The Power of Abstraction

in a Theory, a Project, and an Inspiring Mind

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Why Categories?

* Why category theory?

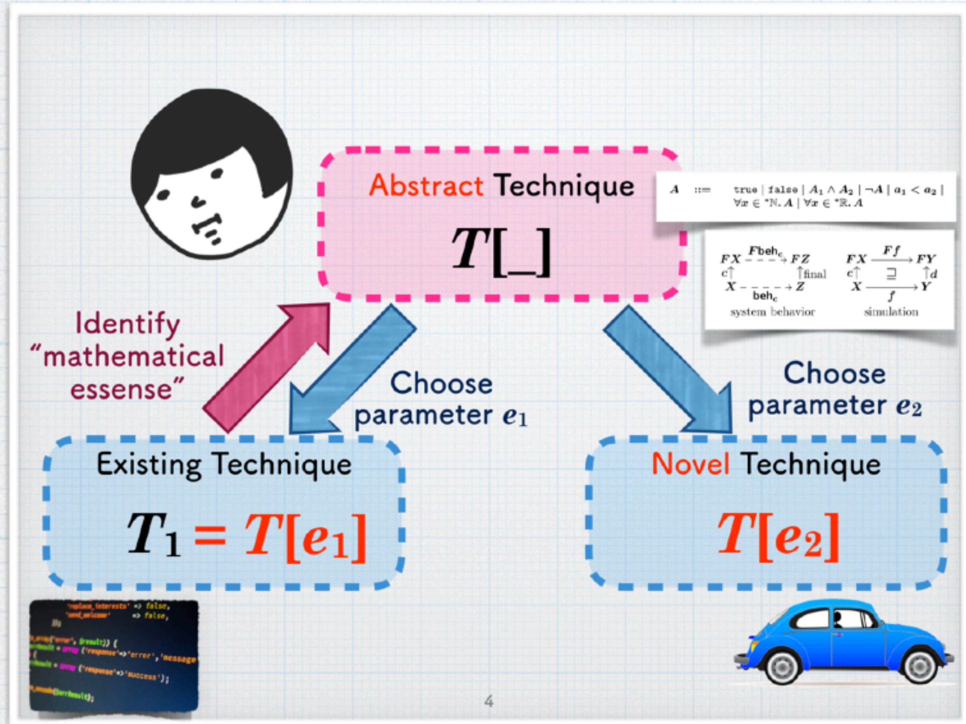
"Unity" as Yoshi talked about (?)

* My answer: for **generalization** & **abstraction**
  * A plain math. language
  * Equipped only with a (typed) **compositional structure**
  * A **nicely extendable** base language: accommodates various additional features (monoidal, topos, locally presentable/accessible, effectus, …)
  * Make **theory builders** aware of what exactly (s)he is using, and thus extract the essence of a theory

"**Scalability** is the end, **compositionality** is the means, **category theory** is the means to the means." (Jules Hedges)

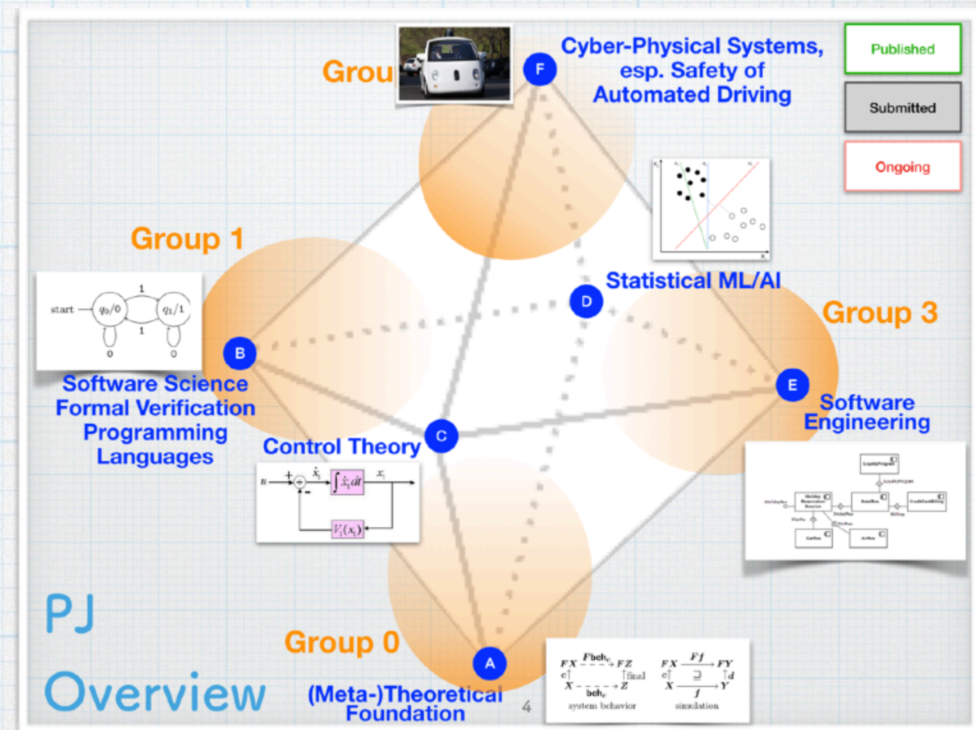
* Then, why **abstraction**??

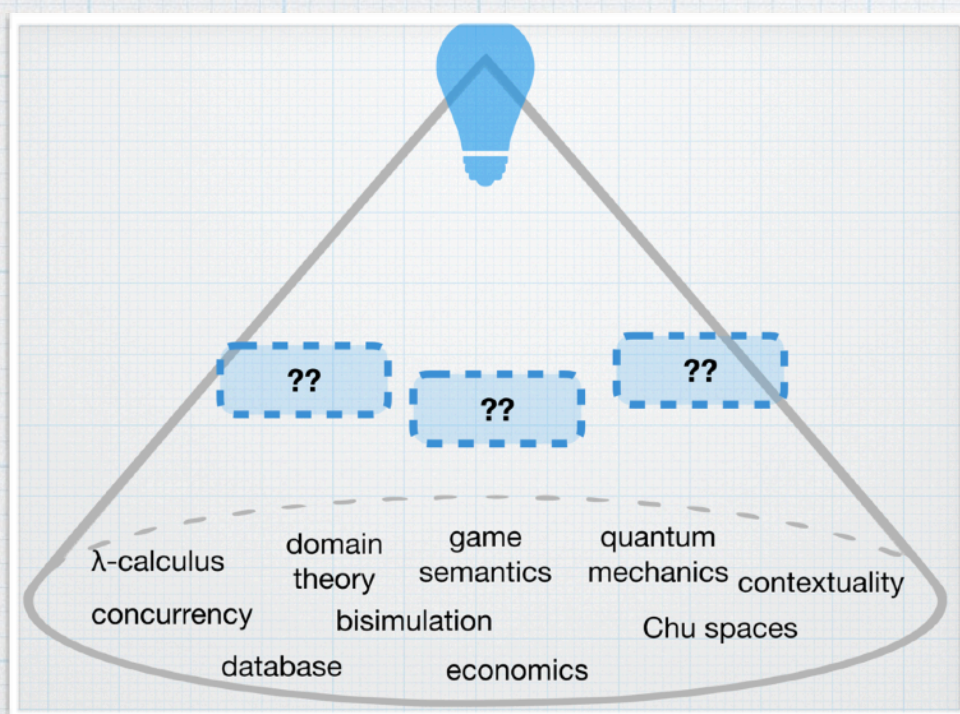
The Power of Abstraction



in a
theory

in a
project





… and
in an
inspiring mind

??

??

??

λ-calculus

domain theory

game semantics

quantum mechanics

contextuality

concurrency

bisimulation

Chu spaces

database

economics

Outline

* **Abstraction in a Theory I**
  Categorical GoI (geometry of interaction)
  [Abramsky, Haghverdi & Scott, MSCS'02] [Hasuo & Hoshino, LICS'11] [Hoshino, Muroya & Hasuo, CSL-LICS'14]
  [Muroya, Hoshino & Hasuo, POPL'16] [Hasuo & Hoshino, APAL'17]

* **Abstraction in a Theory II**
  Codensity Bisimulation Games
  [Katsumata & Sato, CALCO'15] [Sprunger, Katsumata, Dubut & Hasuo, CMCS'18]
  [Komorida, Katsumata, Hu, Klin & Hasuo, submitted (soon on arXiv)]

* **Abstraction in a Project**
  Introducing ERATO MMSD Project:
  from categorical foundations to automated driving

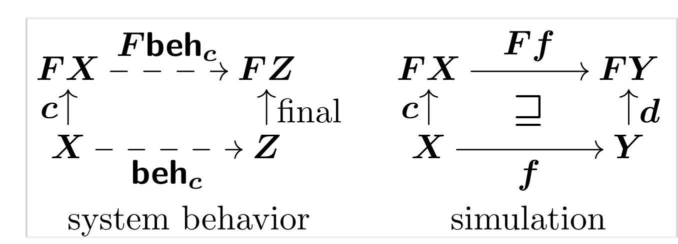
* **Abstraction in an Inspiring Mind**

Abstraction in a Theory:
"Categorical Transfer"

Abstract Technique

$$T[\_]$$

$$A \; ::= \; \mathtt{true} \mid \mathtt{false} \mid A_1 \wedge A_2 \mid \neg A \mid a_1 < a_2 \mid \forall x \in {}^*\mathbb{N}.\, A \mid \forall x \in {}^*\mathbb{R}.\, A$$



Identify "mathematical essense"

Choose parameter e_1

Choose parameter e_2

Existing Technique

$$T_1 = T[e_1]$$

Novel Technique

$$T[e_2]$$

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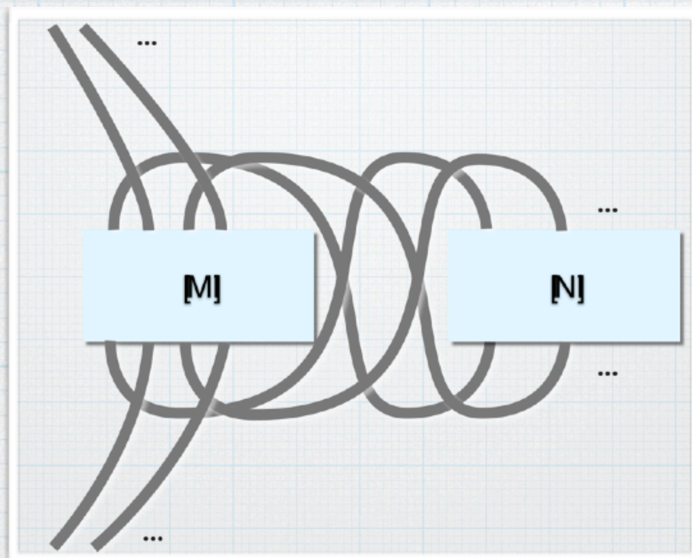
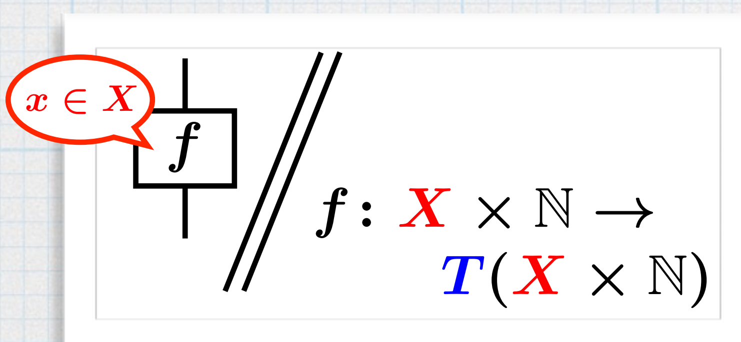
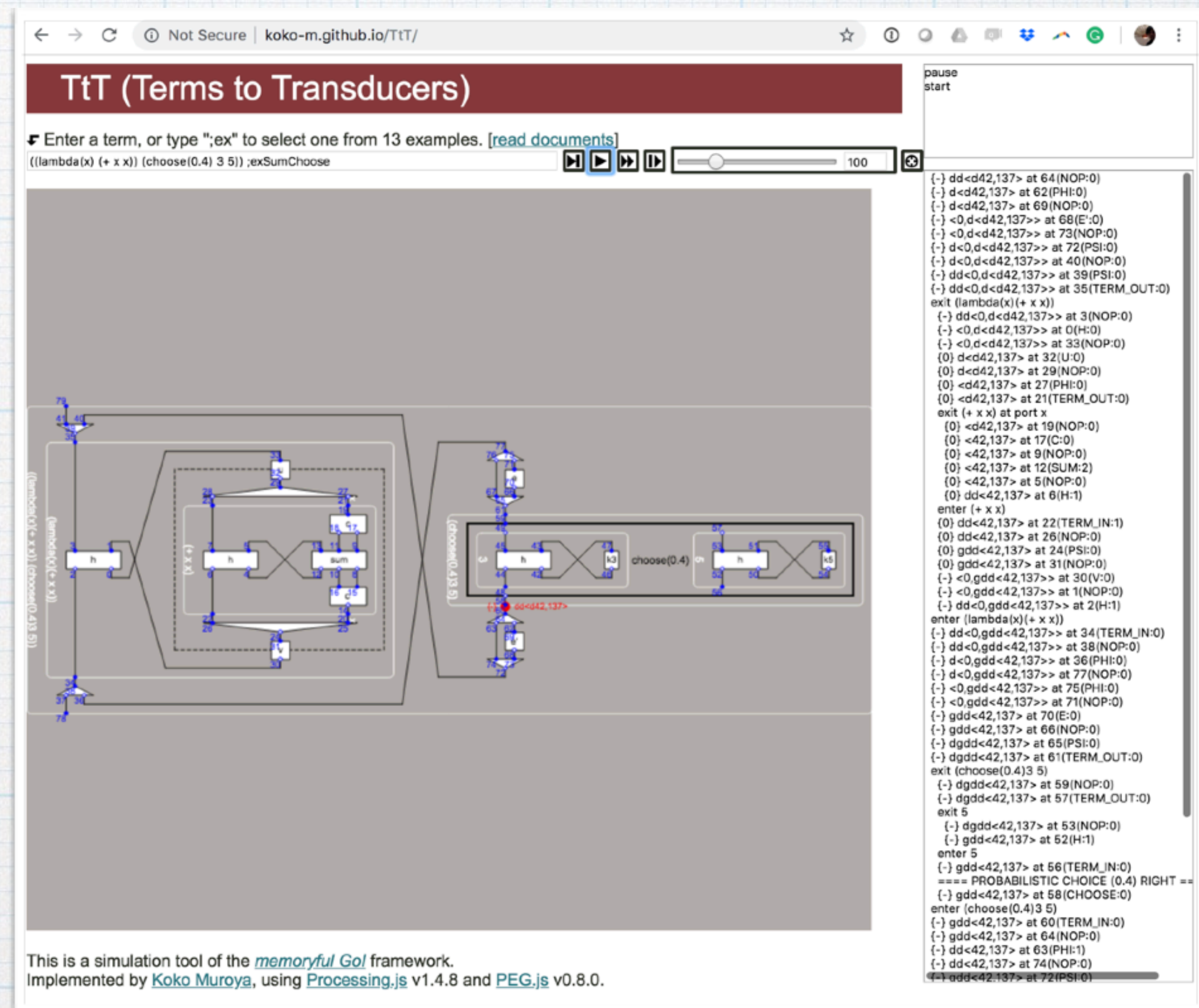
Abstraction in a Theory I
Categorical GoI







