Complexity of Grammar Induction with Quantum Types

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École Normale Supérieure and University of Oxford

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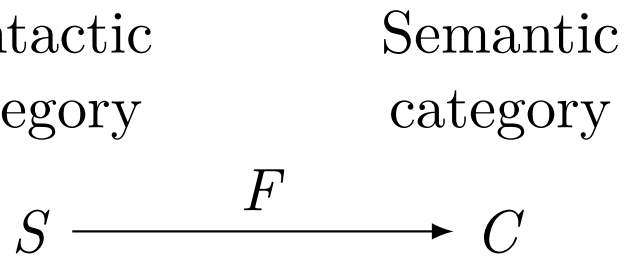
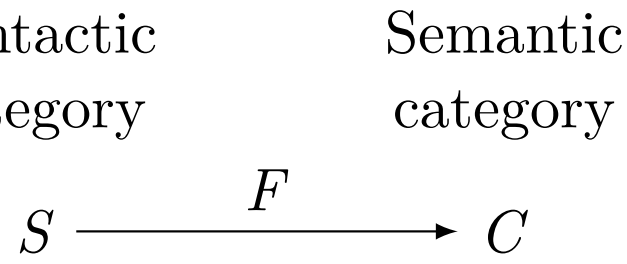
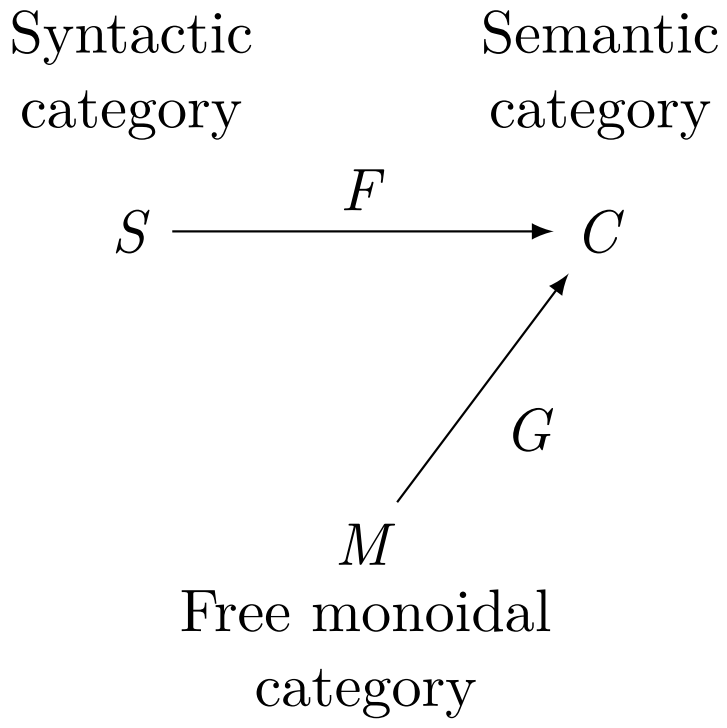
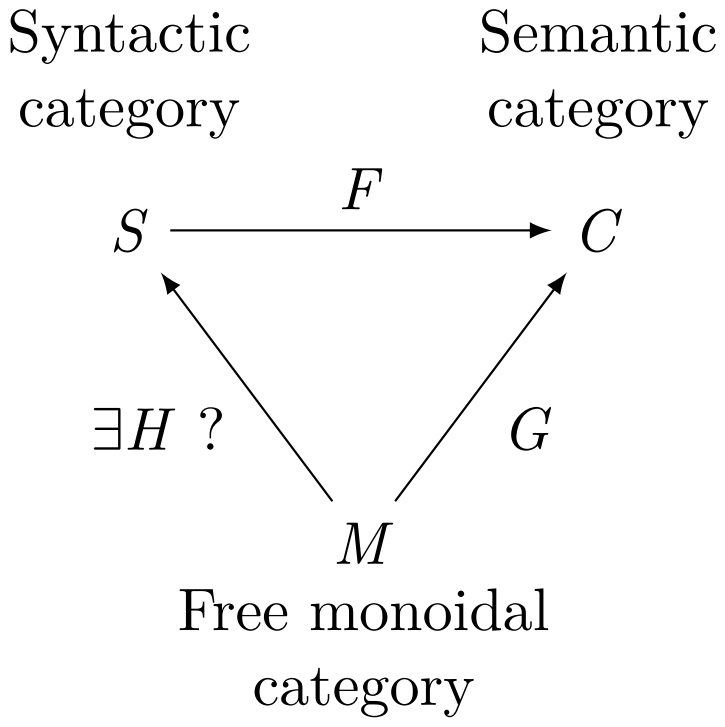
Quantum Physics and Logic, Kyoto


