

Introduction

Mathematical Semantics of Computer Systems

Syntax \leftrightarrow Semantics

- We strive for mathematically clean
semantical models.

- Category theory
图论

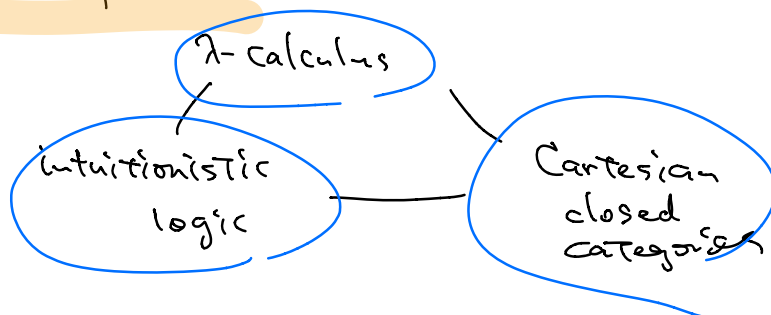
A mathematical language

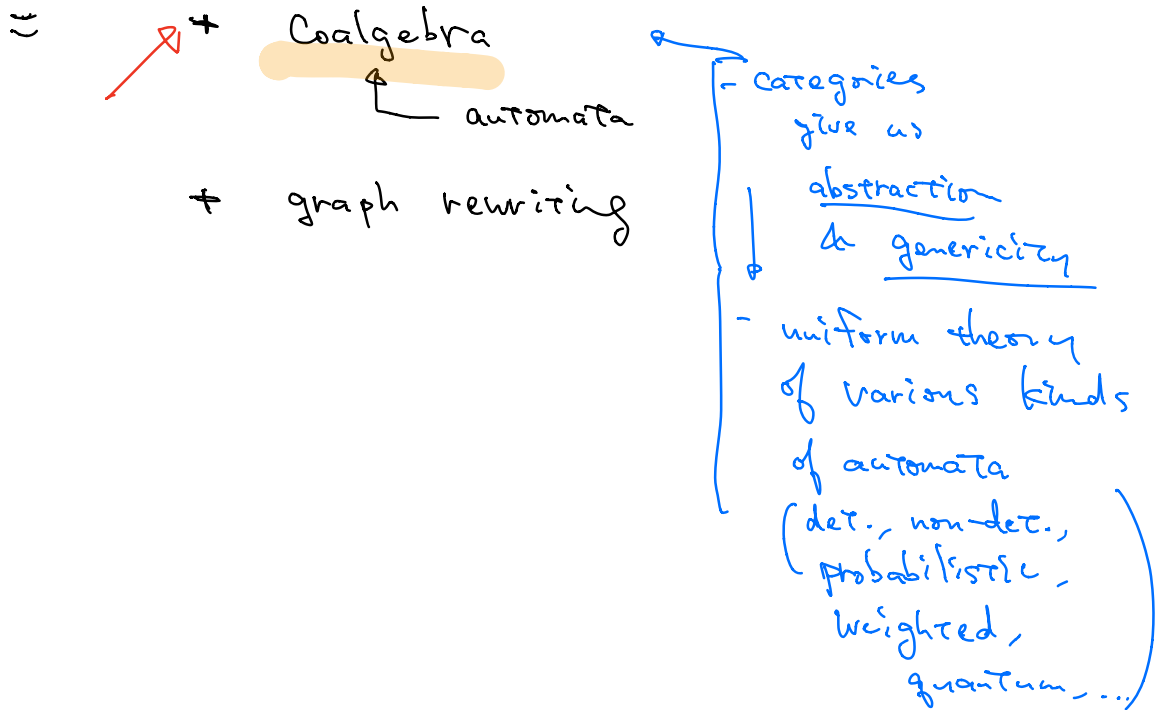
* from algebraic topology, ~'50

* used in many disciplines in
(pure) math.