[Abramsky, Haghverdi & Scott, MSCS'02]
[Hasuo & Hoshino, LICS'11] [Hoshino, Muroya & Hasuo, CSL-LICS'14]
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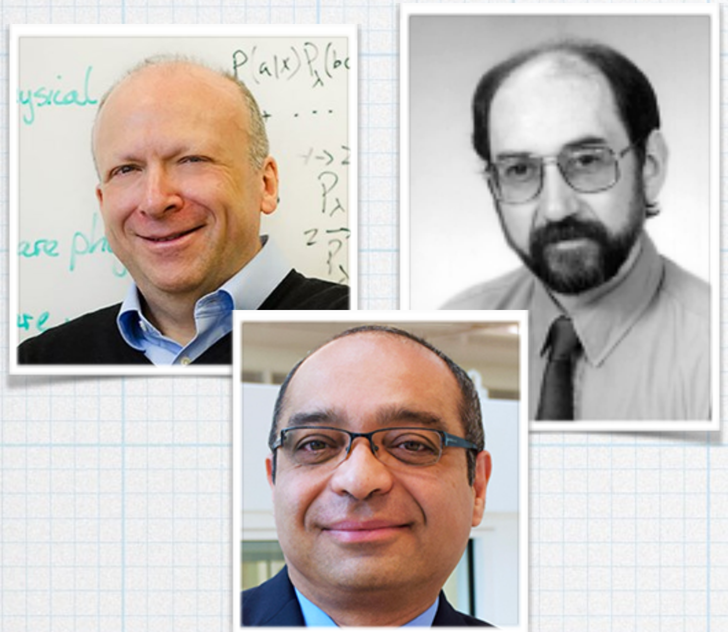

Categorical Geometry of Interaction
[Abramsky, Haghverdi, Scott]

Abstract Technique

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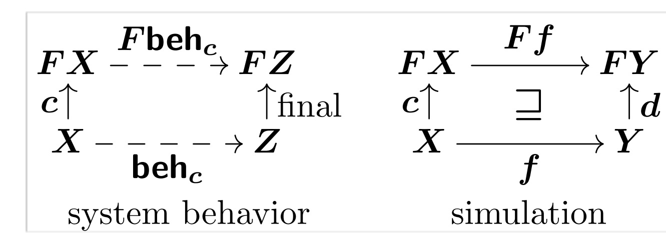

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Choose parameter e_1

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$$T_1 = T[e_1]$$

Geometry of Interaction
[Girard]

Novel Technique

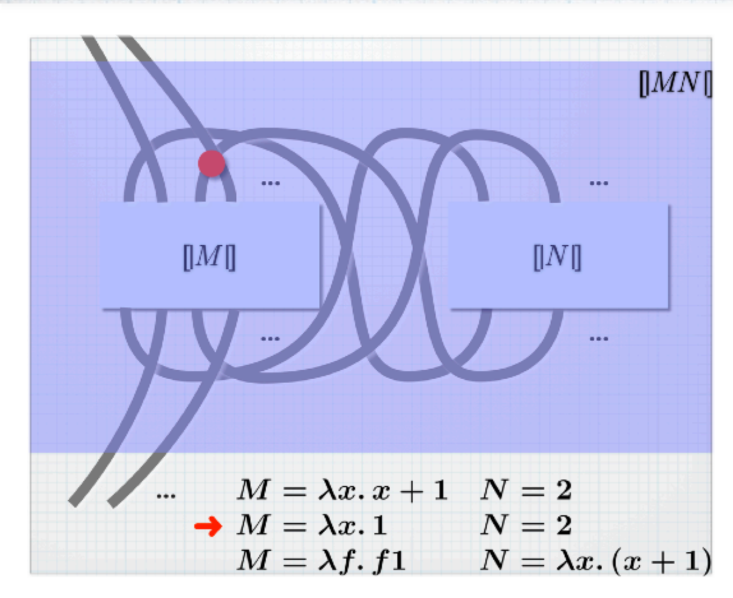
$$T[e_2]$$

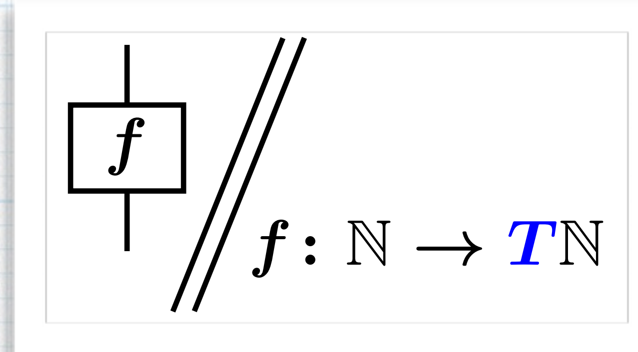
GoI with nondeterministic, probabilistic, quantum, and other effects

··· ➜ Memoryful Geometry of Interaction [Hoshino, Muroya, Hasuo, ···]

Outline

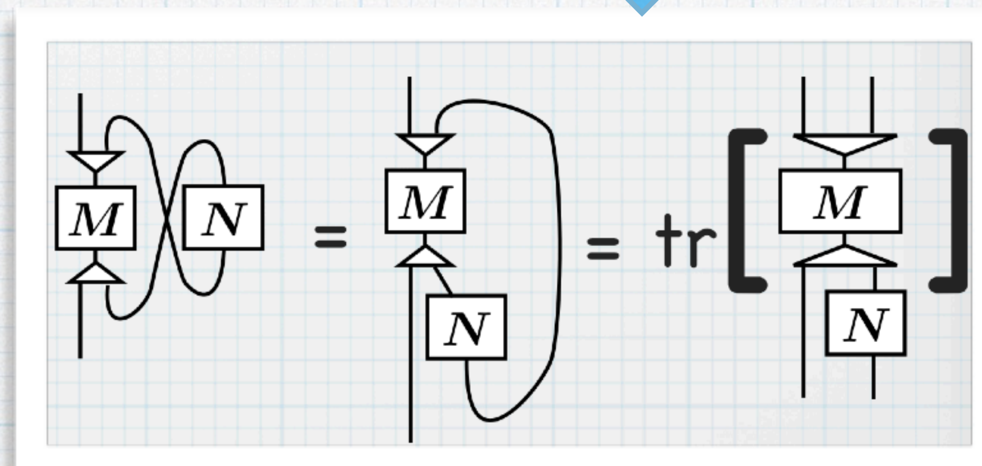
"GoI Animation"





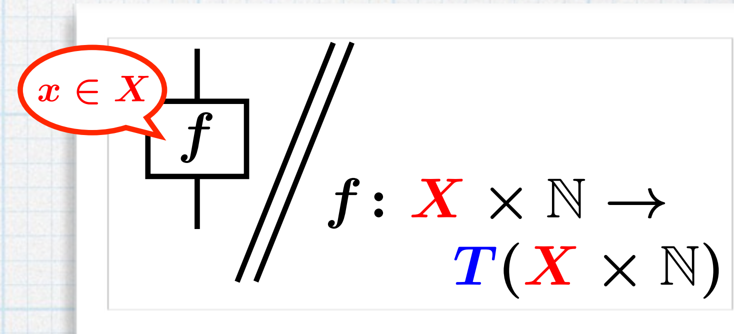
GoI w/ T-branching

[IH & Hoshino, LICS'11]



Categorical GoI

[Abramsky, Haghverdi & Scott, MSCS'02]



Memoryful GoI

[Hoshino, Muroya & IH, CSL-LICS'14 & POPL'16]

Coalgebra meets **higher-order computation** in **Geometry of Interaction** [Girard, LC'88]

Geometry of Interaction (GoI)

- J.-Y. Girard, at Logic Colloquium '88
- Provides **"denotational" semantics** (w/ operational flavor) for linear λ-term M
- As a **compilation technique**

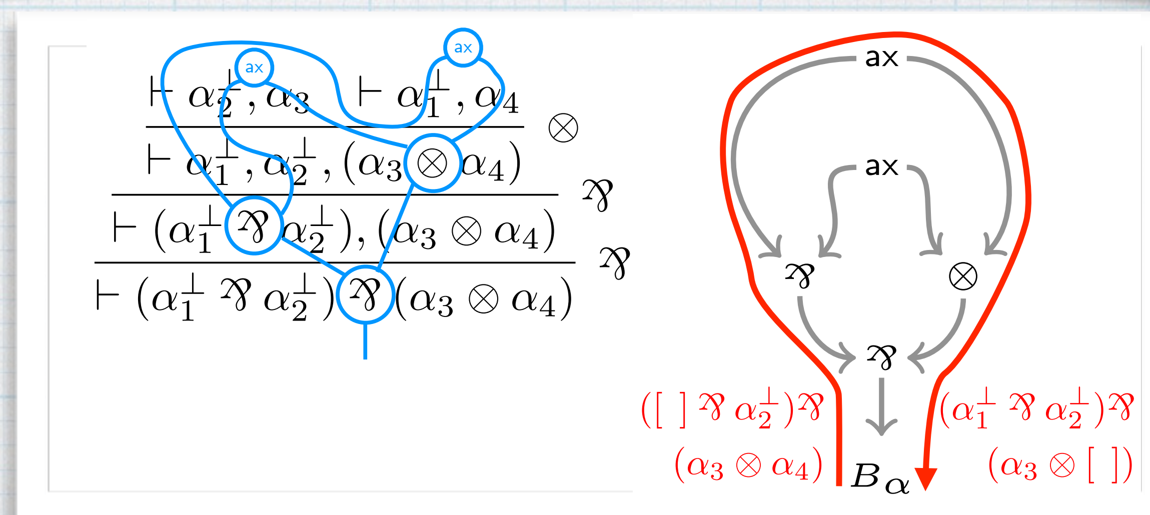
  [Mackie, POPL'95] [Pinto, TLCA'01] [Ghica et al., POPL'07, POPL'11, ICFP'11, ...]

- Two presentations:
  - (Operator-) Algebraic [Girard]
  - Token machines/ interaction abstract machines

    [Danos & Regnier, TCS'99] [Mackie, POPL'95]

$$\dfrac{\dfrac{\dfrac{}{\vdash A,\ A^\perp} \quad \dfrac{}{\vdash A^\perp,\ A}}{\vdash A,\ A^\perp,\ A^\perp \otimes A} \quad \dfrac{\dfrac{}{\vdash A,\ A^\perp}}{\vdash A \parr A^\perp}}{\vdash [A^\perp \otimes A],\ A,\ A^\perp} \qquad \Pi^\bullet = \begin{pmatrix} 0 & 0 & p & q \\ 0 & pq^* + qp^* & 0 & 0 \\ p^* & 0 & 0 & 0 \\ q^* & 0 & 0 & 0 \end{pmatrix} \quad \sigma = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