Syntactic
category
Semantic
category
F
S
C



M
Free monoidal
category


Syntactic category
Semantic category
S
F
C
G
M
Free monoidal category


Syntactic category
Semantic category
F
S
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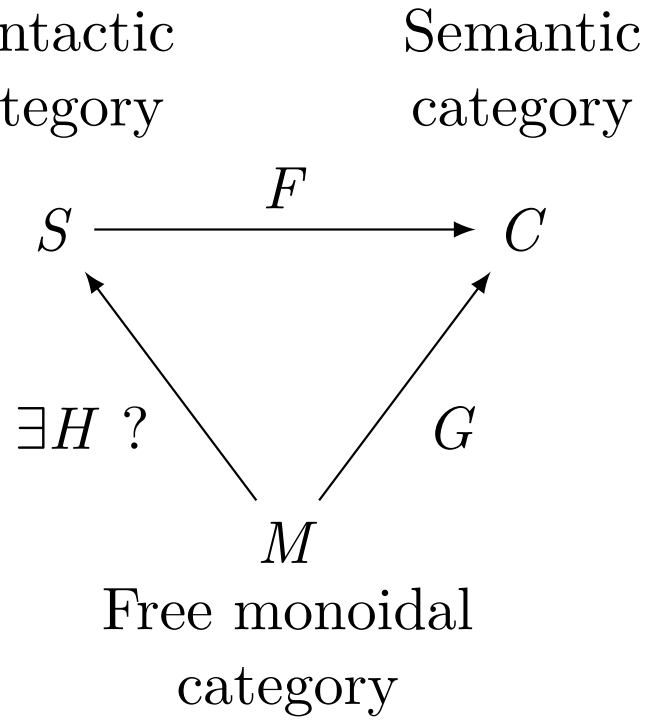
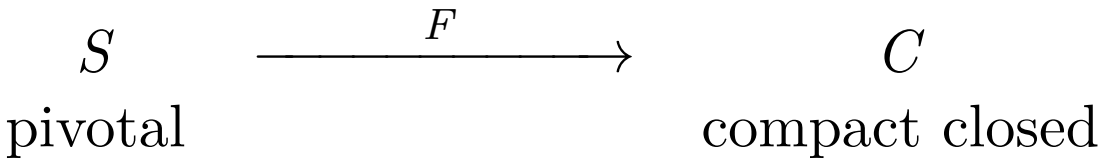
∃H ?
G
M
Free monoidal category



An object $A \in S$ is grammatical $\Leftrightarrow \exists f \in S(A, I)$.
Given G and a finite $P \subset \mathrm{Ob}(M)$, is there an H such that the diagram commutes and $H(P)$ is grammatical?

An example



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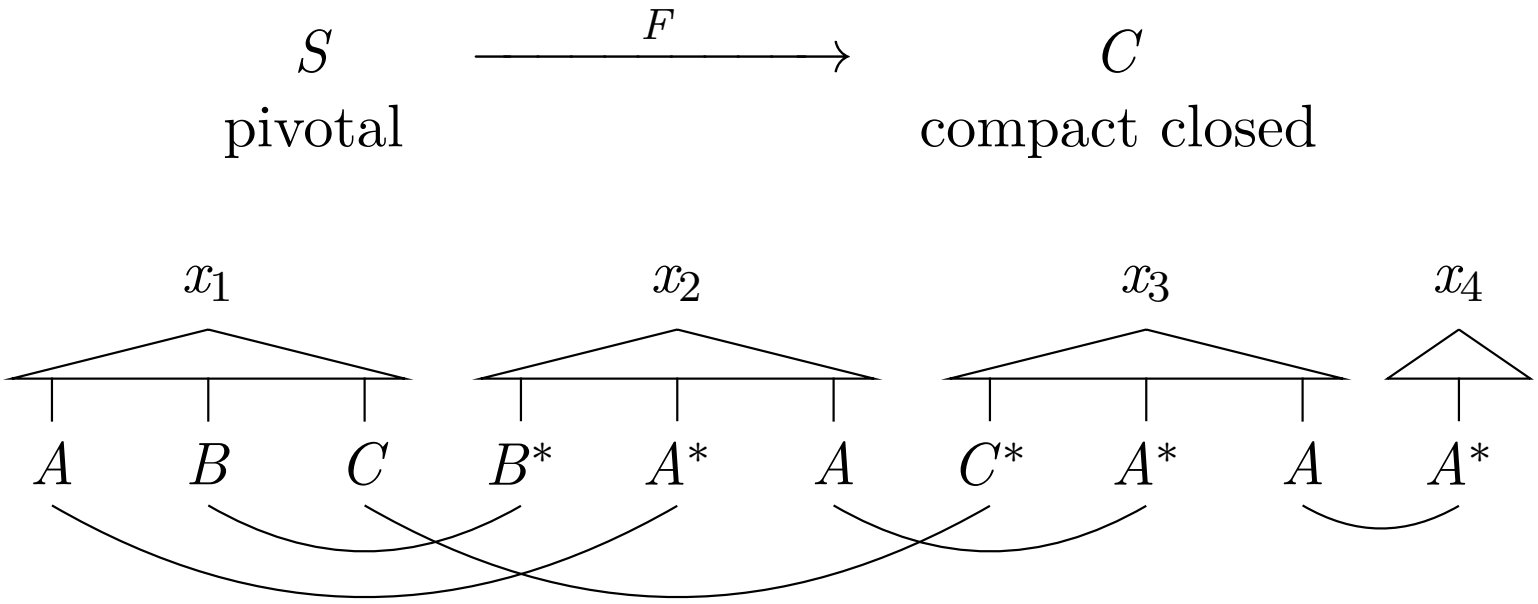