* used in CS / semantics, in the
following principal ways

→ + the Curry-Howard-Lambek
correspondence





Verification

input

a system model M

a specification φ

(a property to be desired)

output

$M \models \varphi$

"yes", and a mathematical proof for

"no", (and $M \not\models \varphi$)

"satisfies"

hopefully a counter example)

Textbooks

first CT book } [Awodey] Easy to Follow
osp. for logicians & CS people
[Leinster]

[MacLane], Categories for the Working
Mathematician.

[Lambek & Scott],

[Jacobs '12], Intro. to Coalg.

[Jacobs '99] fibration

$$\begin{array}{ccccc} Fx & \longleftarrow & FR & \longrightarrow & FY \\ \uparrow & & \uparrow & & \uparrow \\ X & \longleftarrow & R & \longrightarrow & Y \end{array}$$

Commutative
diagrams
in the language
of CT

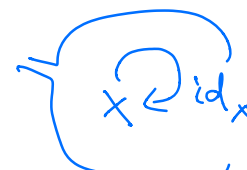
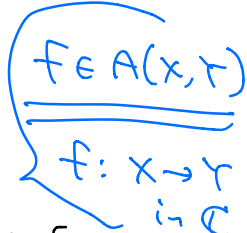
Definition (category)

A category is given by

$$\mathcal{C} = \left(\underbrace{\mathcal{O}, A}_{\text{"sets"}}, \underbrace{(\text{id}_x)_{x \in \mathcal{O}}, (\circ_{x,y,z})_{x,y,z \in \mathcal{O}}}_{\text{"operations"}} \right)$$

where $(A(x,y))_{x,y \in \mathcal{O}}$

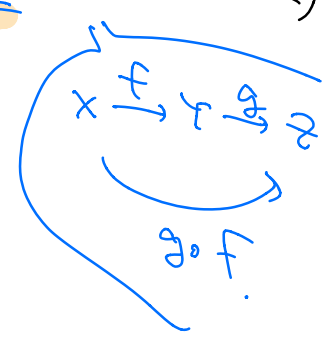
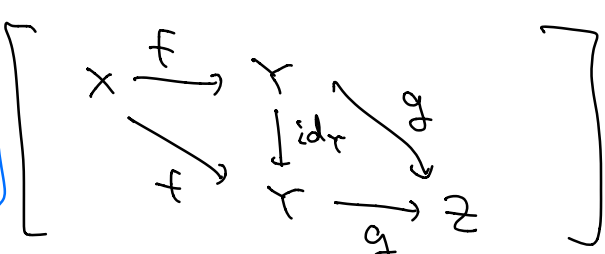
- \mathcal{O} is a "collection" of objects
- $A(x,y)$ is a "collection" of arrows from an object X to another obj. Y
- $\text{id}_x \in A(x,x)$ is called the identity arrow over x , and
- $\circ_{x,y,z} : A(y,z) \times A(x,y) \rightarrow A(x,z)$



composition

subject to

the identity law



equations

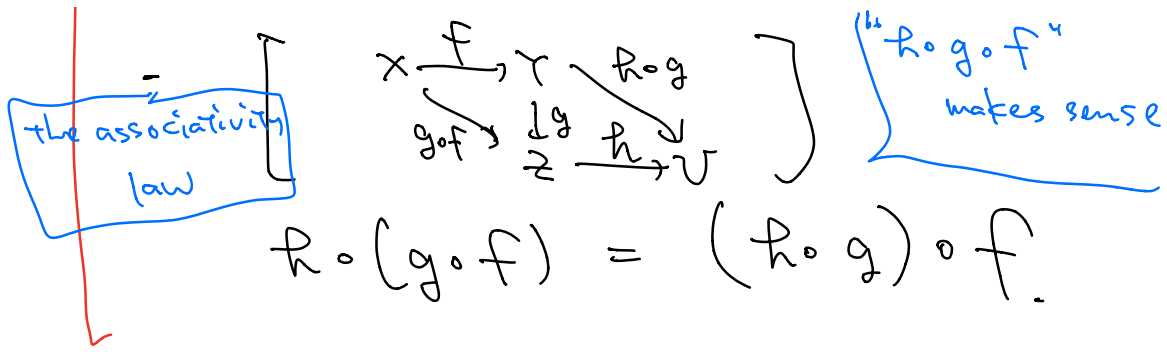
For each $x, y, z \in \mathcal{O}$
 $f \in A(x,y)$
 $g \in A(y,z)$