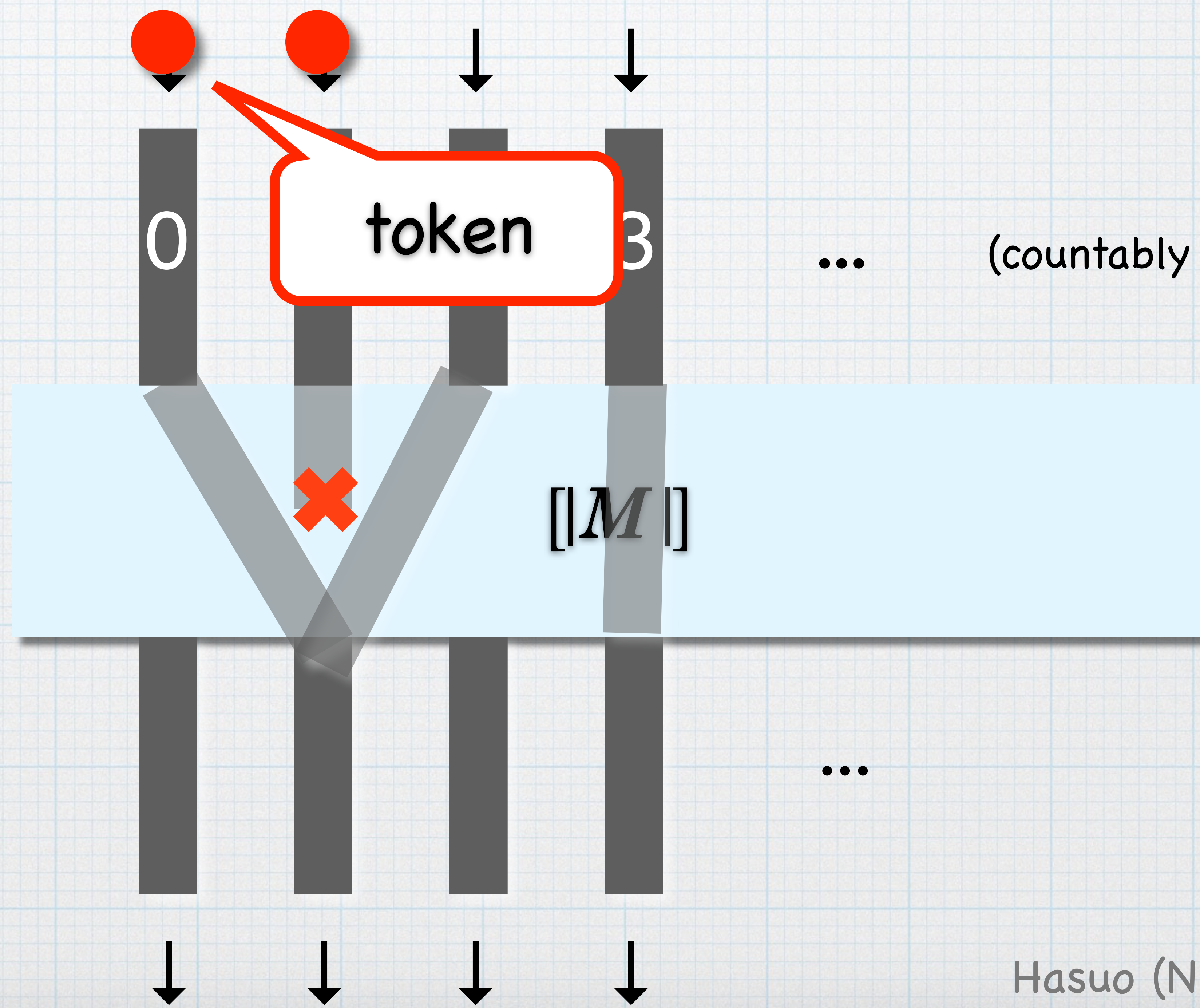
$$(p^*p = q^*q = 1,\ p^*q = q^*p = 0)$$



The GoI Animation

$$[\![M]\!] \;=\; (\,\mathbb{N} \rightharpoonup \mathbb{N},\ \text{a partial function}\,)$$

$$=\;\text{“piping”}$$

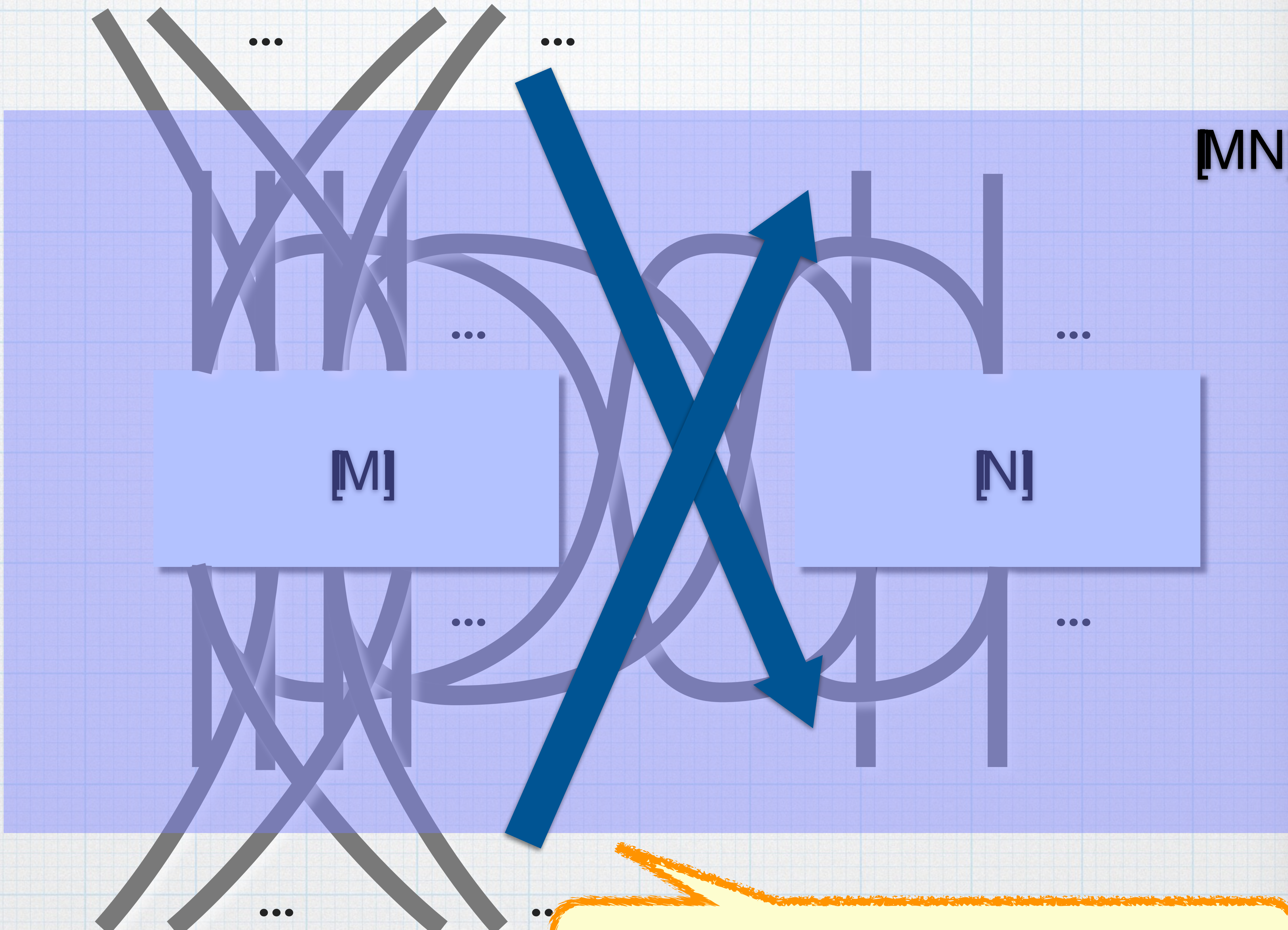


(countably many)

The GoI Animation

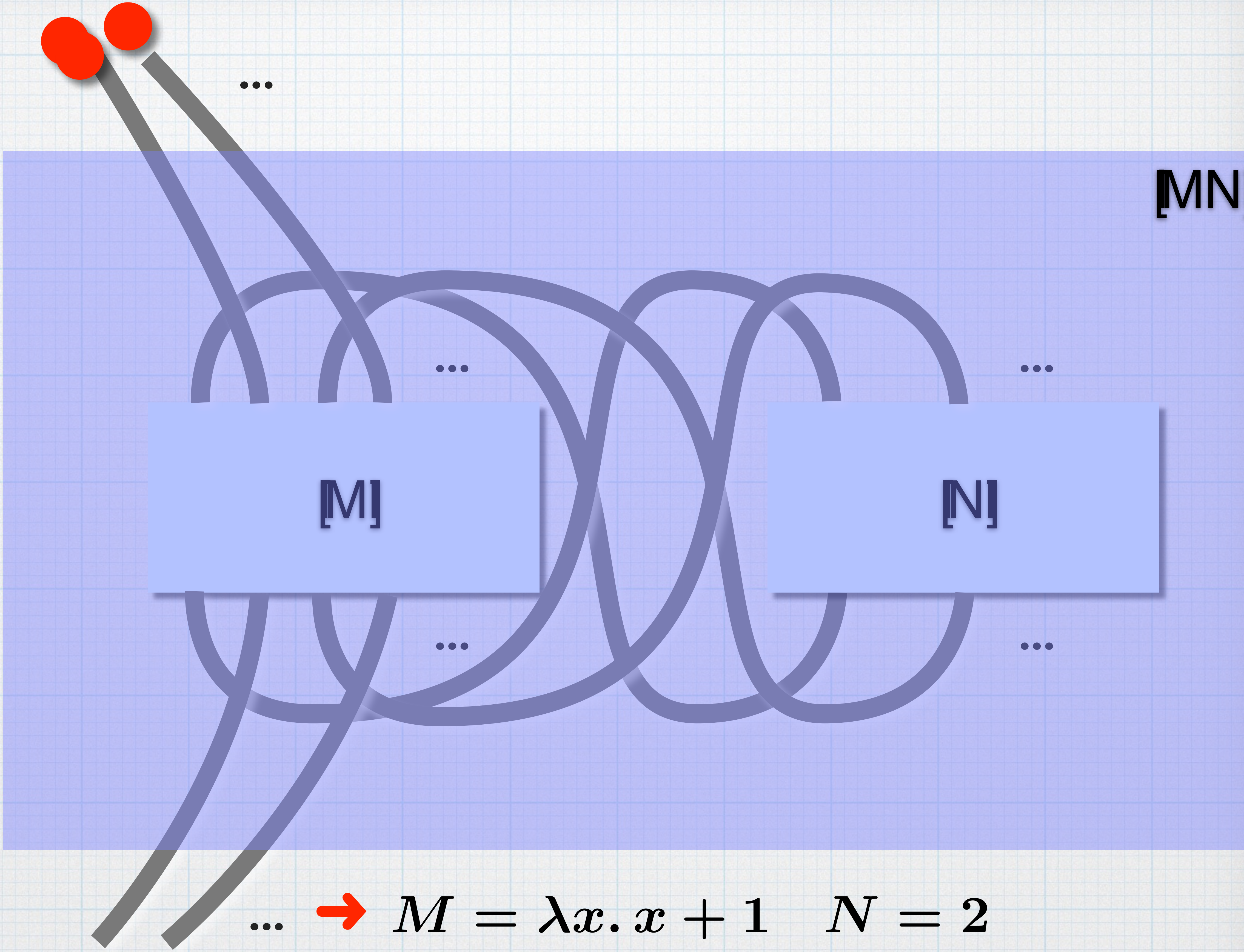
- Function application $[\![M N]\!]$
  - by "parallel composition + hiding"

$$[\![M N]\!] =$$



"parallel composition + hiding"
(cf. AJM games)

$$[\![MN]\!] =$$



$\rightarrow$ $M = \lambda x.\, x+1 \qquad N = 2$

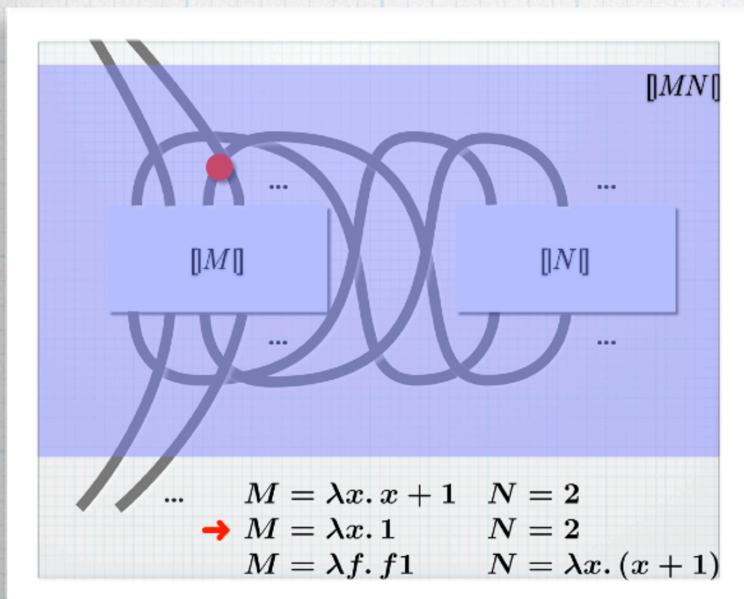
$\rightarrow$ $M = \lambda x.\, 1 \qquad N = 2$

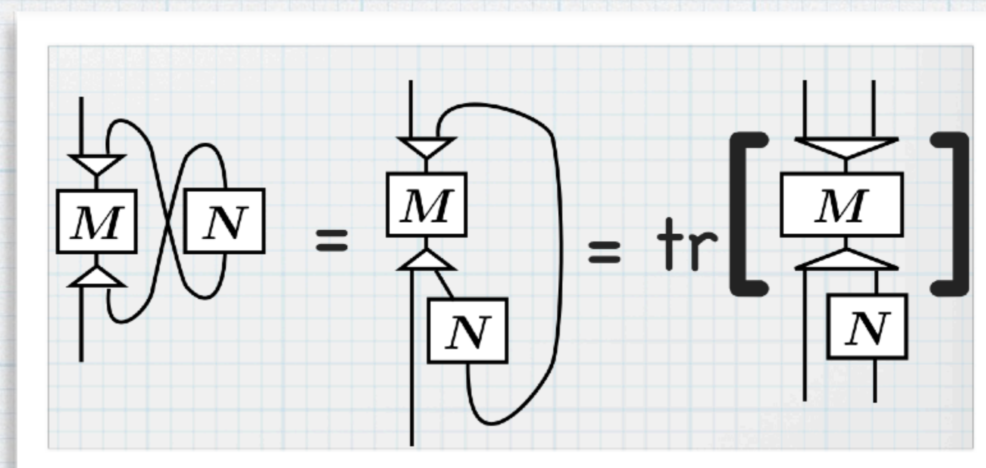
$\rightarrow$ $M = \lambda f.\, f 1 \qquad N = \lambda x.\, (x+1)$

Outline

Coalgebra meets **higher-order computation** in **Geometry of Interaction** [Girard, LC'88]

"GoI Animation"





Categorical GoI

[Abramsky, Haghverdi & Scott, MSCS'02]

Categorical GoI

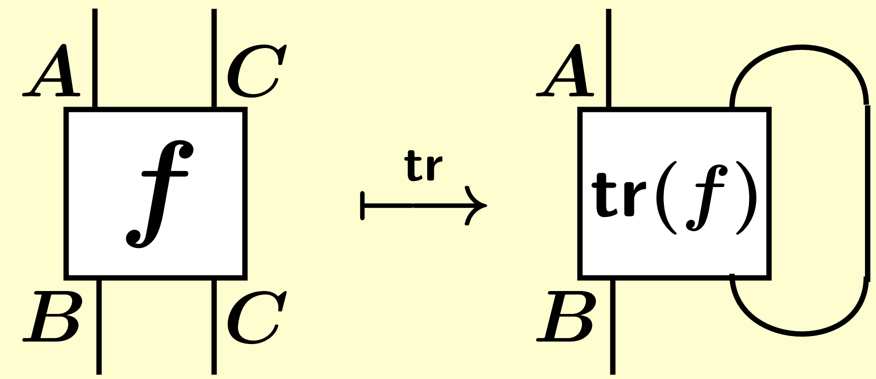
* **Axiomatics of GoI**
  in the categorical language

