Relational Differential Dynamic Logic – Appendix

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1 About KeYmaera X

For compilation of KeYmaera X, refer to:

`https://github.com/LS-Lab/KeYmaeraX-release/blob/master/README.md`.

For proving in KeYmaera X, refer to:

`http://www.ls.cs.cmu.edu/KeYmaeraX/documentation.html`.

2 About our implementation

Our rules are implemented in the file `RelationalProofRules.scala`, which is located, like other rules in KeYmaera X, in:

`/keymaerax-core/src/main/scala/edu/cmu/cs/ls/keymaerax/btactics/`

Our tests for the rules can be found in the file:

`/keymaerax-webui/src/test/scala/btactics/RelationalTests.scala`

To run them, run

test-only *RelationalTests

in sbt mode.

3 Full proof of Proposition 8.

Proof. Note that $\dashv\!\![\alpha]\!\!\rightarrow \subseteq \mathbb{R}^X \times \mathbb{R}^X$ is defined for any set X of variables that contains at least all variables occurring in α. Let us write $\dashv\!\![\alpha]\!\!\rightarrow_X$ to make this set X explicit. It is easy to see that $\omega \dashv\!\![\alpha]\!\!\rightarrow_X \omega'$ implies $\omega(y) = \omega'(y)$ for any variable $y \in X$ that does not occur in α. This means that $(\dashv\!\![\overline{\alpha}]\!\!\rightarrow_{\overline{\mathcal{V}} \uplus \underline{\mathcal{V}}}) = (\dashv\!\![\overline{\alpha}]\!\!\rightarrow_{\overline{\mathcal{V}}} \otimes =_{\underline{\mathcal{V}}})$ and $(\dashv\!\![\underline{\alpha}]\!\!\rightarrow_{\overline{\mathcal{V}} \uplus \underline{\mathcal{V}}}) = (=_{\overline{\mathcal{V}}} \otimes \dashv\!\![\underline{\alpha}]\!\!\rightarrow_{\underline{\mathcal{V}}})$. It follows that

$$\begin{aligned}
\dashv\!\![\overline{\alpha}; \underline{\alpha}]\!\!\rightarrow &= \dashv\!\![\overline{\alpha}]\!\!\rightarrow_{\overline{\mathcal{V}} \uplus \underline{\mathcal{V}}} \circ \dashv\!\![\underline{\alpha}]\!\!\rightarrow_{\overline{\mathcal{V}} \uplus \underline{\mathcal{V}}} \\
&= \left(\dashv\!\![\overline{\alpha}]\!\!\rightarrow_{\overline{\mathcal{V}}} \otimes =_{\underline{\mathcal{V}}}\right) \circ \left(=_{\overline{\mathcal{V}}} \otimes \dashv\!\![\underline{\alpha}]\!\!\rightarrow_{\underline{\mathcal{V}}}\right) \\
&= \left(\dashv\!\![\overline{\alpha}]\!\!\rightarrow_{\overline{\mathcal{V}}} \circ =_{\overline{\mathcal{V}}}\right) \otimes \left(=_{\underline{\mathcal{V}}} \circ \dashv\!\![\underline{\alpha}]\!\!\rightarrow_{\underline{\mathcal{V}}}\right) \\
&= \dashv\!\![\overline{\alpha}]\!\!\rightarrow_{\overline{\mathcal{V}}} \otimes \dashv\!\![\underline{\alpha}]\!\!\rightarrow_{\underline{\mathcal{V}}}
\end{aligned}$$

□