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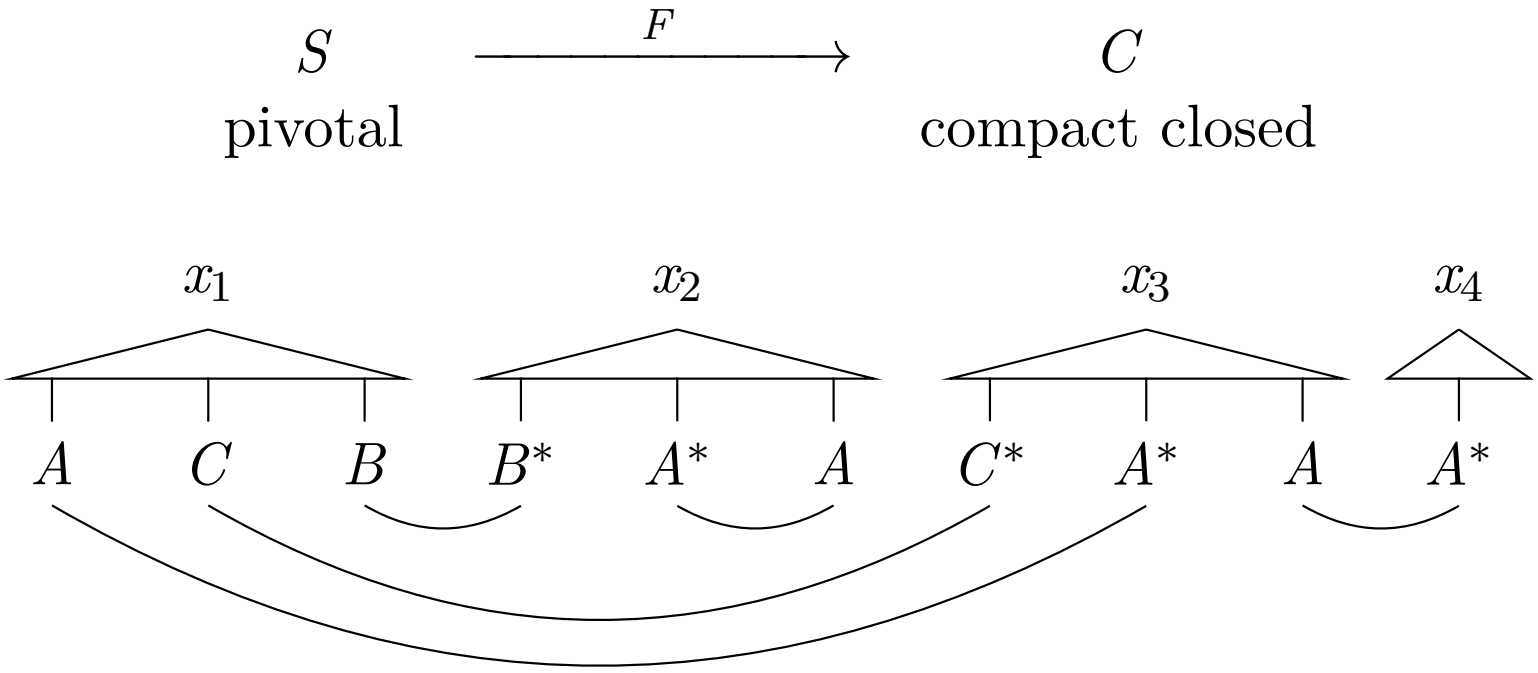

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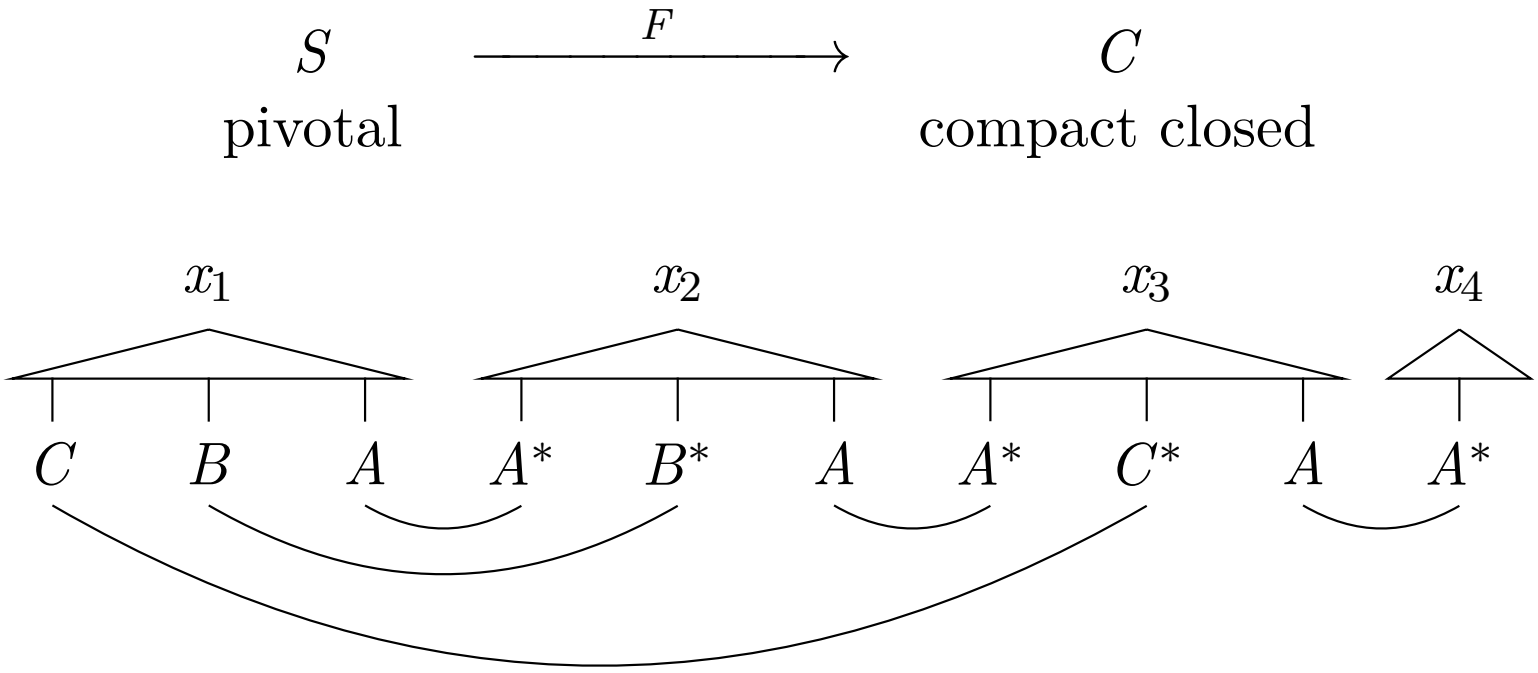