* Our main reference:
  * [AHS02] S. Abramsky, E. Haghverdi, and P. Scott, **Geometry of interaction and linear combinatory algebras**, Math. Str. Comp. Sci, 2002
  * Especially its technical report version (Oxford CL), since it's a bit more detailed
* See also:
  * IH and Naohiko Hoshino. **Semantics of Higher-Order Quantum Computation via Geometry of Interaction**. Annals Pure & Applied Logic 2017.
    arxiv.org/abs/1605.05079

The Categorical GoI Workflow

Traced monoidal category $\mathbb{C}$

+ other constructs ➜ "GoI situation" [AHS02]



Categorical GoI [AHS02]

Linear combinatory algebra

* Applicative str. + combinators
* Model of untyped calculus

Realizability

* PER, ω-set, assembly, ...
* "Programming in untyped λ"

Linear category

Model of typed calculus

What we use (ingredient)

GoI situation

Defn. (GoI situation [AHS02])
A *GoI situation* is a triple $(\mathbb{C}, \boldsymbol{F}, \boldsymbol{U})$ where

- $\mathbb{C} = (\mathbb{C}, \otimes, \boldsymbol{I})$ is a traced symmetric monoidal category (TSMC);
- $\boldsymbol{F} : \mathbb{C} \to \mathbb{C}$ is a traced symmetric monoidal functor, equipped with the following retractions (which are monoidal natural transformations).

$$
\begin{array}{rcl}
\boldsymbol{e} : \boldsymbol{F}\boldsymbol{F} \lhd \boldsymbol{F} : \boldsymbol{e}' & & \text{Comultiplication} \\
\boldsymbol{d} : \mathbf{id} \lhd \boldsymbol{F} : \boldsymbol{d}' & & \text{Dereliction} \\
\boldsymbol{c} : \boldsymbol{F} \otimes \boldsymbol{F} \lhd \boldsymbol{F} : \boldsymbol{c}' & & \text{Contraction} \\
\boldsymbol{w} : \boldsymbol{K_I} \lhd \boldsymbol{F} : \boldsymbol{w}' & & \text{Weakening}
\end{array}
$$

  Here $\boldsymbol{K_I}$ is the constant functor into the monoidal unit $\boldsymbol{I}$;
- $\boldsymbol{U} \in \mathbb{C}$ is an object (called *reflexive object*), equipped with the following retractions.

$$
\begin{array}{c}
\boldsymbol{j} : \boldsymbol{U} \otimes \boldsymbol{U} \lhd \boldsymbol{U} : \boldsymbol{k} \\
\boldsymbol{I} \lhd \boldsymbol{U} \\
\boldsymbol{u} : \boldsymbol{F}\boldsymbol{U} \lhd \boldsymbol{U} : \boldsymbol{v}
\end{array}
$$

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I \lhd U \\
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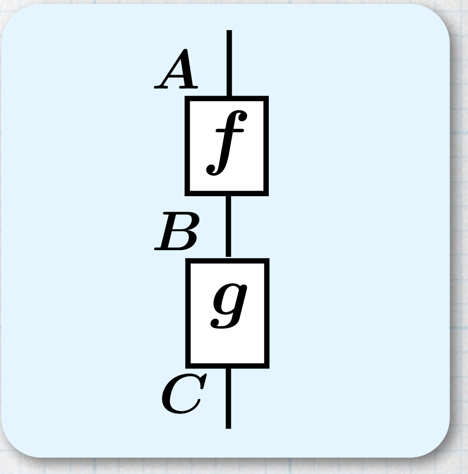
\end{array}
$$

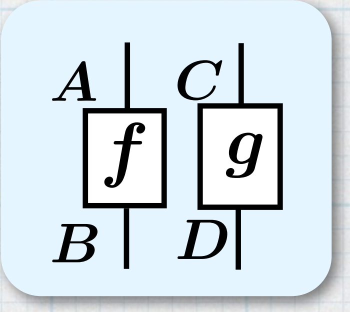
- **Monoidal category** $(\mathbb{C}, \otimes, I)$

  - **String diagrams**

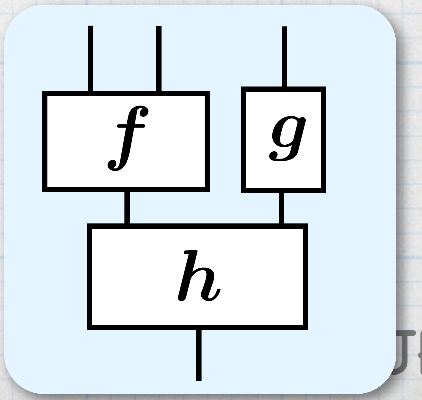
$$\frac{A \xrightarrow{f} B \quad B \xrightarrow{g} C}{A \xrightarrow{g \circ f} C}$$



$$\frac{A \xrightarrow{f} B \quad C \xrightarrow{g} D}{A \otimes C \xrightarrow{f \otimes g} B \otimes D}$$



$$h \circ (f \otimes g)$$



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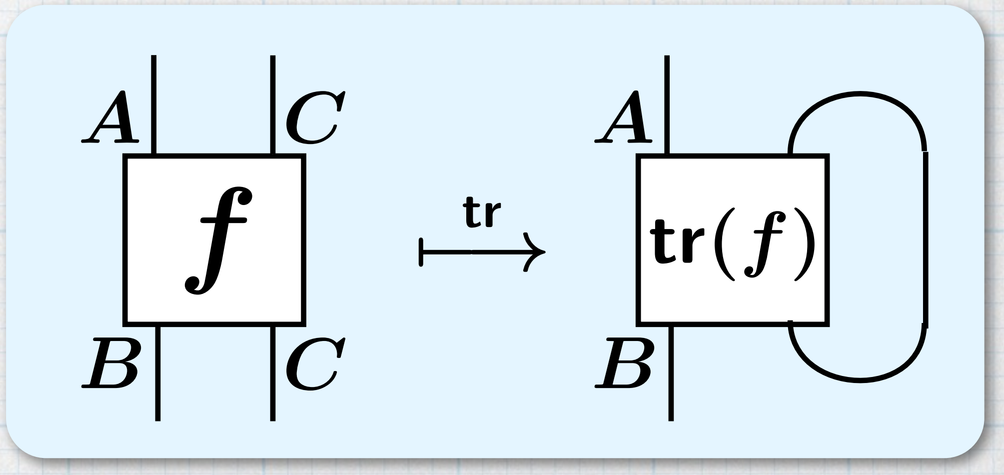
* **Traced** **monoidal category**
  * **"feedback"**

$$\frac{A \otimes C \xrightarrow{f} B \otimes C}{A \xrightarrow{\mathsf{tr}(f)} B}$$

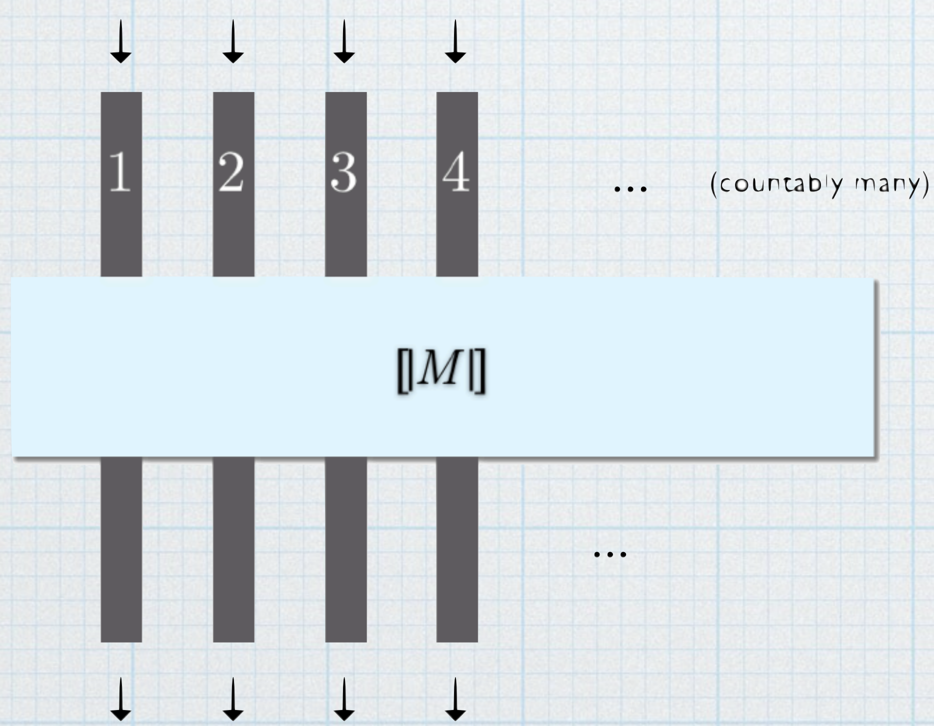
that is



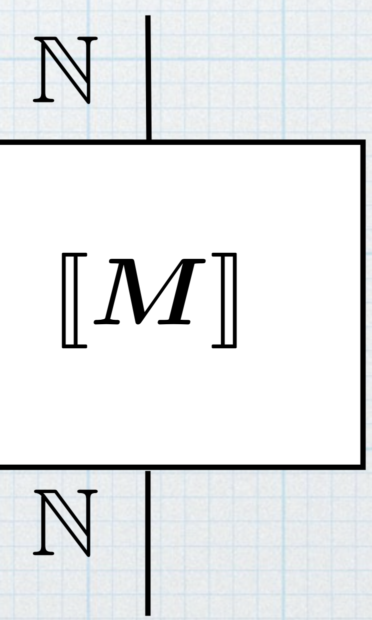
String Diagram vs. "Pipe Diagram"

* I use two ways of depicting partial functions $\mathbb{N} \rightharpoonup \mathbb{N}$

In the monoidal category $(\mathbf{Pfn}, +, 0)$



Pipe diagram

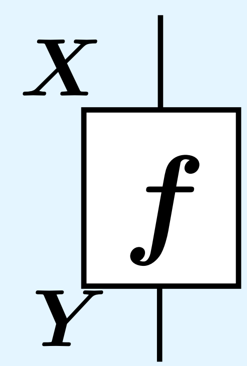


String diagram

Traced Sym. Monoidal Category $(\mathbf{Pfn}, +, 0)$

* Category **Pfn** of **partial functions**
  * <u>**Obj.**</u> A set X
  * <u>**Arr.**</u> A partial function

$$\frac{\boldsymbol{X} \to \boldsymbol{Y} \ \text{ in } \mathbf{Pfn}}{\boldsymbol{X} \rightharpoonup \boldsymbol{Y}, \text{ partial function}}$$

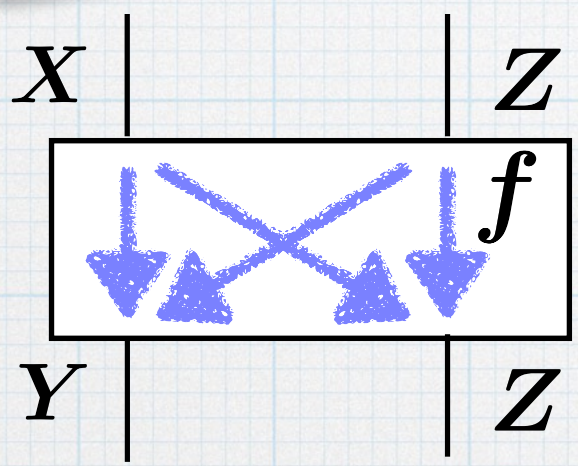


* is traced symmetric monoidal

Traced Sym. Monoidal Category $(\mathbf{Pfn}, +, 0)$

- $$\frac{X + Z \xrightarrow{f} Y + Z \quad \text{in } \mathbf{Pfn}}{X \xrightarrow{\mathrm{tr}(f)} Y \quad \text{in } \mathbf{Pfn}}$$

How?

