **so he deduces Alice's input**.

– causal stuff –

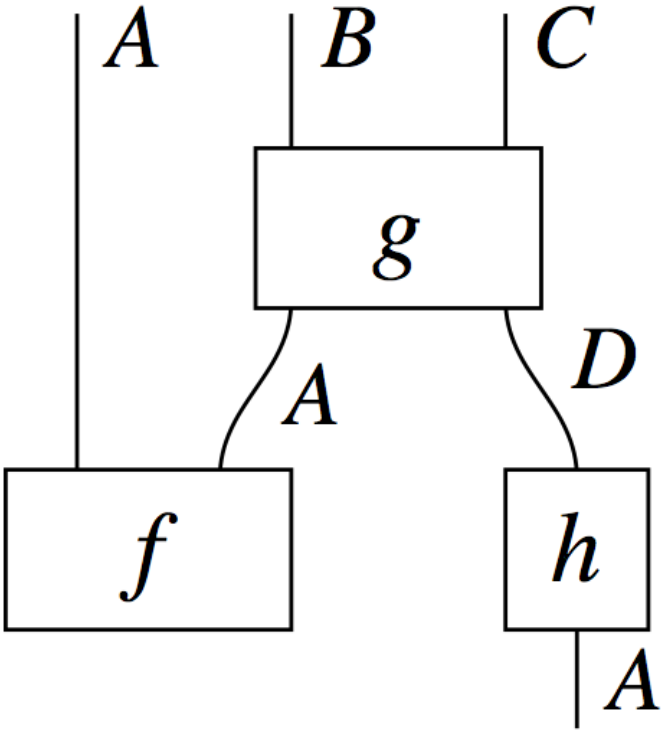
1. ***causal structure*** := space-time reachability poset

2. ***non-signalling*** := resulting operational restriction

3. ***causality axiom*** := non-signalling from the future

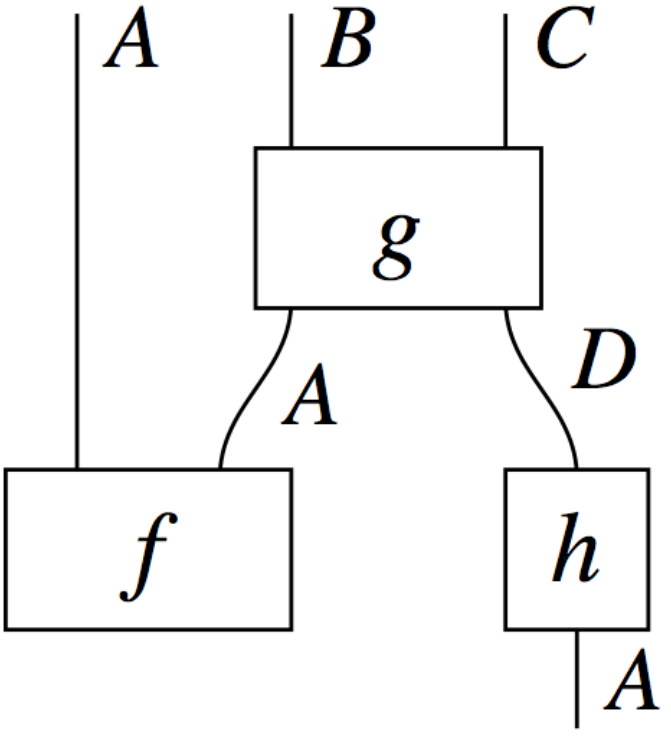
So how are these notions related?

– process theory = diagrams –



(— & A. Kissinger (2014) *Picturing Quantum Processes*, CUP)

– process theory = diagrams –



(or, a symmetric strict monoidal category)

– process theory = diagrams –

states have no inputs

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effects have no outputs

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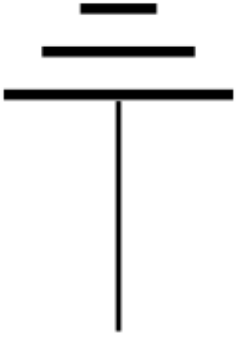
discarding :=



– terminality –

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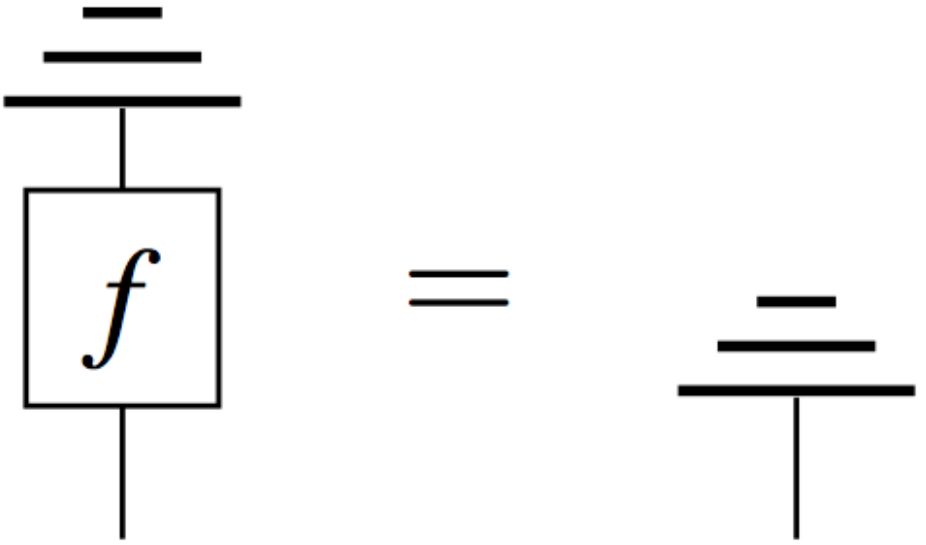
Def. *terminality* of a process theory :=



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Prop. TFAE:

(a) terminality,

(b) all effects are discarding,

(c) for each system there is only one effect.

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(c) for each system there is only one effect.