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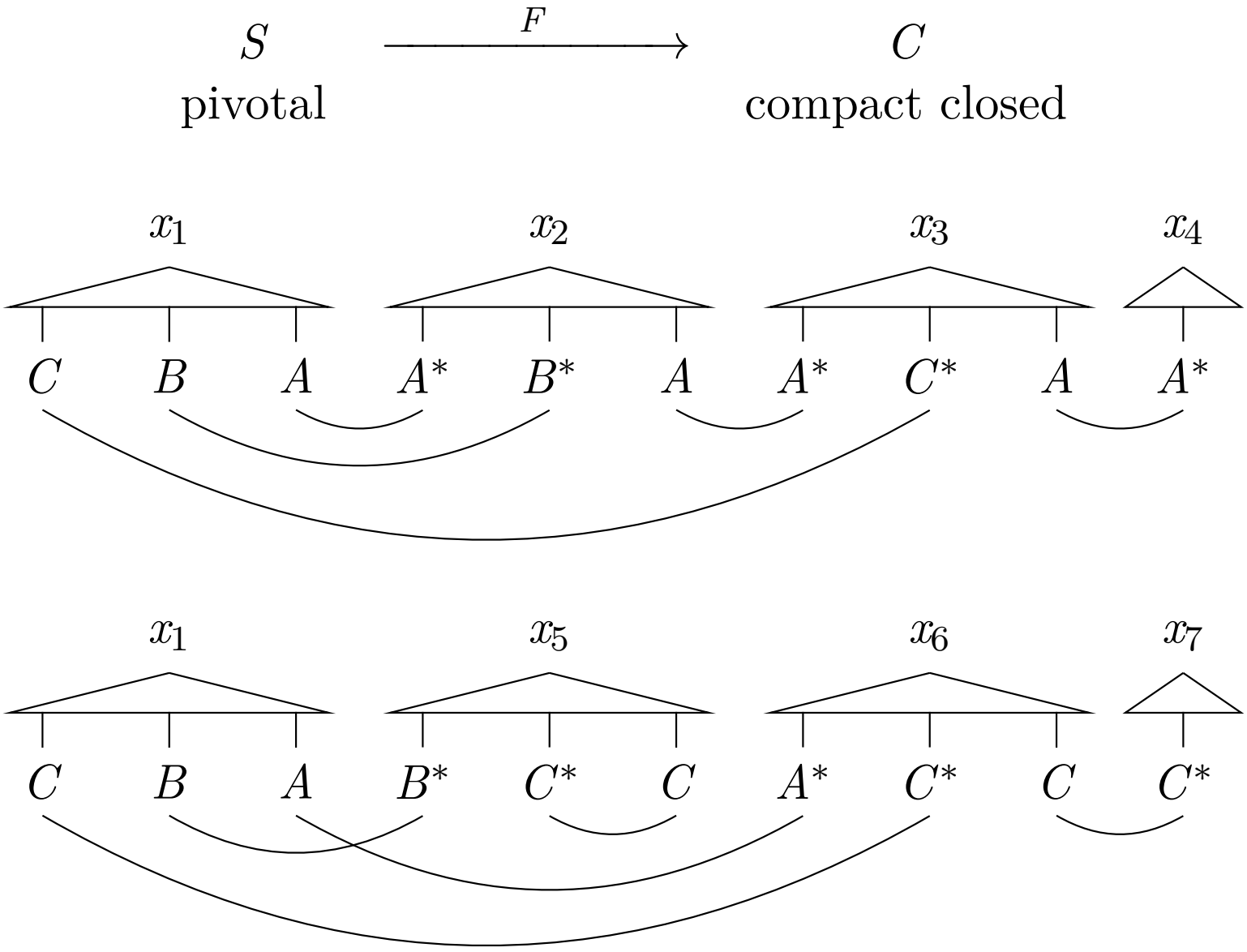
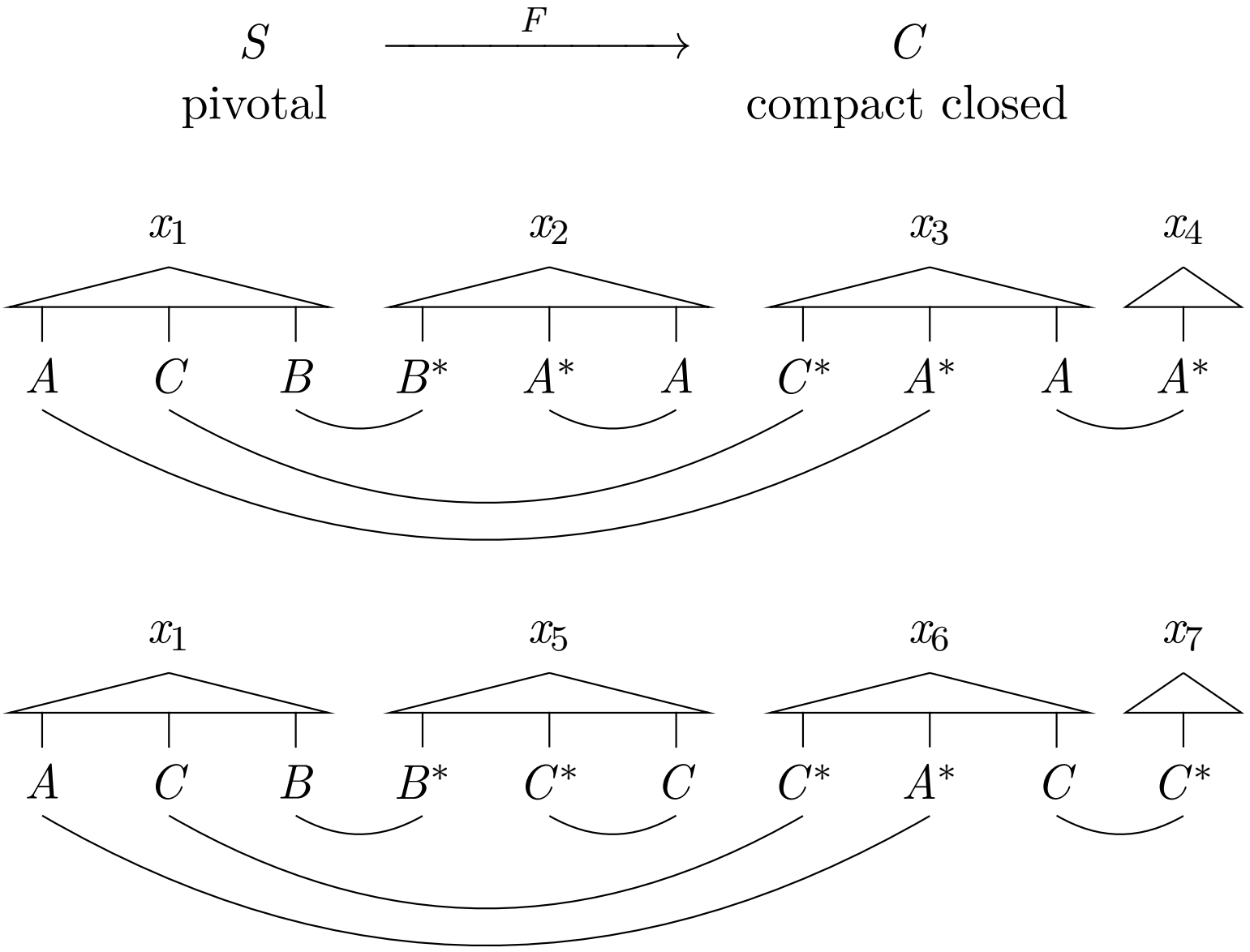