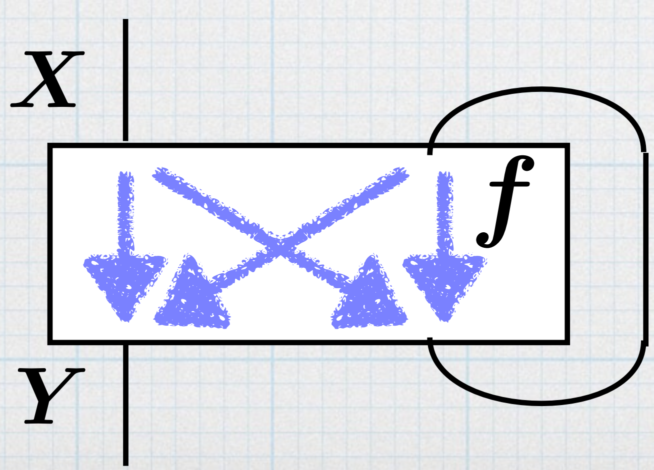
- 

$$f_{XY}(x) := \begin{cases} f(x) & \text{if } f(x) \in Y \\ \bot & \text{o.w.} \end{cases}$$

Similar for f_{XZ}, f_{ZY}, f_{ZZ}

- **Execution formula** (Girard)
- Partiality is essential (infinite loop)

- Trace operator:



$$\mathrm{tr}(f) = f_{XY} \sqcup \left(\coprod_{n \in \mathbb{N}} f_{ZY} \circ (f_{ZZ})^n \circ f_{XZ} \right)$$

GoI situation

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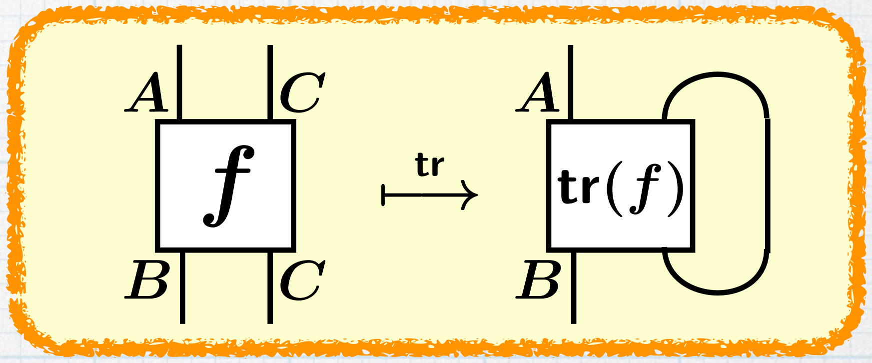
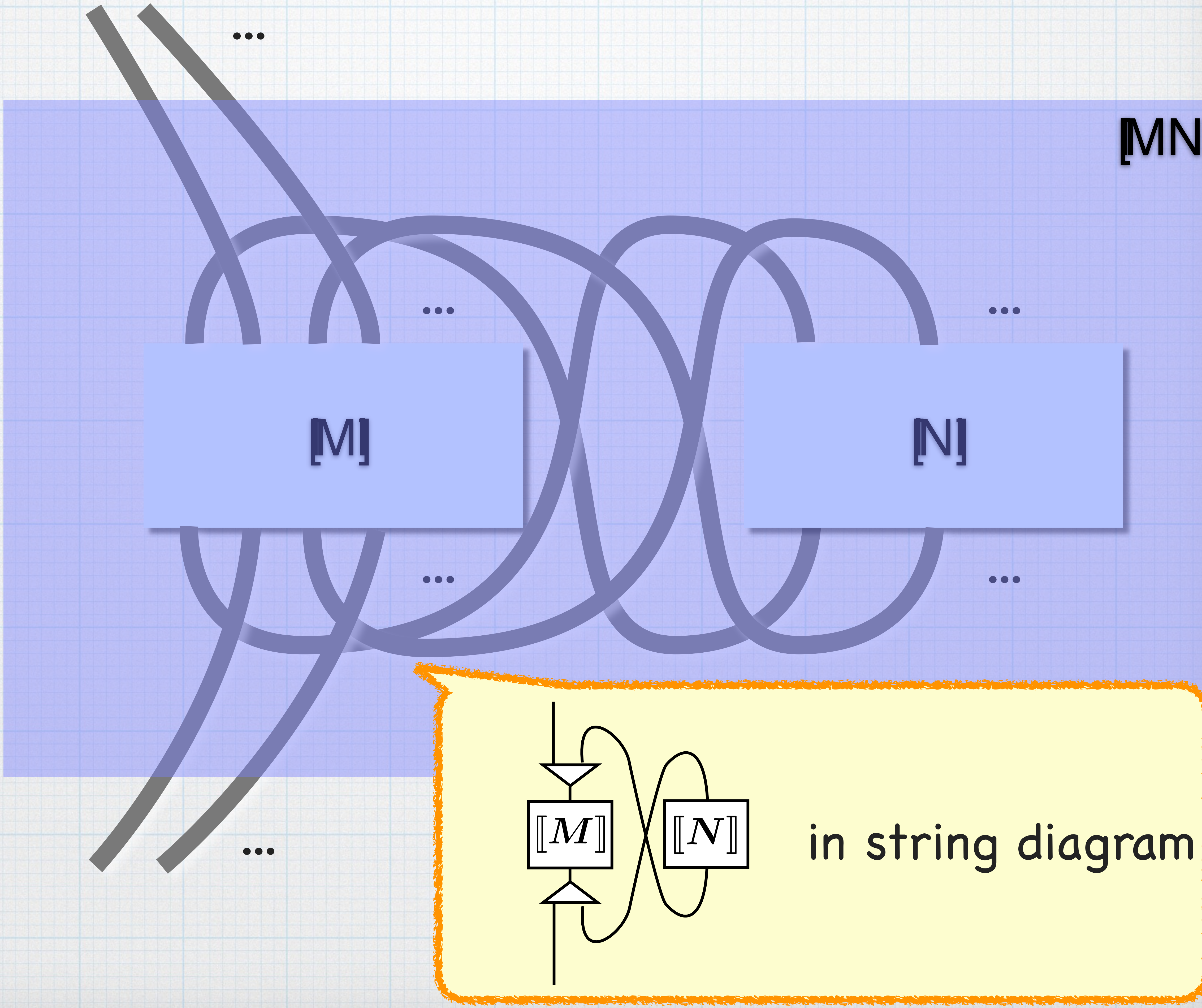
- Traced sym. monoidal cat.
  - Where one can "feedback"



  - Why for GoI?

$\llbracket MN \rrbracket$

$=$



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$w : K_I \lhd F : w'$	Weakening

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$$I \lhd U$$
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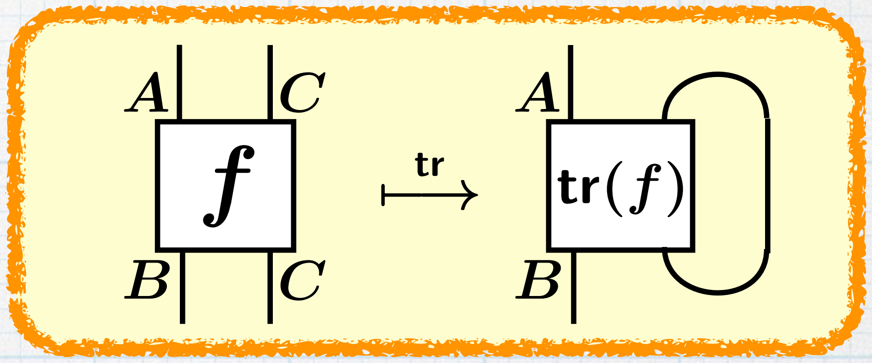
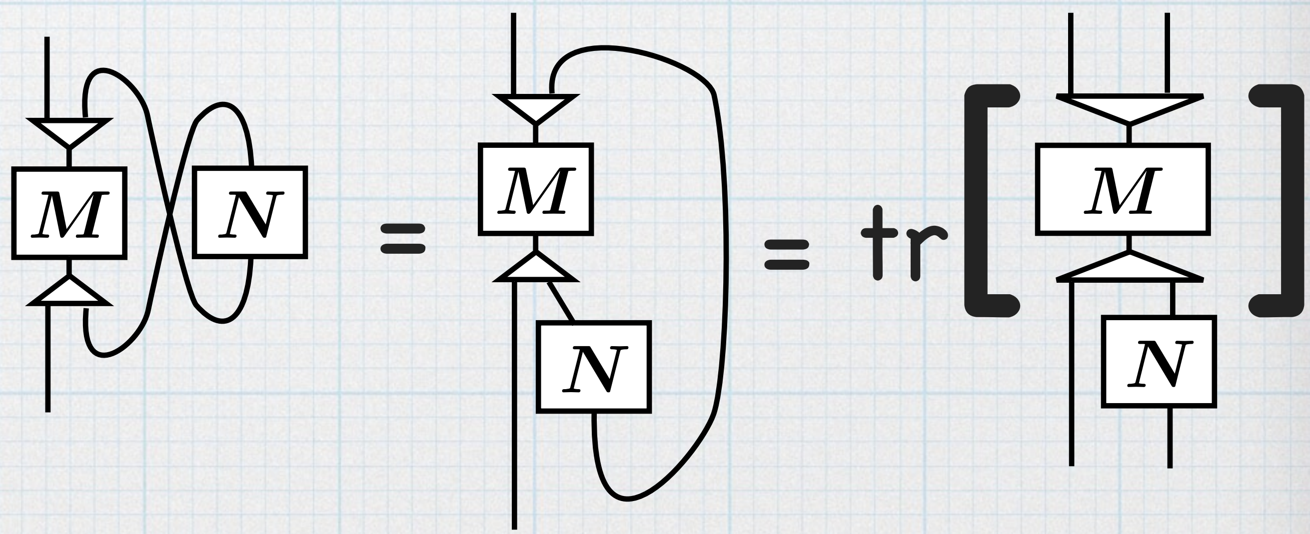
* Traced sym. monoidal cat.
  * Where one can "feedback"



* Why for GoI?



* Leading example: **Pfn**

GoI situation

Defn. (GoI situation [AHS02])
A *GoI situation* is a triple $(\mathbb{C}, F, U)$ where

- $\mathbb{C} = (\mathbb{C}, \otimes, I)$ is a traced symmetric monoidal category (TSMC);
- $F : \mathbb{C} \to \mathbb{C}$ is a traced symmetric monoidal functor, equipped with the following retractions (which are monoidal natural transformations).

$$e : FF \lhd F : e' \quad \text{Comultiplication}$$
$$d : \mathrm{id} \lhd F : d' \quad \text{Dereliction}$$
$$c : F \otimes F \lhd F : c' \quad \text{Contraction}$$
$$w : K_I \lhd F : w' \quad \text{Weakening}$$

Here K_I is the constant functor into the monoidal unit I;

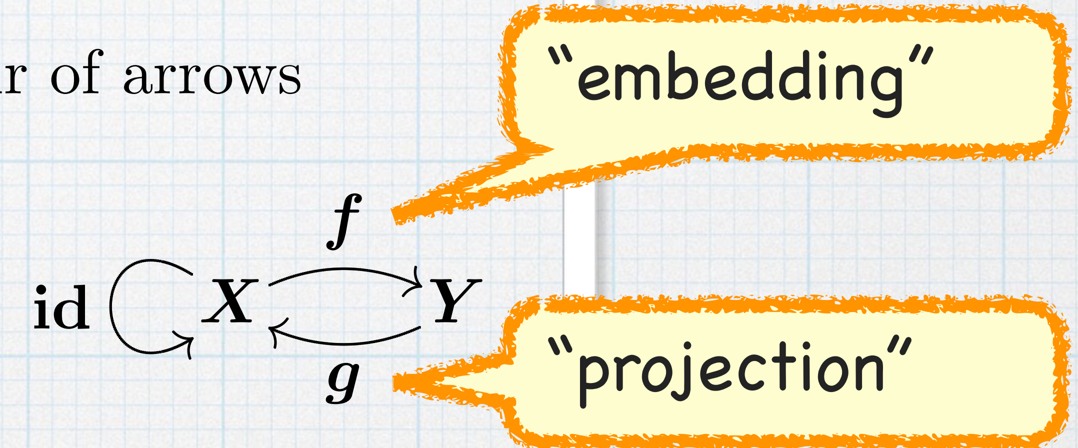
- $U \in \mathbb{C}$ is an object (called *reflexive object*), equipped with the following retractions.

$$j : U \otimes U \lhd U : k$$
$$I \lhd U$$
$$u : FU \lhd U : v$$

Defn. (Retraction)
A *retraction* from X to Y,

$$f : X \lhd Y : g\ ,$$

is a pair of arrows



such that $g \circ f = \mathrm{id}_X$.

- Functor F
- For obtaining $! : A \to A$

GoI situation

Defn. (GoI situation [AHS02])
A *GoI situation* is a triple $(\mathbb{C}, \boldsymbol{F}, \boldsymbol{U})$ where

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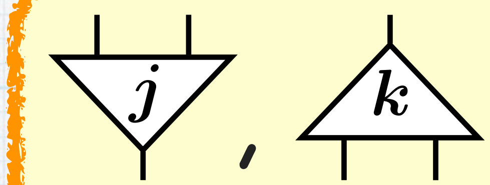
$$\begin{array}{rcll} \boldsymbol{e} : \boldsymbol{FF} & \lhd & \boldsymbol{F} : \boldsymbol{e'} & \text{Comultiplication} \\ \boldsymbol{d} : \mathbf{id} & \lhd & \boldsymbol{F} : \boldsymbol{d'} & \text{Dereliction} \\ \boldsymbol{c} : \boldsymbol{F} \otimes \boldsymbol{F} & \lhd & \boldsymbol{F} : \boldsymbol{c'} & \text{Contraction} \\ \boldsymbol{w} : \boldsymbol{K_I} & \lhd & \boldsymbol{F} : \boldsymbol{w'} & \text{Weakening} \end{array}$$

  Here $\boldsymbol{K_I}$ is the constant functor into the monoidal unit $\boldsymbol{I}$;
- $\boldsymbol{U} \in \mathbb{C}$ is an object (called *reflexive object*), equipped with the following retractions.

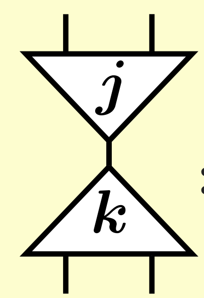
$$\begin{array}{c} \boldsymbol{j} : \boldsymbol{U} \otimes \boldsymbol{U} \lhd \boldsymbol{U} : \boldsymbol{k} \\ \boldsymbol{I} \lhd \boldsymbol{U} \\ \boldsymbol{u} : \boldsymbol{FU} \lhd \boldsymbol{U} : \boldsymbol{v} \end{array}$$

* The reflexive object U

* Retr. $\boldsymbol{U} \otimes \boldsymbol{U} \underset{\boldsymbol{k}}{\overset{\boldsymbol{j}}{\rightleftarrows}} \boldsymbol{U}$



with



= id

GoI situation

Defn. (GoI situation [AHS02])
A *GoI situation* is a triple $(\mathbb{C}, \boldsymbol{F}, \boldsymbol{U})$ where

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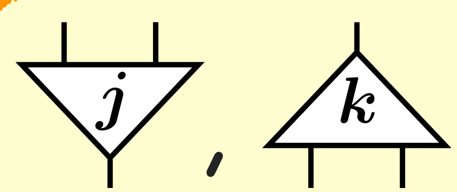
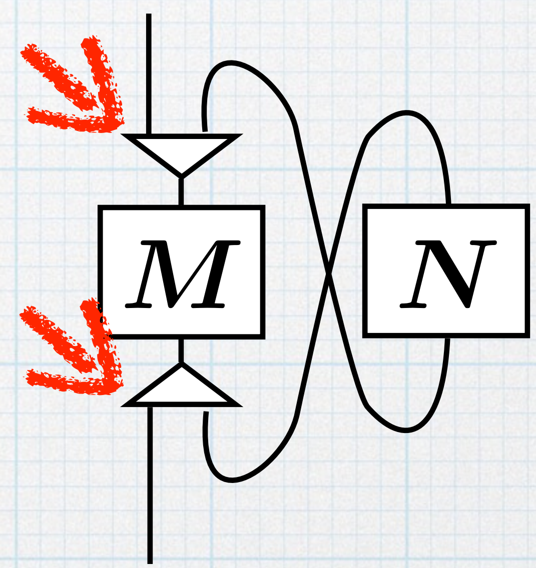


Here K_I is the constant functo

- $U \in \mathbb{C}$ is an object (called *reflexive* *object*), equipped with the following retractions.