(or, terminality $\equiv$ tensor unit is terminal)

– terminality –

E.g.

- normalisation for probabilities and states
- trace-preserving for CP-maps
- resolution of identity for quantum measurements

– non-signalling –

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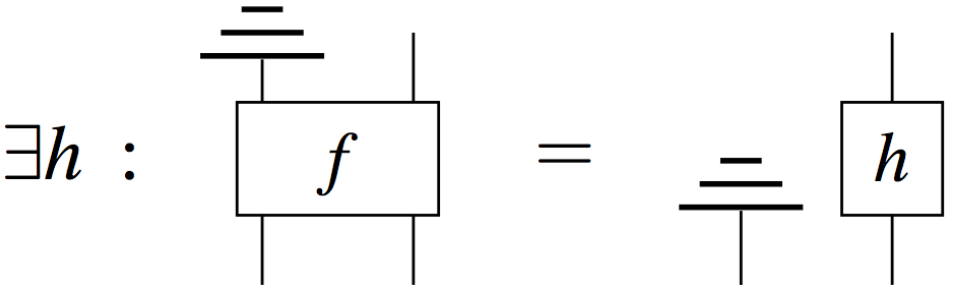
Def. *A-B non-signalling* of a process :=



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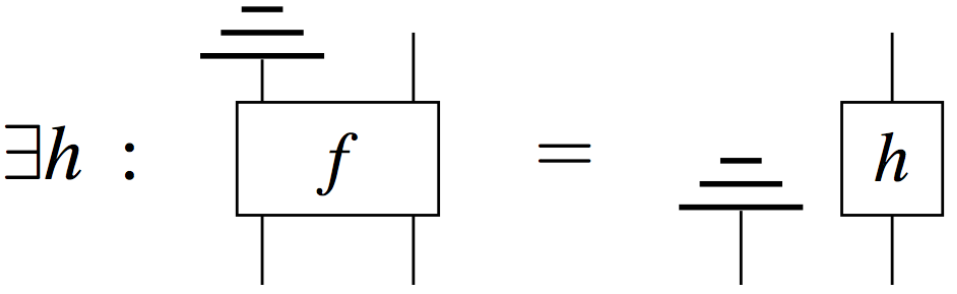
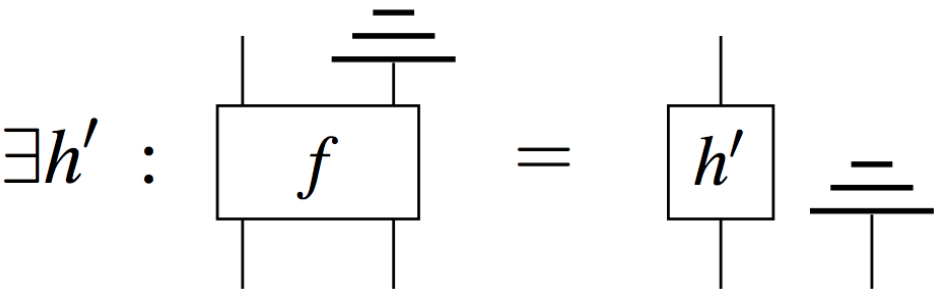


Def. *B-A non-signalling* of a process :=



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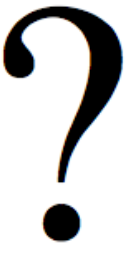

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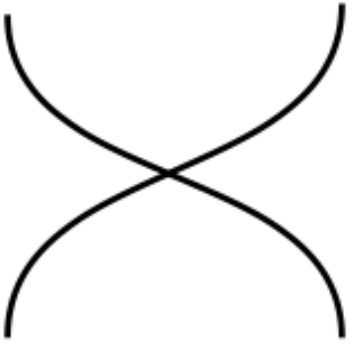

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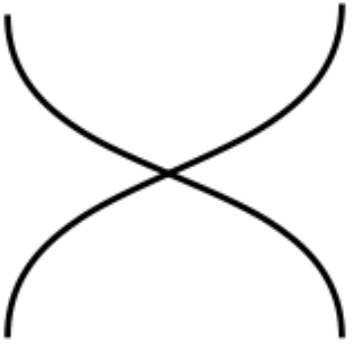

$\Rightarrow$ restrict to processes without explicit signalling

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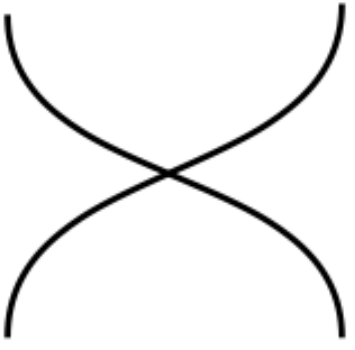


$\Rightarrow$ restrict to processes without explicit signalling

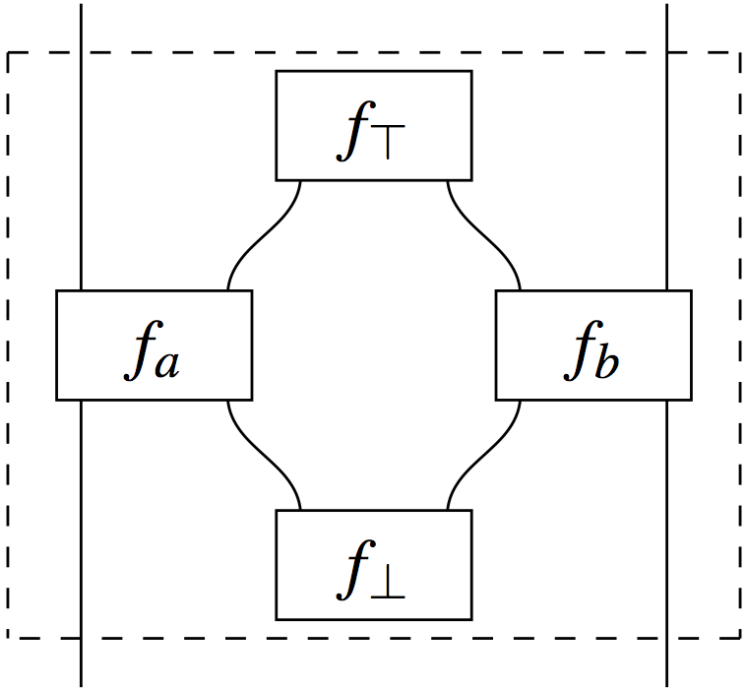
$\Rightarrow$ consider internal causal structure of processes