S
pivotal
F
C
compact closed
x1
x2
x3
x4
C
B
A
A*
B*
A
A*
C*
A
A*
x1
x5
x6
x7
C
B
A
B*
C*
C
A*
C*
C
C*

An example


S
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x_1 x_2 x_3 x_4
A C B B* A* A C* A* A A*
x_1 x_5 x_6 x_7
A C B B* C* C C* A* C C*

A : finite set

$C \subset A^3$: finite set of triples

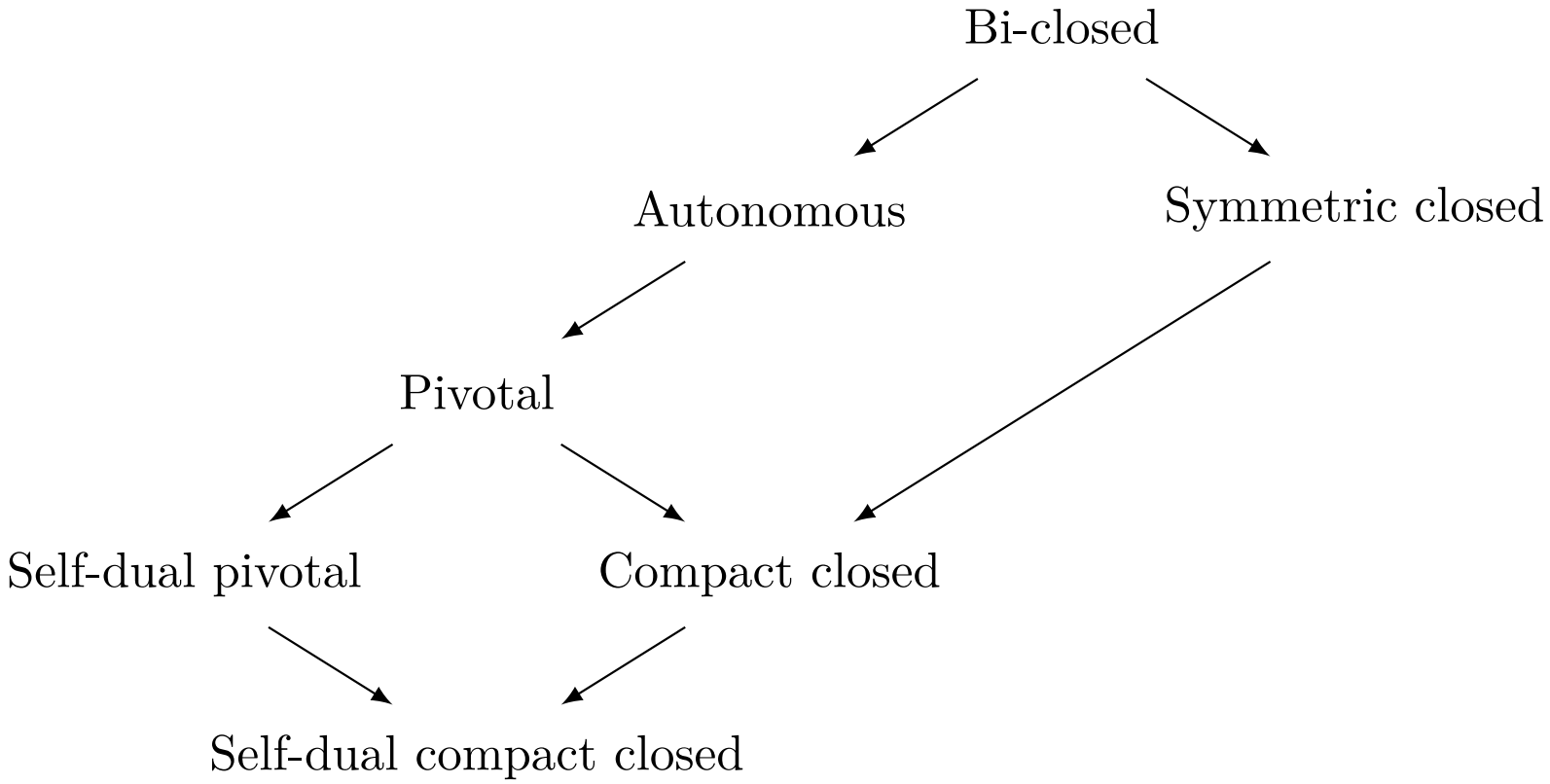
Problem: find a total ordering $<$ of A such that

$$\forall (a, b, c) \in C, a < b < c \text{ or } c < b < a$$

This problem is NP-complete[1] and reduces to grammar inference from pivotal to compact closed categories.