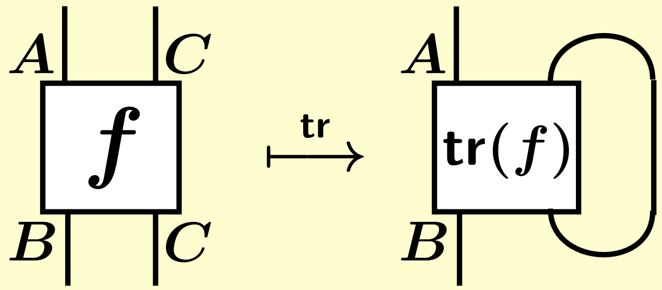
$$\begin{aligned} j &: U \otimes U \lhd U : k \\ & I \lhd U \\ u &: FU \lhd U : v \end{aligned}$$

- The reflexive object U
- Why for GoI?



- Example in **Pfn**:

$$\mathbb{N} \in \mathbf{Pfn}, \text{ with } \quad \mathbb{N} + \mathbb{N} \cong \mathbb{N}, \quad \mathbb{N} \cdot \mathbb{N} \cong \mathbb{N}$$



tuation: Summary

Defn. (GoI situation [AHS02])
A *GoI situation* is a triple $(\mathbb{C}, F, U)$ wh… e

- $\mathbb{C} = (\mathbb{C}, \otimes, I)$ is a traced symmetric monoidal category (TSMC);
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$$w : K_I \lhd F : w' \qquad \text{We}$$

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- $U \in \mathbb{C}$ is an object (called *reflexive obj…*), … the following retractions.

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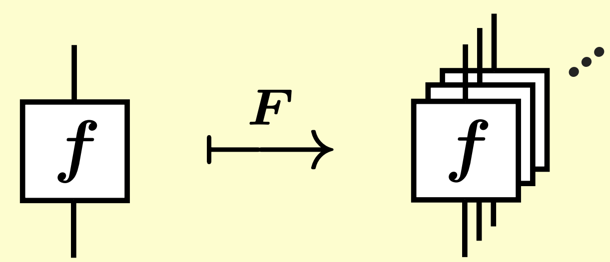
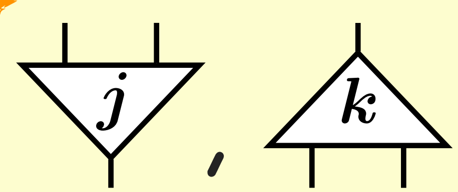
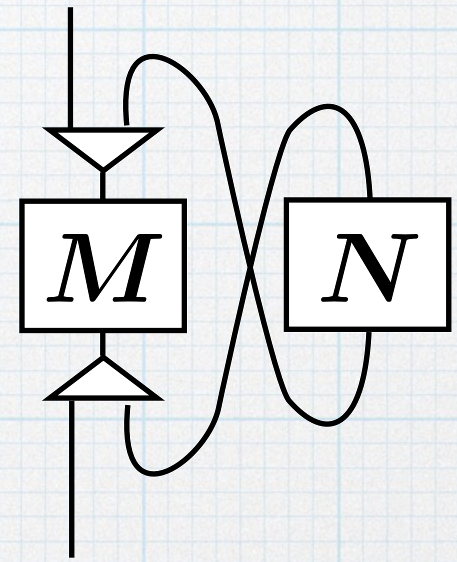
For ! , via



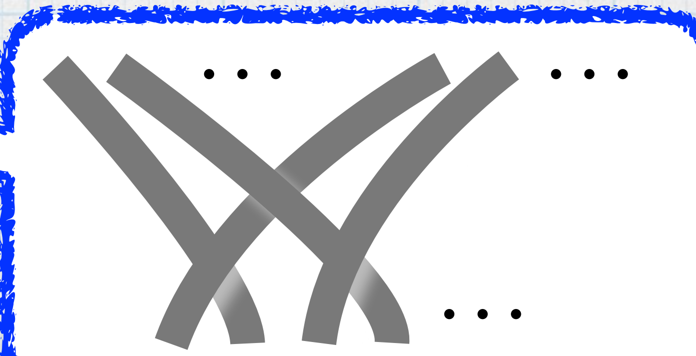


- Categorical axiomatics of the "GoI animation"



- Example:

$(\mathbf{Pfn}, \mathbb{N} \cdot \_\,, \mathbb{N})$



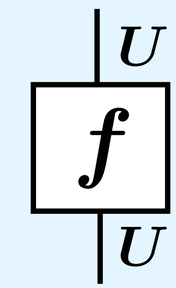
Categorical GoI: Constr. of an LCA

Thm. ([AHS02])
Given a GoI situation $(\mathbb{C}, F, U)$, the homset

$$\mathbb{C}(U, U)$$

carries a canonical LCA structure.

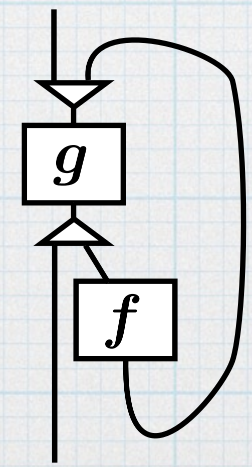
- Applicative str. $\cdot$
- ! operator
- Combinators B, C, I, ...

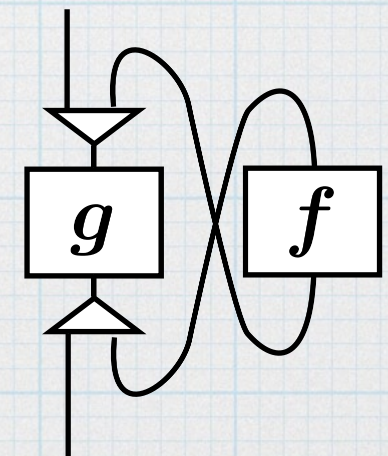


$\in \mathbb{C}(U, U)$

- $g \cdot f$

$$:= \mathbf{tr}\big((U \otimes f) \circ k \circ g \circ j \big)$$

$=$


$=$


Summary: Categorical GoI

Defn. (GoI situation [AHS02])
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$$
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\boldsymbol{c} : \boldsymbol{F} \otimes \boldsymbol{F} & \lhd & \boldsymbol{F} : \boldsymbol{c}' & \text{Contraction} \\
\boldsymbol{w} : \boldsymbol{K_I} & \lhd & \boldsymbol{F} : \boldsymbol{w}' & \text{Weakening}
\end{array}
$$

  Here $\boldsymbol{K_I}$ is the constant functor into the monoidal unit $\boldsymbol{I}$;

- $\boldsymbol{U} \in \mathbb{C}$ is an object (called *reflexive object*), equipped with the following retractions.

$$
\begin{array}{rcl}
\boldsymbol{j} : \boldsymbol{U} \otimes \boldsymbol{U} & \lhd & \boldsymbol{U} : \boldsymbol{k} \\
\boldsymbol{I} & \lhd & \boldsymbol{U} \\
\boldsymbol{u} : \boldsymbol{FU} & \lhd & \boldsymbol{U} : \boldsymbol{v}
\end{array}
$$

Thm. ([AHS02])
Given a GoI situation $(\mathbb{C}, \boldsymbol{F}, \boldsymbol{U})$, the homset

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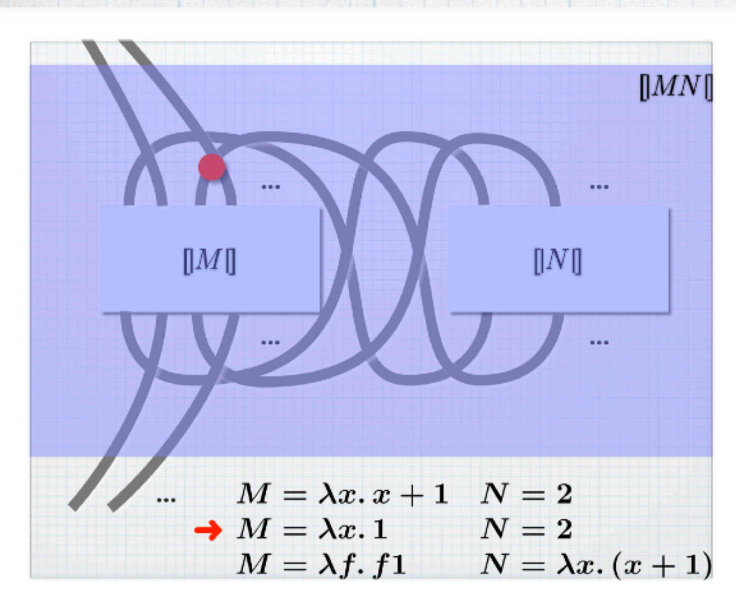
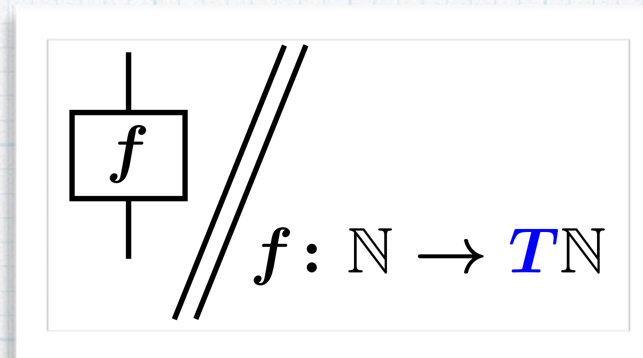
carries a canonical LCA structure.

Outline

Coalgebra meets **higher-order computation** in **Geometry of Interaction** [Girard, LC'88]

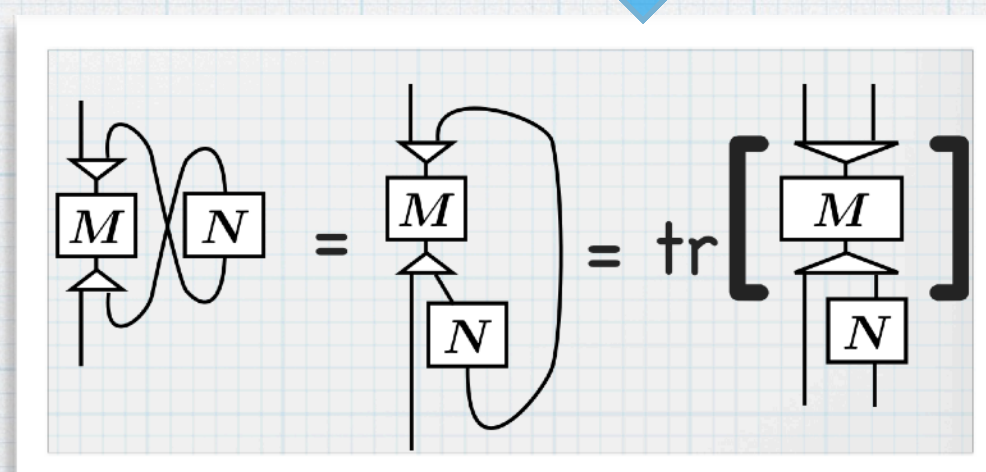
"GoI Animation"





GoI w/ T-branching

[IH & Hoshino, LICS'11]



Categorical GoI

[Abramsky, Haghverdi & Scott, MSCS'02]

Why Catego
Examples

$Kl(T)$ for different branching monads T

(Potential) non-termination

* **Pfn** (partial functions)

$$\dfrac{\dfrac{X \to Y \text{ in } \mathbf{Pfn}}{X \rightharpoonup Y, \text{ partial function}}}{X \to \mathcal{L}Y \text{ in } \mathbf{Sets}} \quad \text{where } \mathcal{L}Y = \{\bot\} + Y$$

Non-determinism

* **Rel** (relations)

$$\dfrac{\dfrac{X \to Y \text{ in } \mathbf{Rel}}{R \subseteq X \times Y, \text{ relation}}}{X \to \mathcal{P}Y \text{ in } \mathbf{Sets}} \quad \text{where } \mathcal{P} \text{ is the powerset monad}$$