– non-signalling –

Prop. Most general internal structure $\equiv$

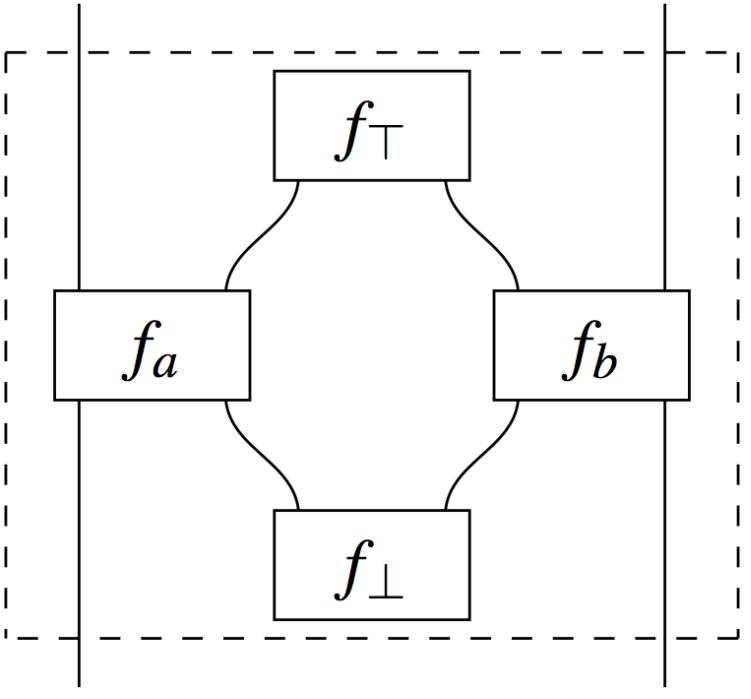
– non-signalling –

Prop. Most general internal structure $\equiv$



– non-signalling –

Def. *non-signalling* of a process theory := A-B & B-A non-signalling of all processes of the form:



– main result –

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Thm. For all process theories:

$$\boxed{\text{terminality} \Rightarrow \text{non-signalling}}$$

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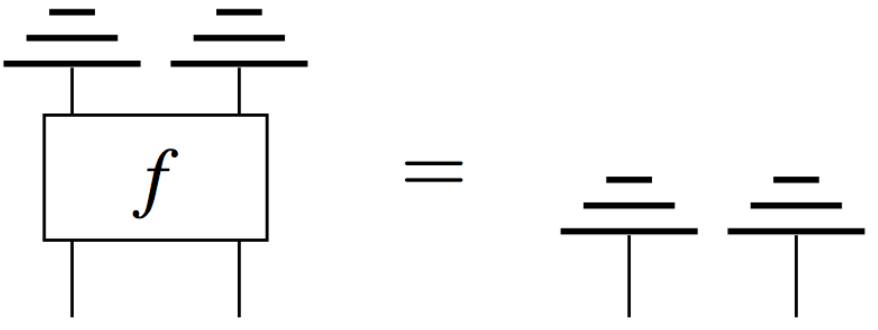
Pf. Recall terminality:



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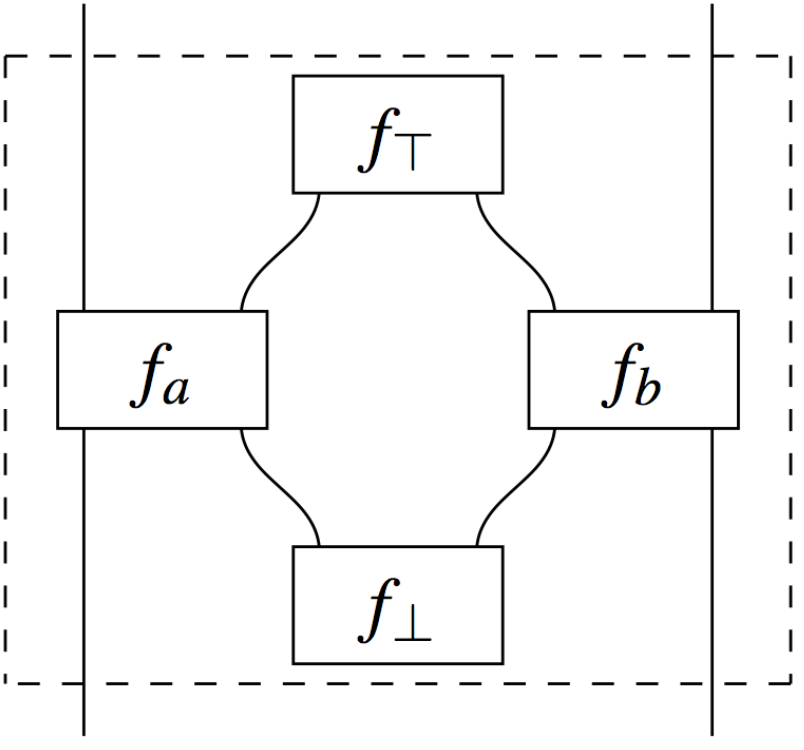
So:



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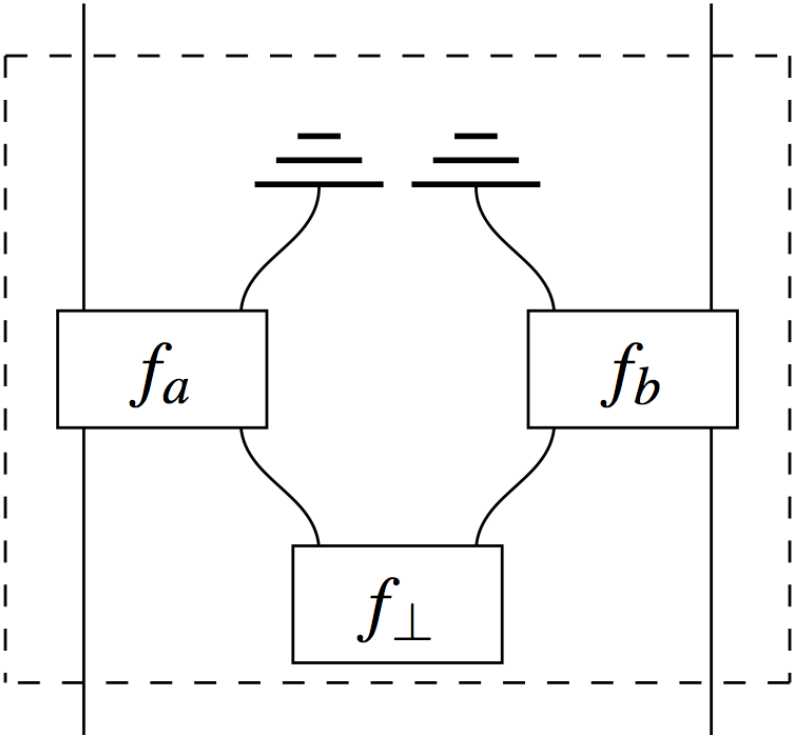

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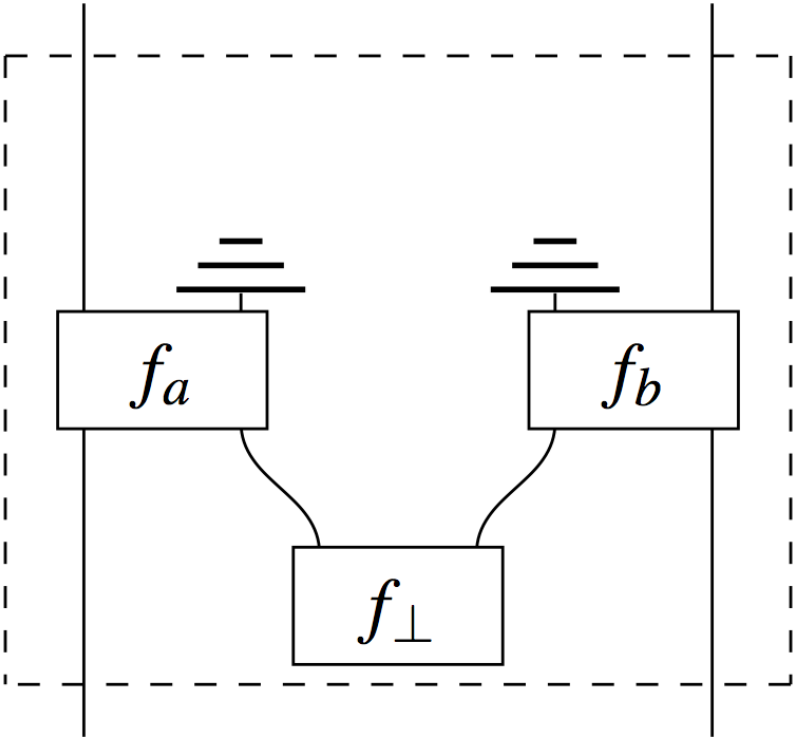

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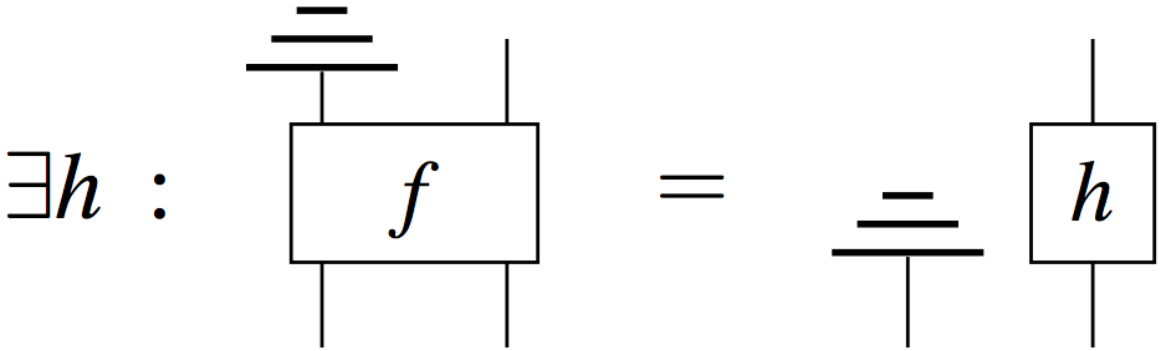
Recall A-B non-signalling:



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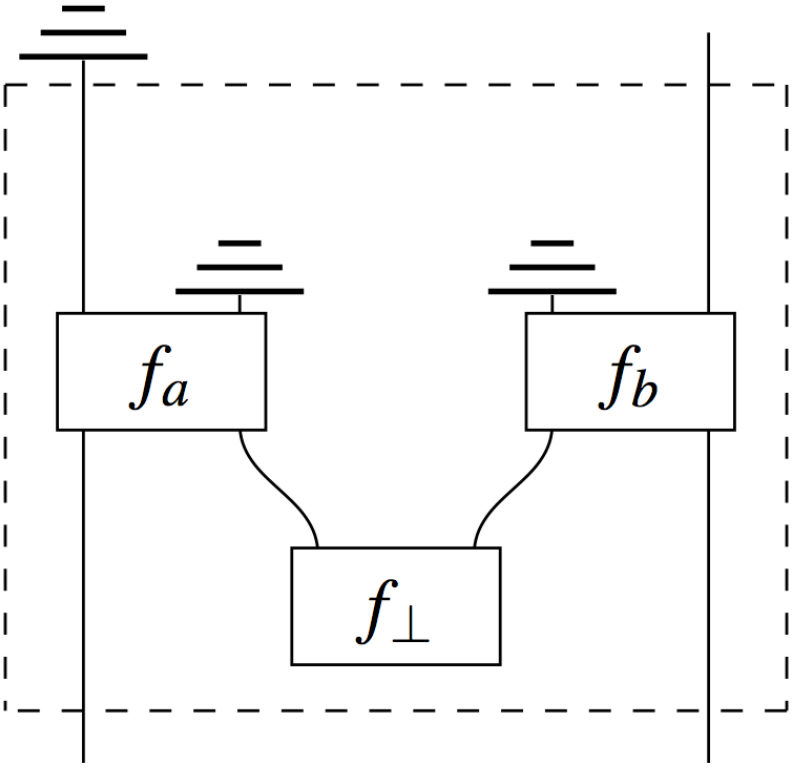