[1] Guttmann and Maucher (2006)

A hierarchy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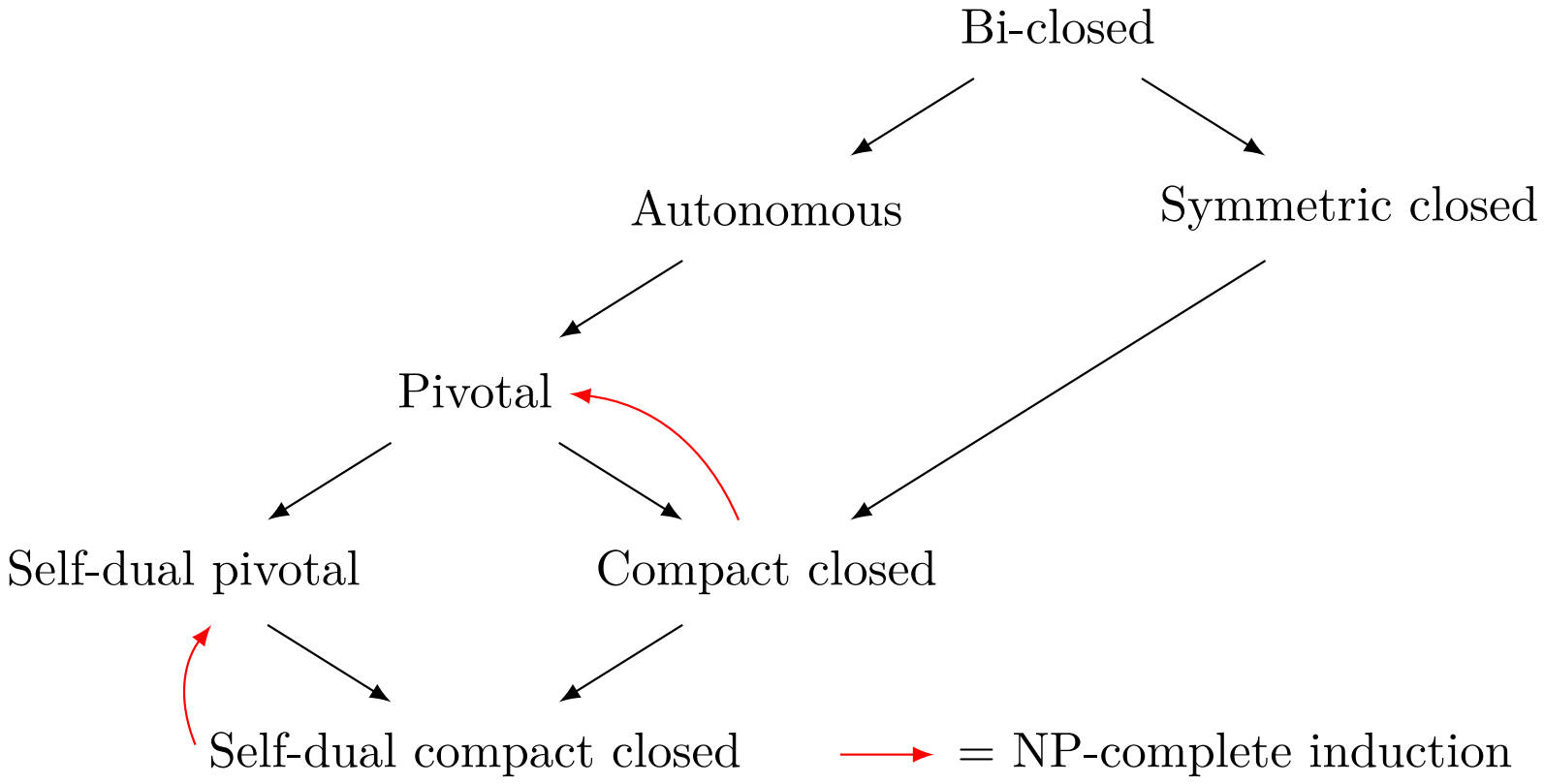
Symmetric closed
Pivotal
Self-dual pivotal
Compact closed
Self-dual compact closed