Probabilistic branching

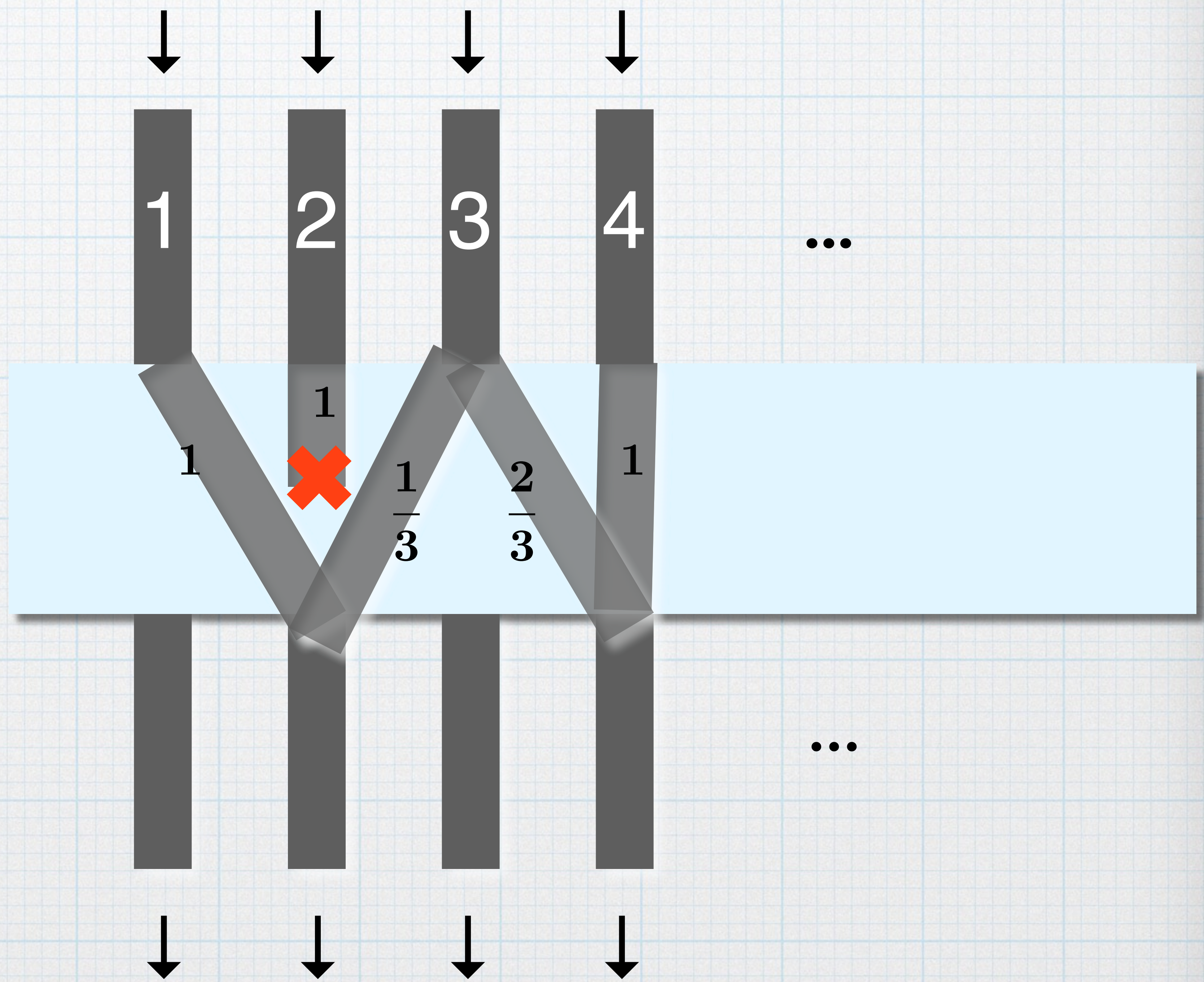
* **DSRel**

$$\dfrac{X \to Y \text{ in } \mathbf{DSRel}}{X \to \mathcal{D}Y \text{ in } \mathbf{Sets}}$$

where $\mathcal{D}Y = \{d : Y \to [0,1] \mid \sum_y d(y) \le 1\}$

Different Branching in The GoI Animation

- **Pfn** (partial functions)
  - Pipes can be stuck
- **Rel** (relations)
  - Pipes can branch
- **DSRel**
  - Pipes can branch probabilistically



Branching Monad: Source of Particle-Style GoI Situations

Thm. ([Jacobs,CMCS10])
Given a "branching monad" T on **Sets**, the monoidal category

$$(\mathcal{K}\ell(T), +, \mathbf{0})$$

is

- a *unique decomposition category* [Haghverdi,PhD00], hence is
- a traced symmetric monoidal category.

Cor.
$\big(\,(\mathcal{K}\ell(T), +, \mathbf{0}),\ \mathbb{N}\cdot\_\,,\ \mathbb{N}\,\big)$ is a GoI situation.

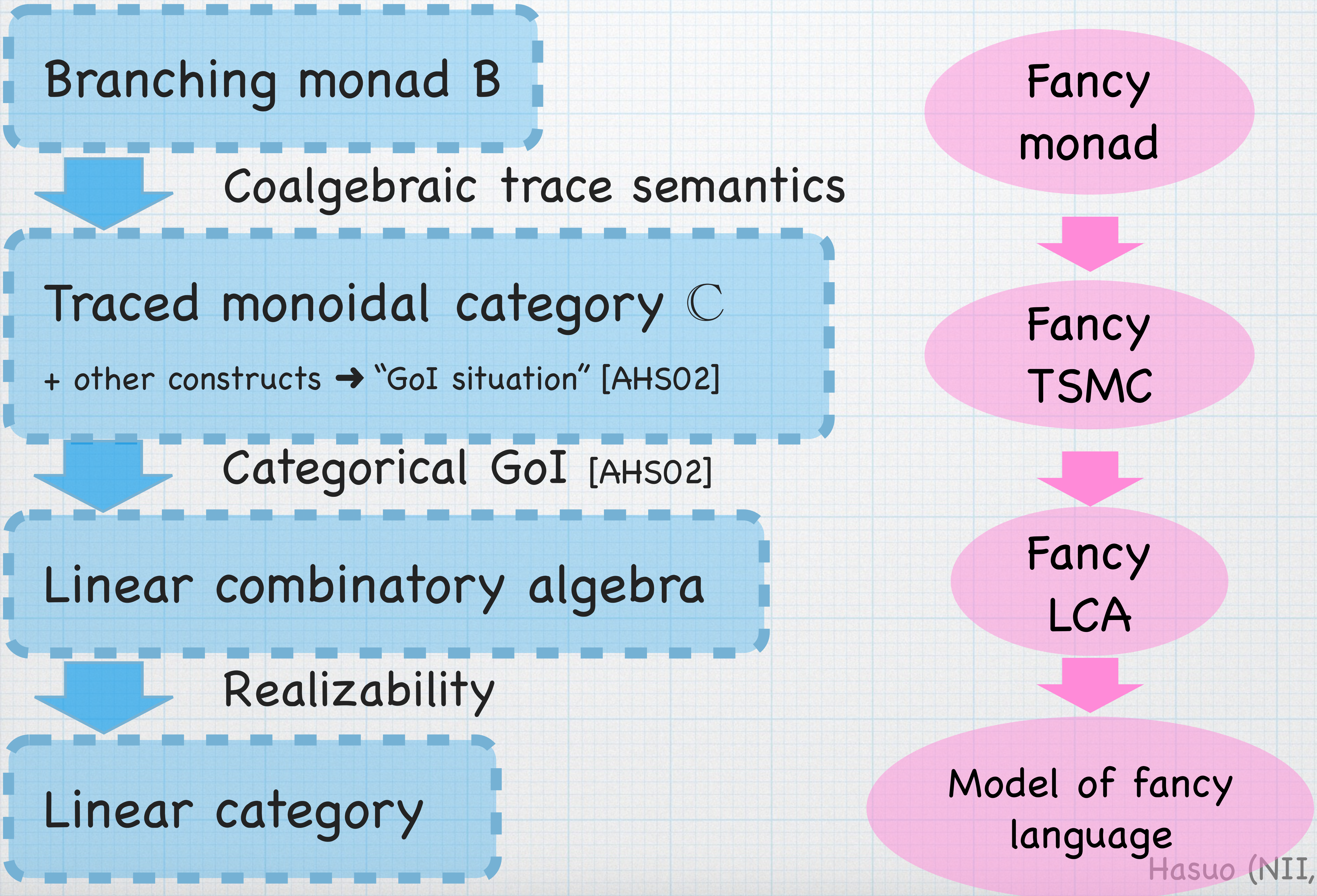
Monads in [Hasuo,Jacobs&Sokolova07]

- Kl(T) is $\mathbf{Cpo}_{\perp}$-enriched

Particle-style: trace via the execution formula

$$\mathbf{tr}(f) = f_{XY} \sqcup \left(\coprod_{n\in\mathbb{N}} f_{ZY} \circ (f_{ZZ})^n \circ f_{XZ}\right)$$

The Categorical GoI Workflow

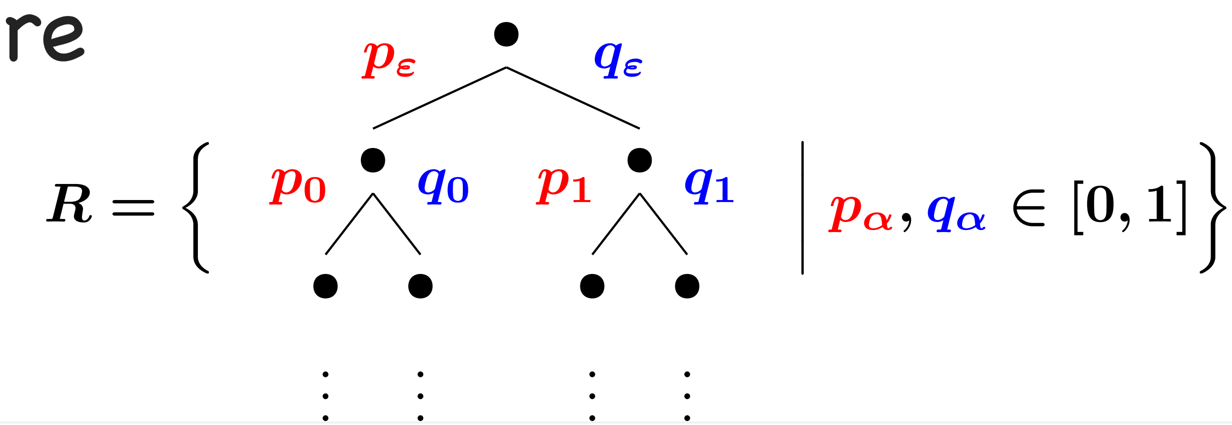


* Model for (a variant of) the Selinger-Valiron **quantum λ-calculus**

  (linear λ + prep./Unitary/meas.)
  [Hasuo & Hoshino, LICS'11 & APAL'16]
* via the **quantum branching monad**
* ... with considerable complication :(

$$[\![\Gamma \vdash M : \tau]\!] : \; [\![\Gamma]\!] \longrightarrow ([\![\tau]\!] \multimap R) \multimap R$$

where



Realizability

Linear category

Workflow

Fancy monad

* Records measurement outcomes
* R as a suitable **final coalgebra** in the realizability category

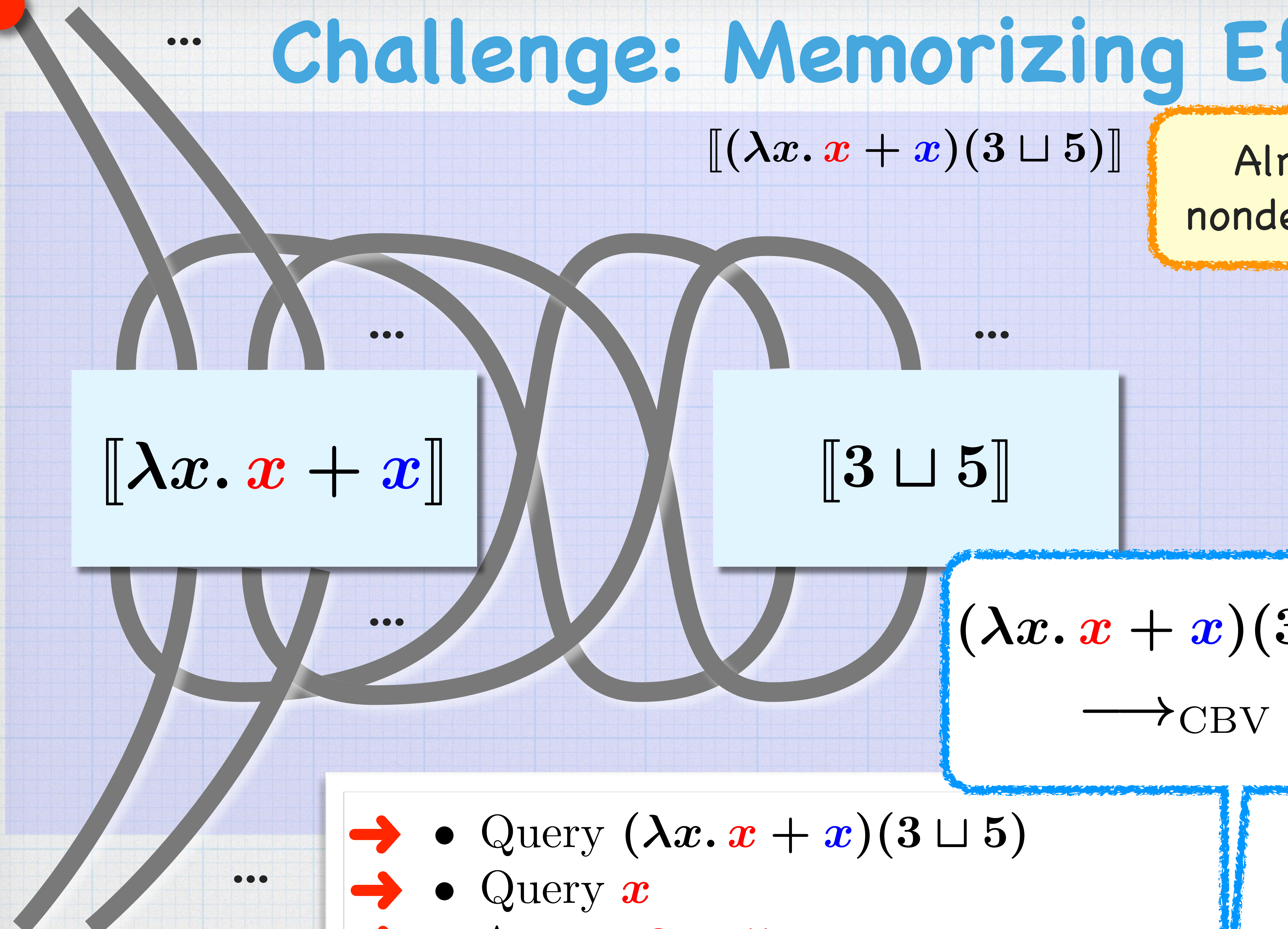
Fancy LCA

Model of fancy language

Challenge: Memorizing Effects

Already w/ nondeterminism!

Challenge: Memorizing Effects



Already w/ nondeterminism!