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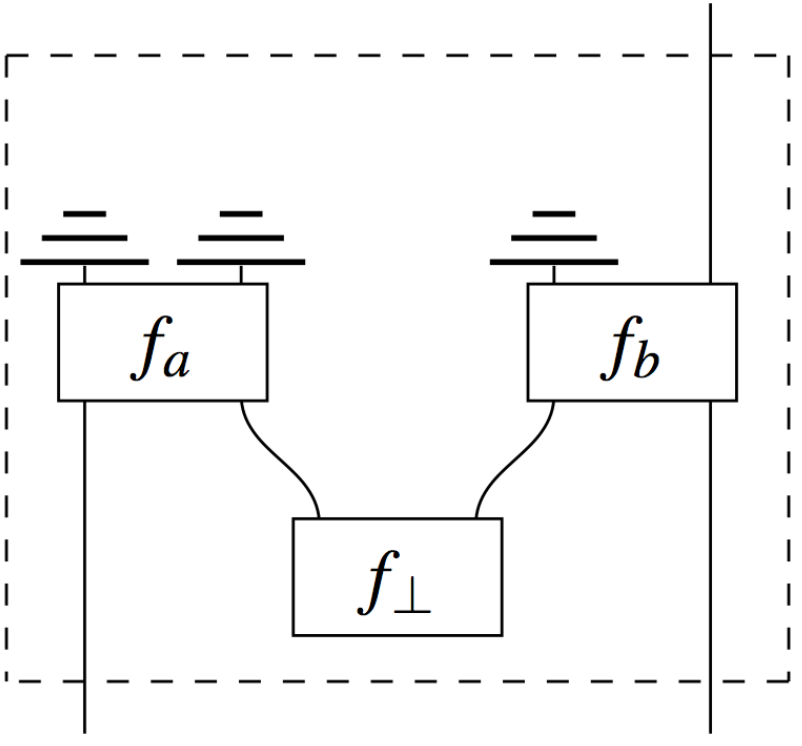

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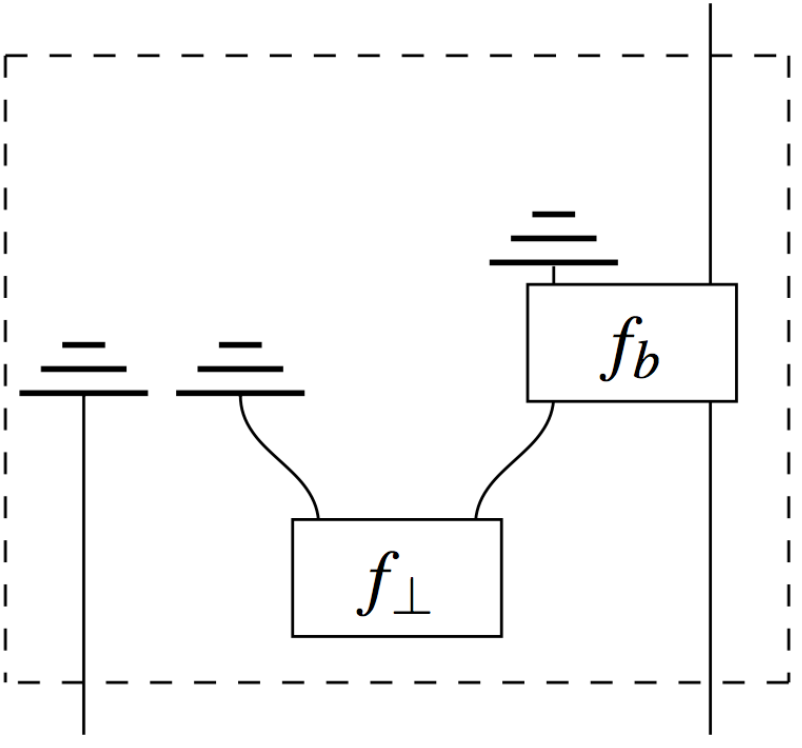

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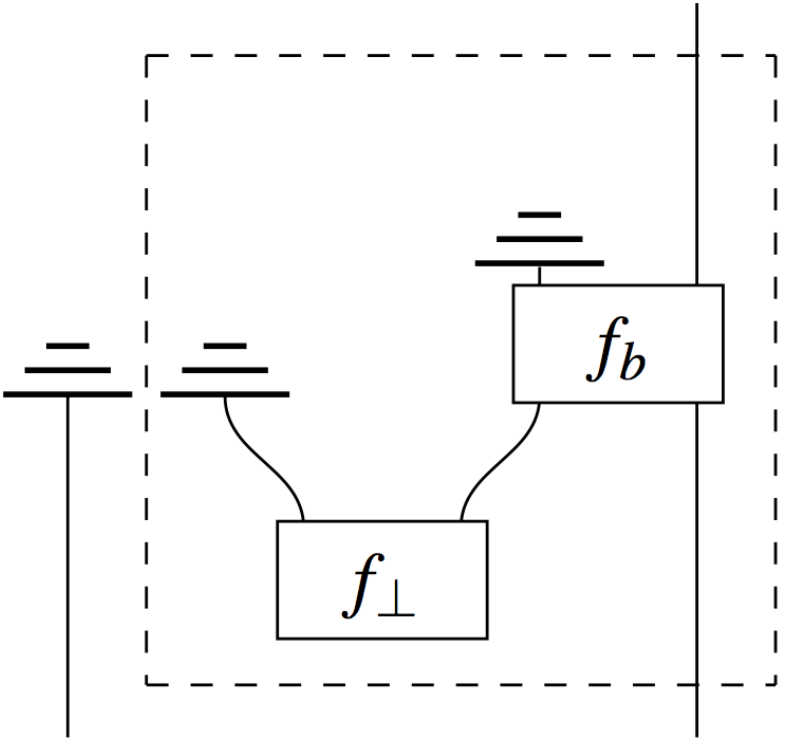