A hierarchy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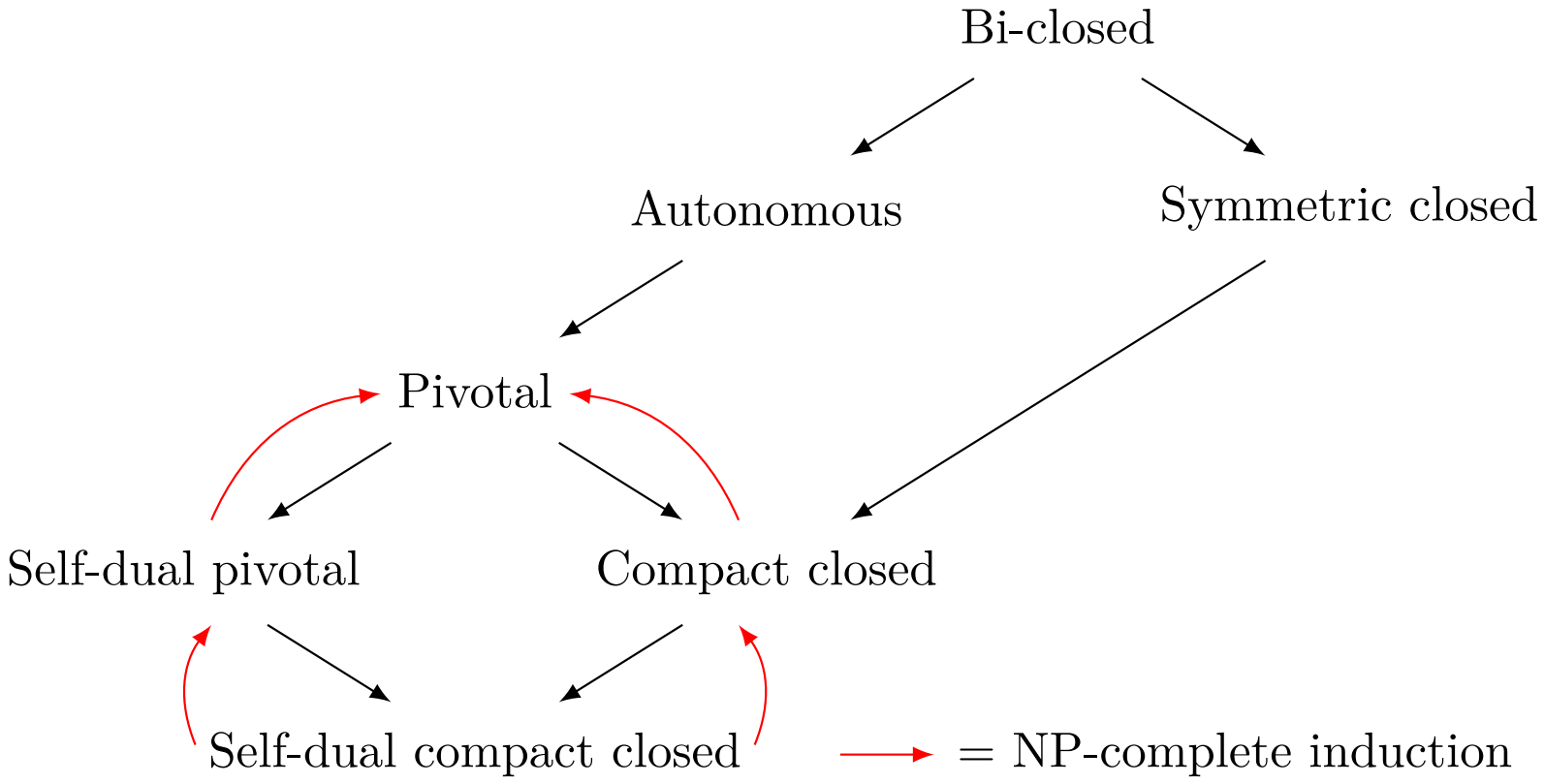
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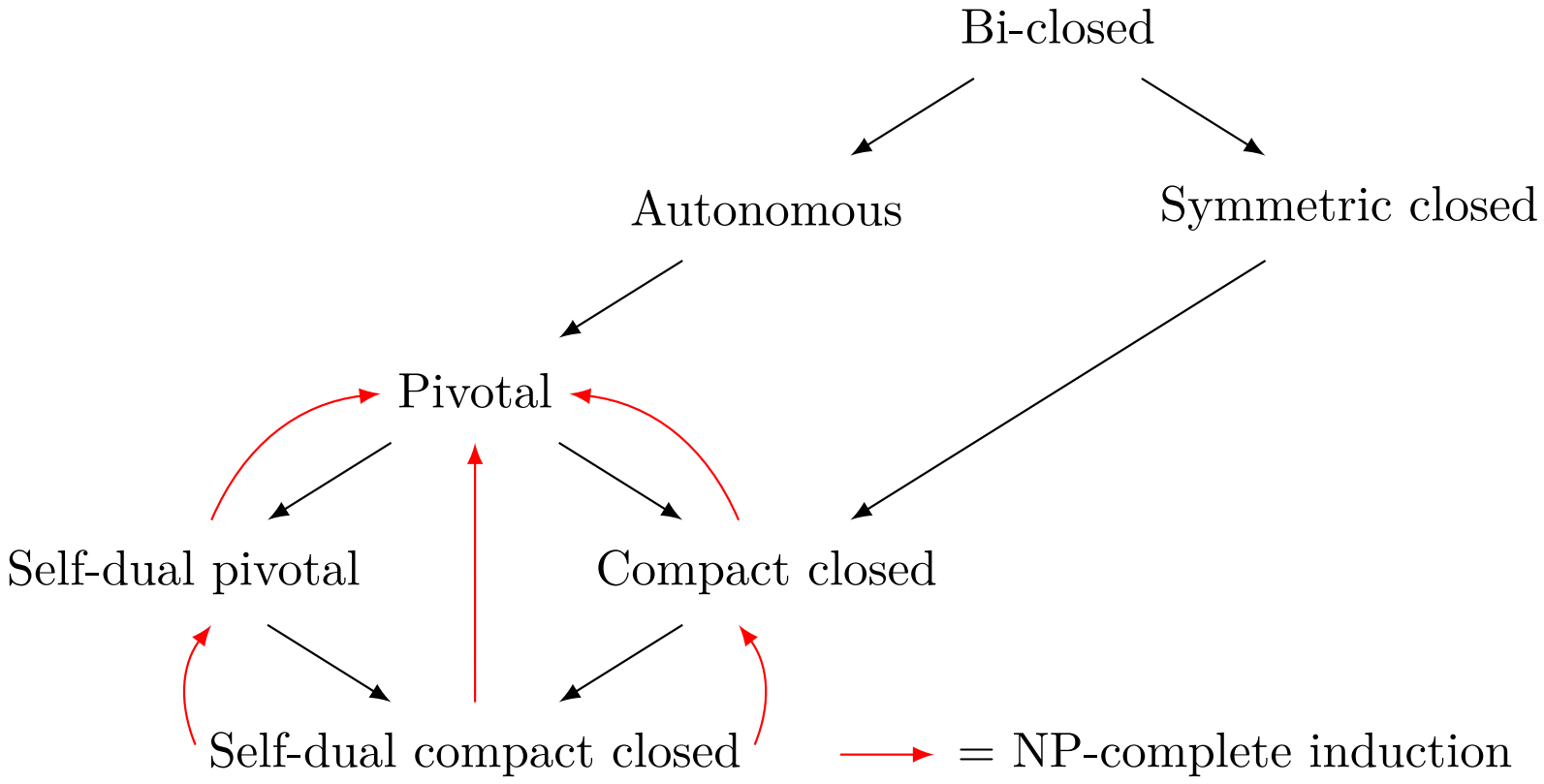
