

# Mathematical Semantics of Computer Systems, *MSCS* (4810-1168) Handout for Lecture 12 (2017/1/16)

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Video recording of the lectures is available at: <http://www-mmm.is.s.u-tokyo.ac.jp/videos/mscs2016>

## 1 Cartesian Closed Categories as Models of Typed $\lambda$ -Calculus

Starting from the last lecture where we motivated exponentials...

**Definition.** Exponentials. Cartesian closed categories.

**Lemma.** *Exponentials induce adjunctions  $A \times (-) \dashv (-)^A$*

Exercise: work out the unit, the counit, and the triangular equalities.

We then review the syntax, typing rules, capture-avoiding substitution, conversion rules and the  $\beta\eta$ -equality, in the typed  $\lambda$ -calculus. NB: we use the term calculus *a la Church* (where bound variables have explicit types, that is, like  $\lambda x^U. t$  instead of  $\lambda x. t$ ).

**Lemma.** *Subject reduction lemma.*

**Lemma.** *Each derivable type judgment has a unique derivation tree.*

**Definition.** Interpretation  $\llbracket \_ \rrbracket$  of typed  $\lambda$ -calculus. Interpreting types, type derivation trees, type judgments, and terms.

**Definition.** Substitution lemma: interpretation of  $s[t/x]$  is given by composition of arrows.

**Theorem.** *Soundness of categorical semantics: if  $s =_{\beta\eta} t$ , then  $\llbracket s \rrbracket = \llbracket t \rrbracket$ .*

**If we have time:**

- The Curry-Howard correspondence; terms as proofs; conversion as proof normalization

## 2 Lectures Remaining

Coalgebras and State-Based Transition Systems. We will follow [Jacobs 2012, Chapters 1-3]

- 2017.1.23 (Mon)
- 2017.1.30 (Mon)