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Thm. For all process theories with unique scalar:

non-signalling $\Rightarrow$ terminality

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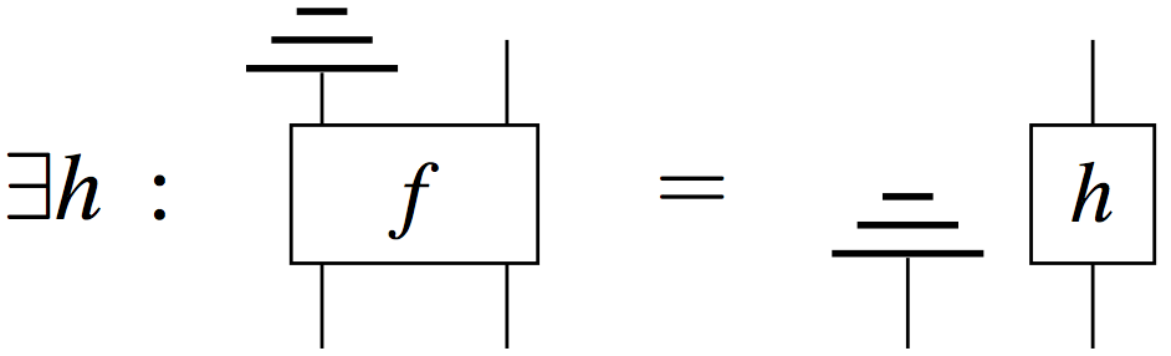
Pf. Take 2nd system trivial in:



– discussion –

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• non-signalling requires causality to even be meaningful, ...

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⇒ **Term 'causality axiom' was very well chosen!**

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• terminality is both conceptually and mathematically very simple, ... hence, is the more fundamental notion

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• causality can be time-reversed, ...

– discussion –

• non-signalling requires causality to even be meaningful, ... but then it just follows from terminality

• terminality is both conceptually and mathematically very simple, ... hence, is the more fundamental notion

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• causality can be time-reversed, ... however, by time-asymmetry of terminality, processes cannot

– covariance –

— & R. Lal (2012) *Causal categories: relativistically interacting processes*. Foundations of Physics **43**, 458-501. arXiv:1107.6019

R. Blute, I. T. Ivanov & Prakash Panangaden (2003) *Discrete quantum causal dynamics*. International Journal of Theoretical Physics **42**, 2025-2041. arXiv:gr-qc/0109053

F. Markopoulou (2000) *Quantum causal histories*. Classical and Quantum Gravity, **17**, 2059. arXiv:hep-th/9904009

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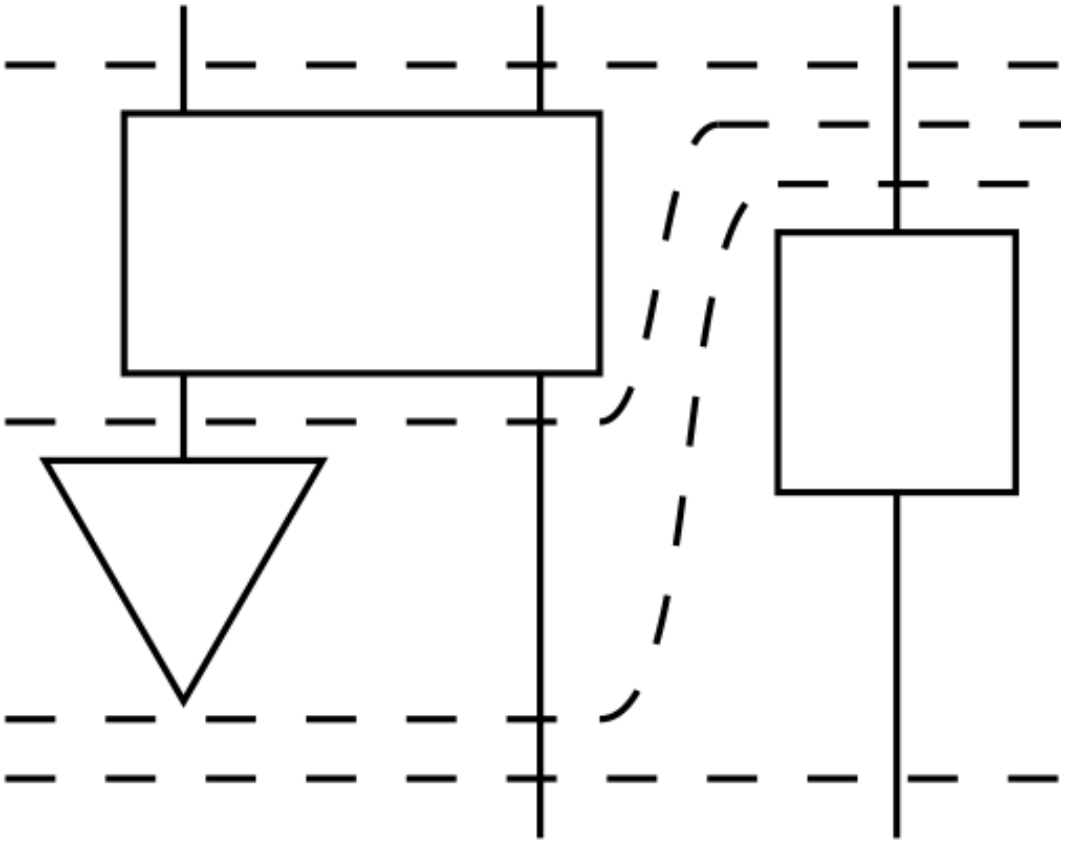