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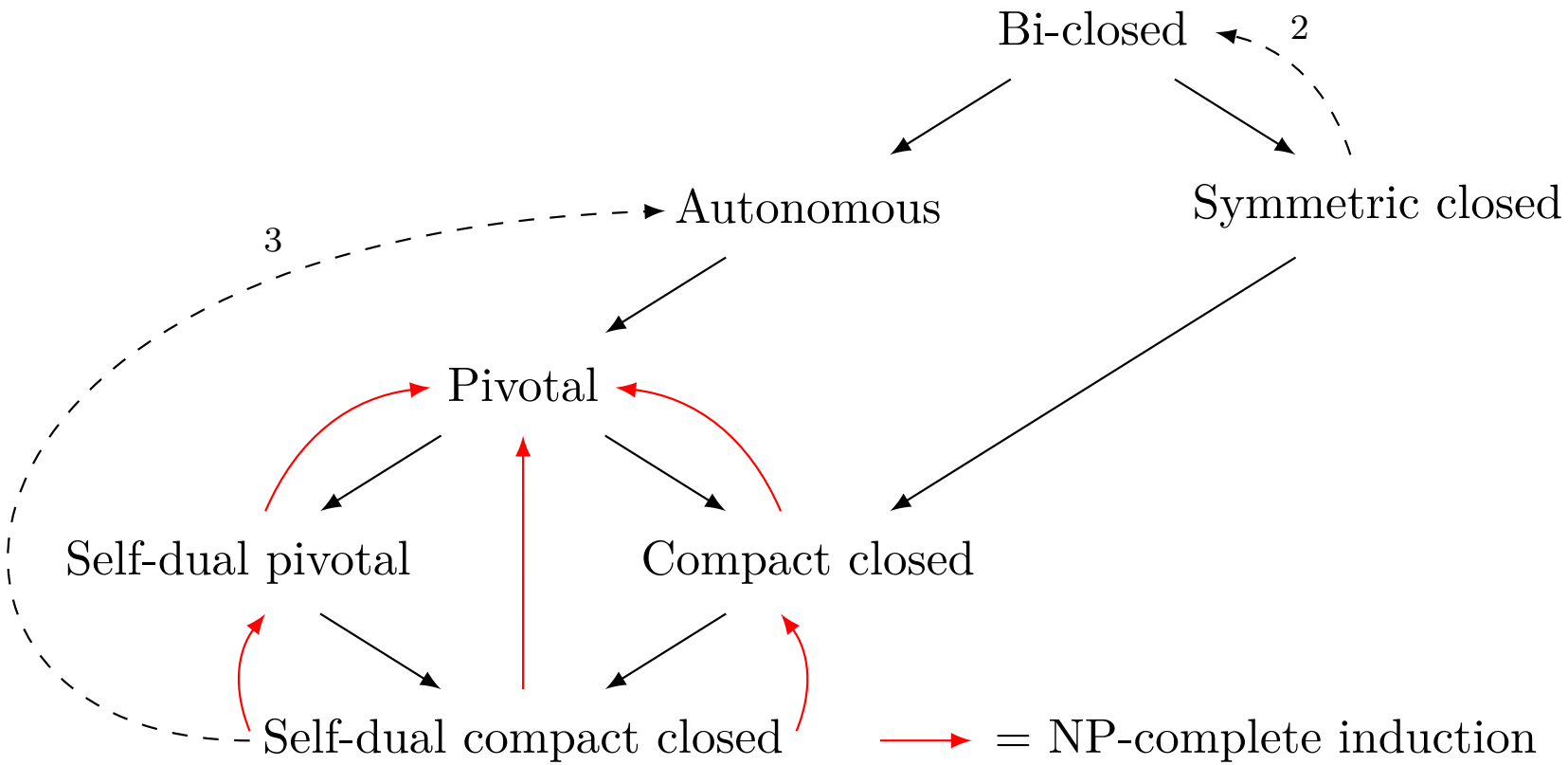
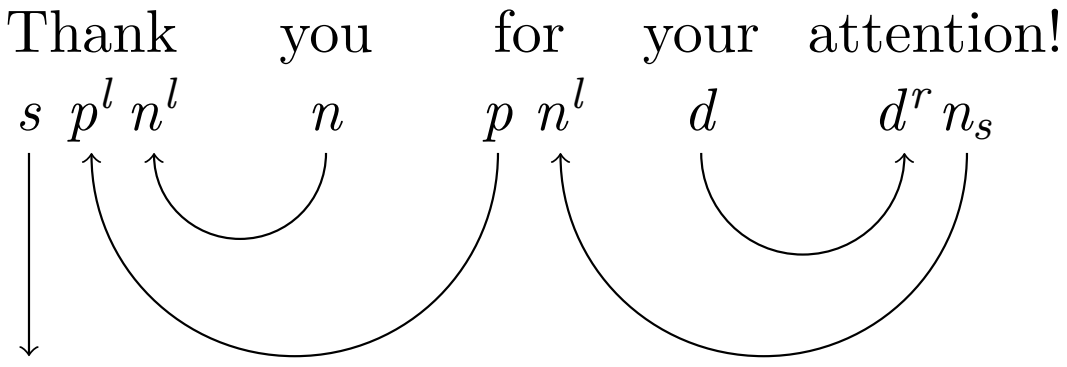


[2] Dudau-Sofronie, Tellier, and Tommasi (2001)

[3] Béchet, Foret, and Tellier (2007)

Thank you