$$\text{id}_y \circ f = f$$



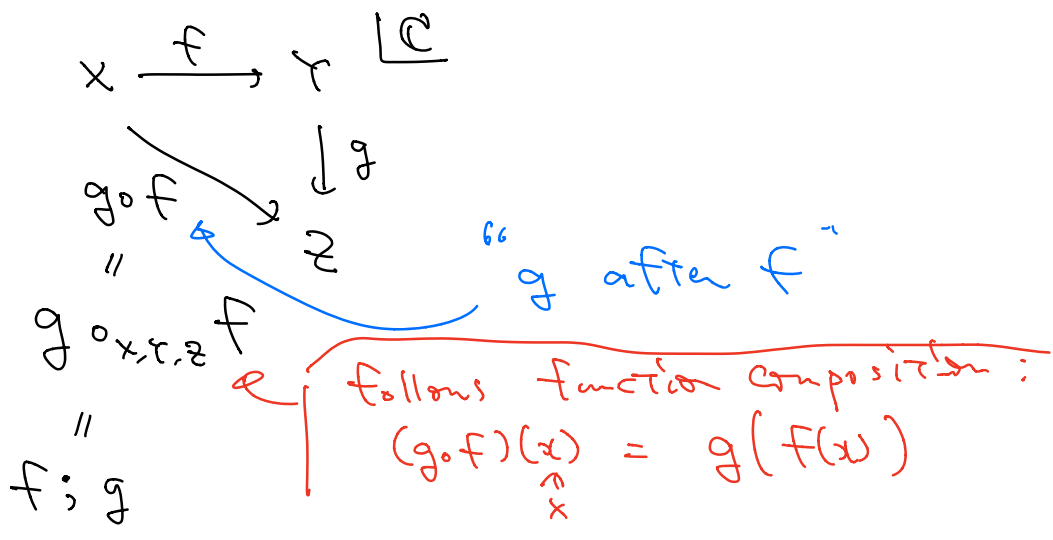
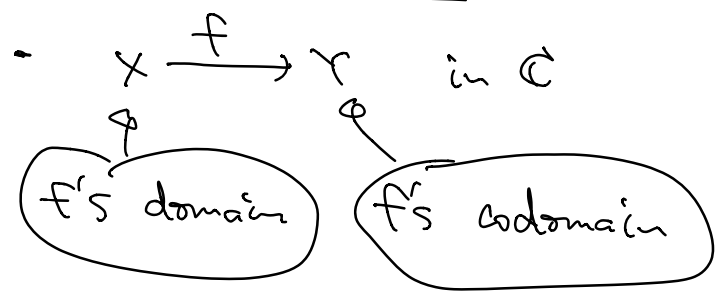
$$g \circ \text{id}_y = g$$



$$h \circ (g \circ f) = (h \circ g) \circ f$$

an algebra = a set
 + operations
 + equations

Some terminologies



Example

- The category Sets of sets and functions,

obj. sets

arr. functions between them.

id. $\text{id}_X : X \rightarrow X$ in Sets

is given by the function

$$\begin{array}{ccc} X & \longrightarrow & X \\ \in & & \in \\ x & \longmapsto & x \end{array}$$

comp. Given $X \xrightarrow{f} Y \xrightarrow{g} Z$ in Sets

we define $g \circ f : X \rightarrow Z$ in Sets

by the function

$$\begin{array}{ccc} X & \longrightarrow & Z \\ \in & & \in \\ x & \longmapsto & g(f(x)) \end{array}$$

NB - $\{\text{sets}\}$ is not a set.

it is a proper class. a small set