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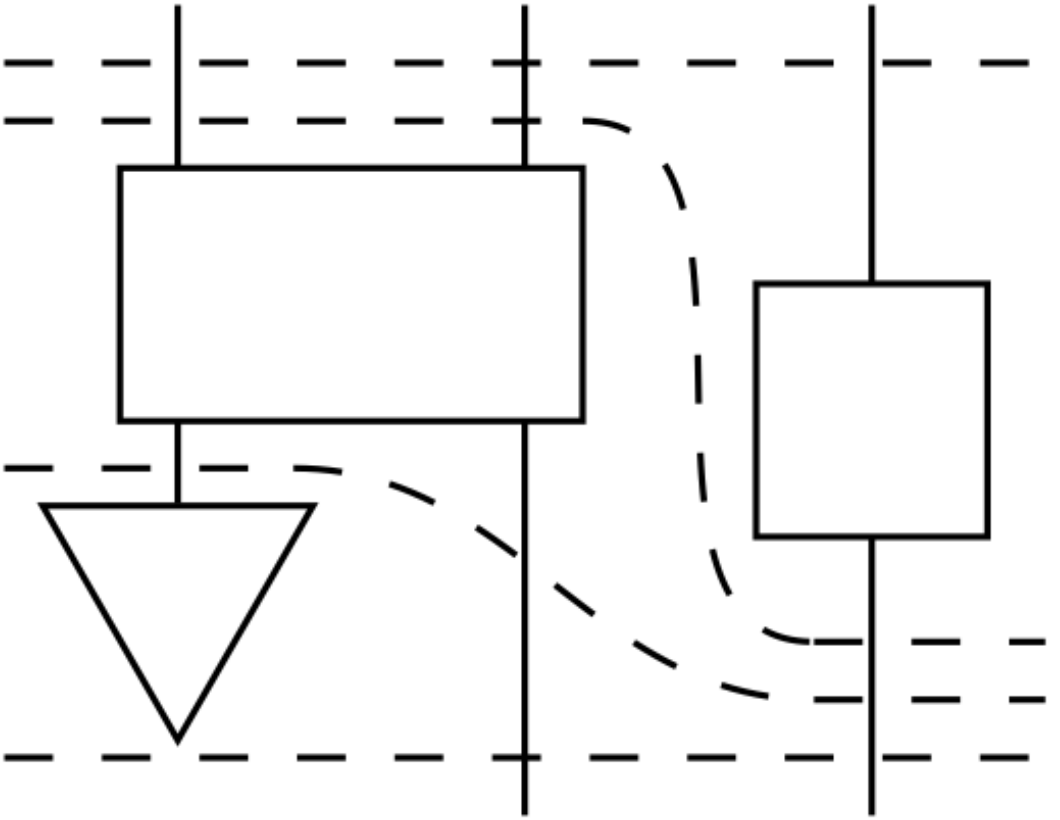

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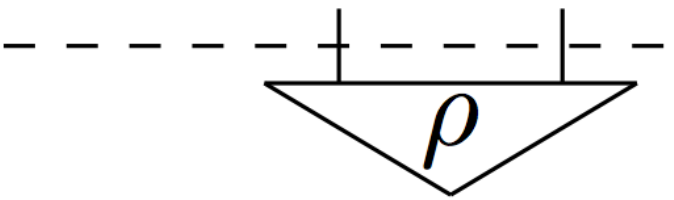

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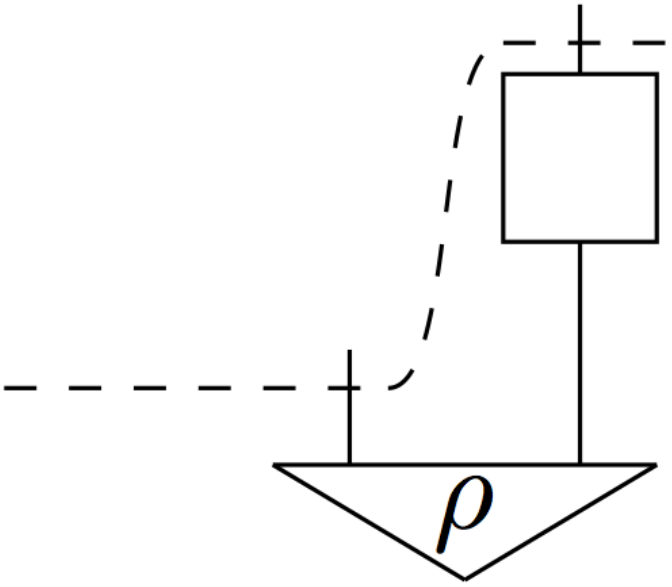

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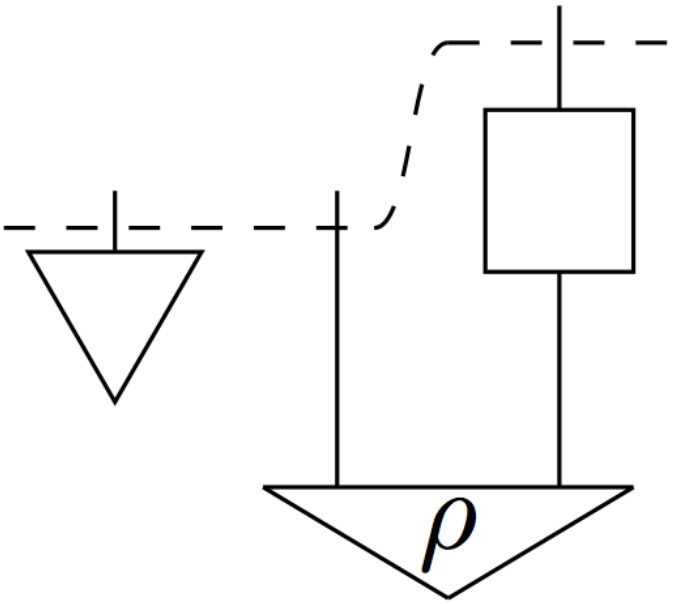

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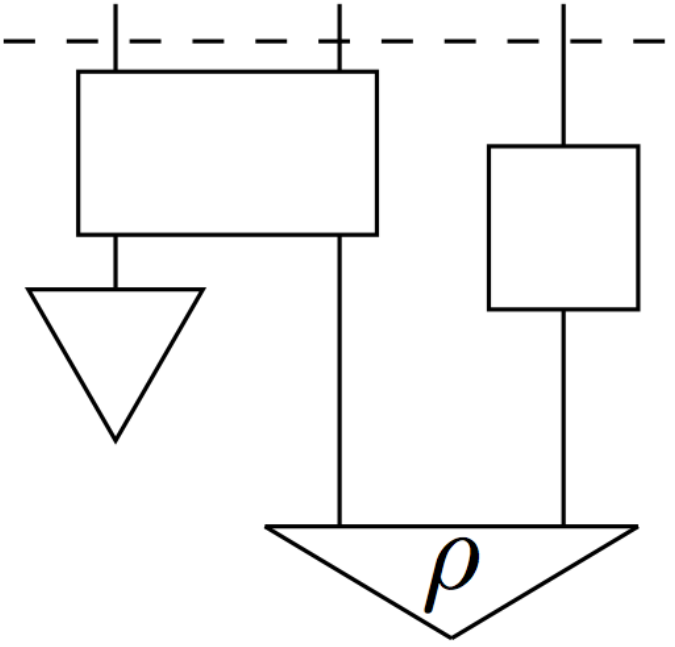

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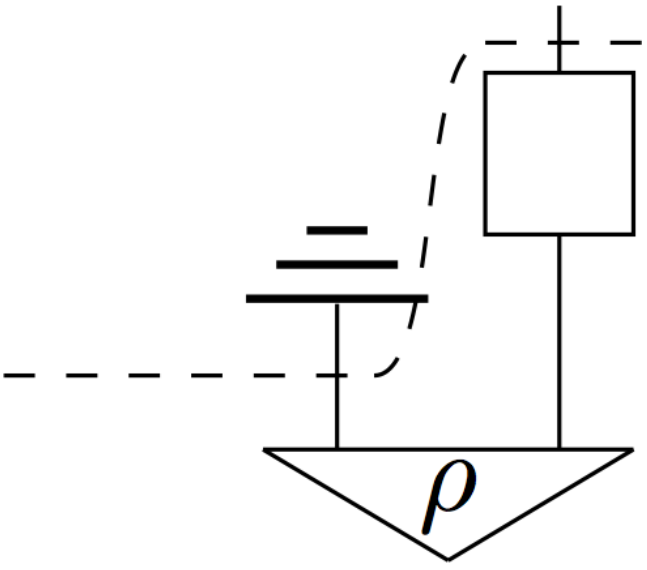

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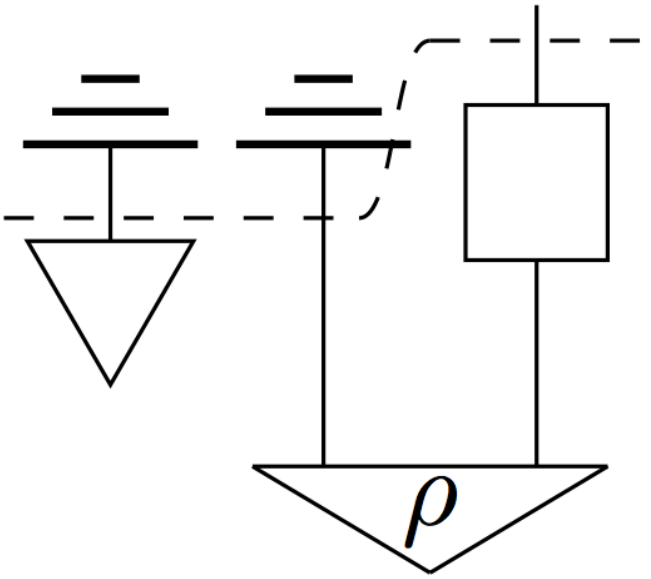
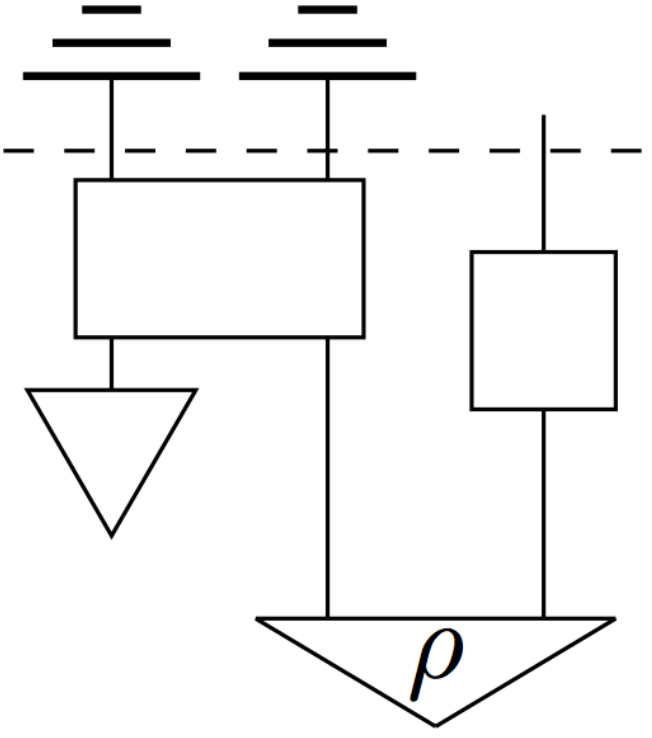
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