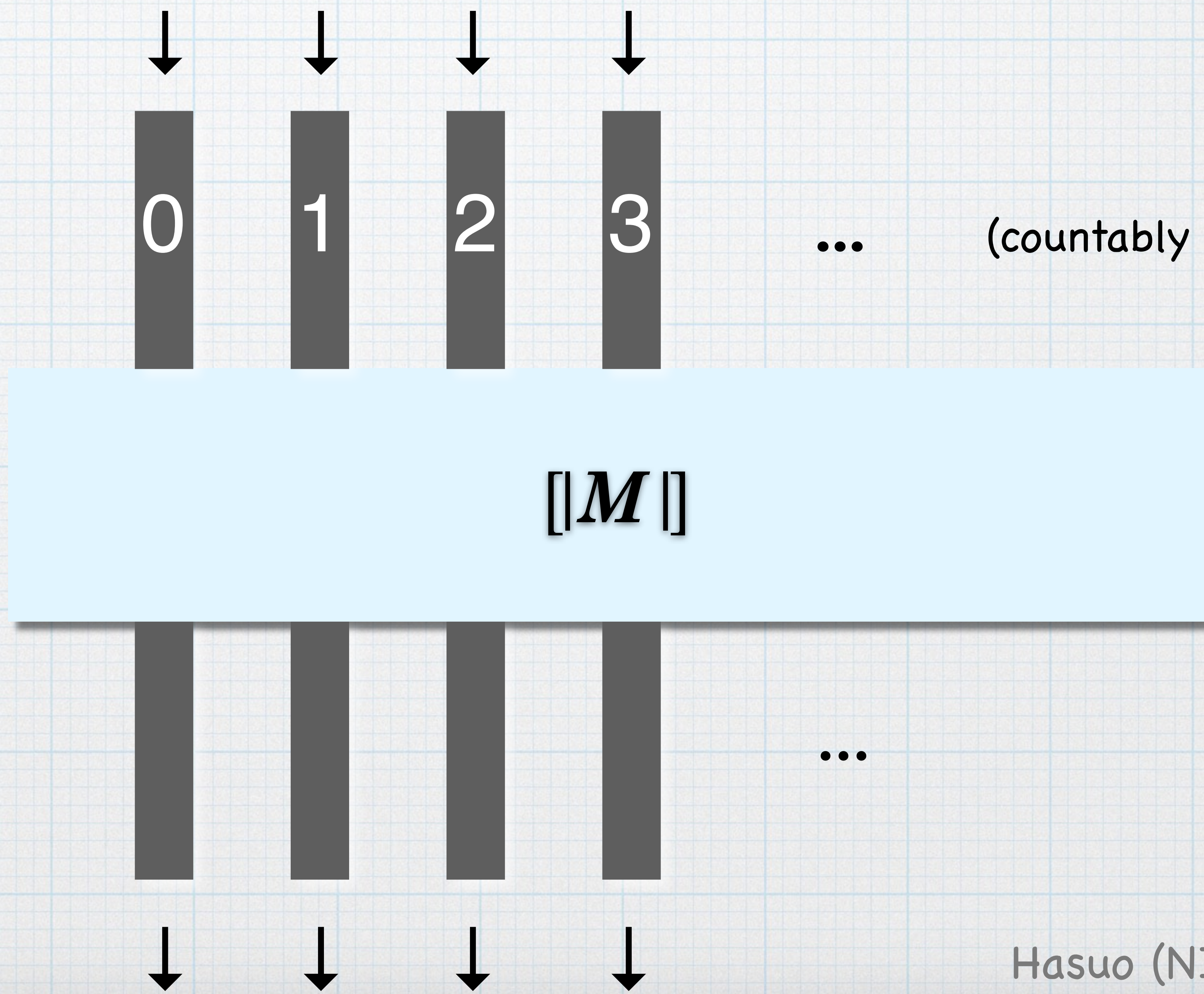
$(\lambda x.\, x + x)(3 \sqcup 5) \longrightarrow_{\text{CBV}} \mathbf{6}$ or $\mathbf{10}$??

- Query $(\lambda x.\, x + x)(3 \sqcup 5)$
- Query x
- Answer $\mathbf{3}$ or $\mathbf{5}$
- Query x
- Answer $\mathbf{3}$ or $\mathbf{5}$
- Answer $\mathbf{3} + \mathbf{3}$, $\mathbf{3} + \mathbf{5}$, $\mathbf{5} + \mathbf{3}$ or $\mathbf{5} + \mathbf{5}$

The GoI Animation

$[\![M]\!] = (\, \mathbb{N} \rightharpoonup \mathbb{N},\ \text{a partial function}\,)$

$= \text{“piping”}$



(countably many)

An Idea

* Let a traversing token rearrange piping!

Memoryful GoI

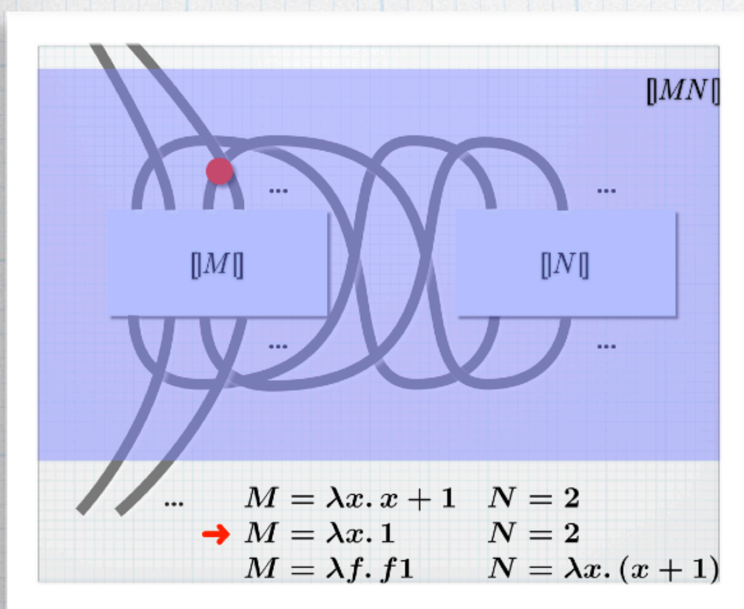
* Let a traversing token rearrange piping!

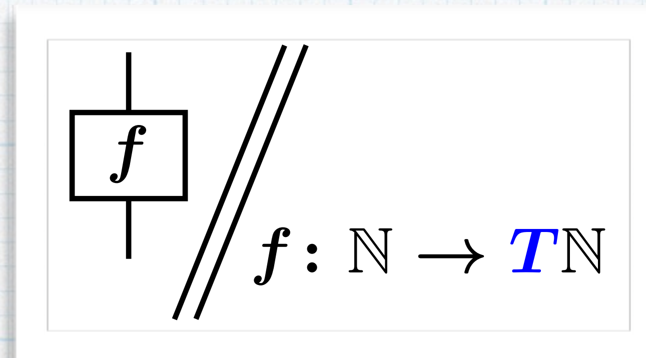
PythagoraSwitch, NHK Education
(Created by (another) Masahiko Sato)

Outline

Coalgebra meets **higher-order computation** in **Geometry of Interaction** [Girard, LC'88]

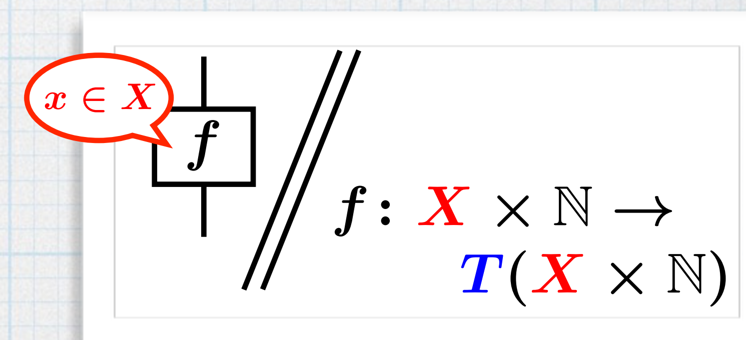
"GoI Animation"





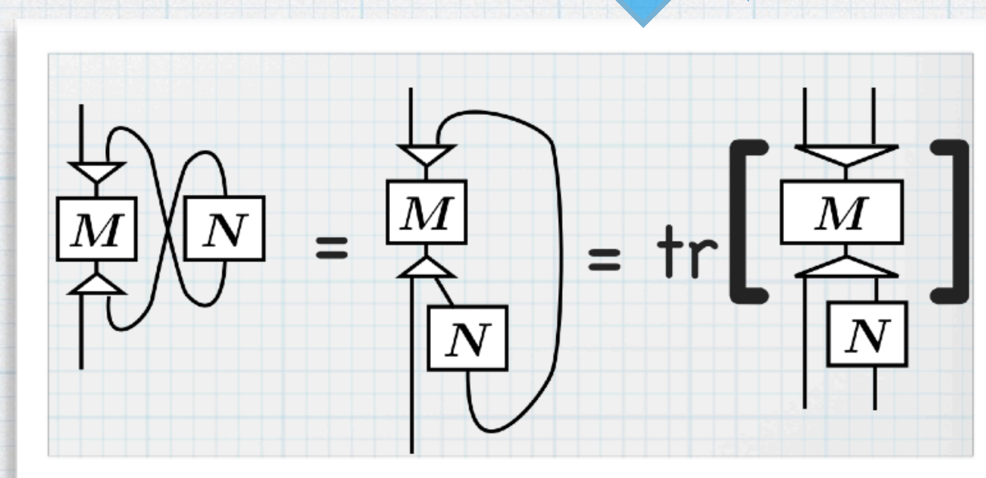
GoI w/ T-branching

[IH & Hoshino, LICS'11]



Memoryful GoI

[Hoshino, Muroya & IH, CSL-LICS'14 & POPL'16]

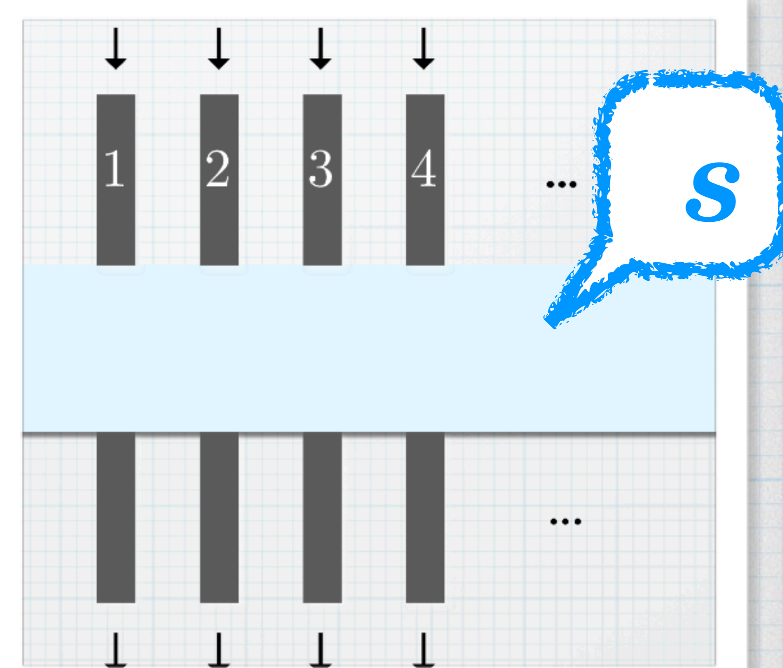


Categorical GoI

[Abramsky, Haghverdi & Scott, MSCS'02]

Memoryful GoI

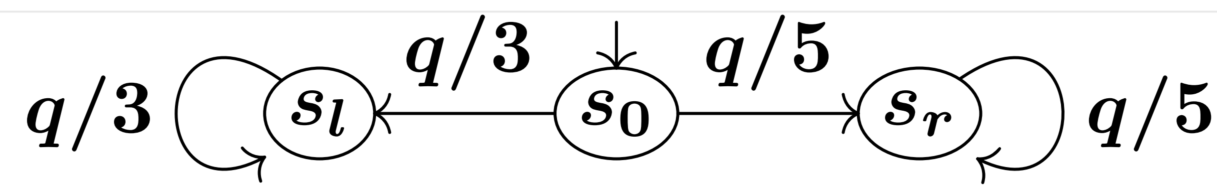
* Equip piping with internal states, or **memory**



* **not** $[\![3 \sqcup 5]\!] : \mathbb{N} \longrightarrow \mathcal{P}\mathbb{N}\ , \quad q \longmapsto \{3, 5\}$

but a transducer (Mealy machine)

$[\![3 \sqcup 5]\!] : X \times \mathbb{N} \longrightarrow \mathcal{P}(X \times \mathbb{N})\ ,$



* Not a new idea:
  * Slices in GoI for additives [Laurent, TLCA'01]
  * Resumption GoI [Abramsky, CONCUR'96]

Memoryful GoI

* We introduce memory in a **structured manner...**
  ➜
  the "traced monoidal category" of transducers

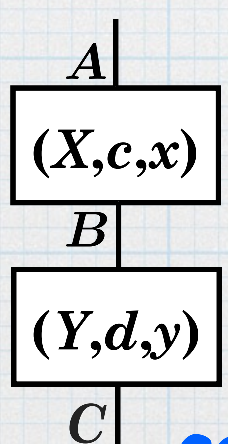
Trans(T)

Objects: sets $A, B, \ldots$

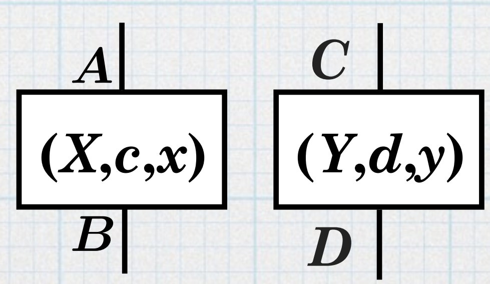
Arrows:

$$\frac{A \longrightarrow B \text{ in } \mathbf{Trans}(T)}{\overline{\left(\, X,\ X \times A \xrightarrow{c} T(X \times B),\ x_0 \in X \,\right),\ T\text{-transducer}}}$$

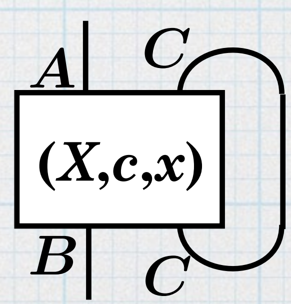
* with operations like



composition ∘



tensor ⊗



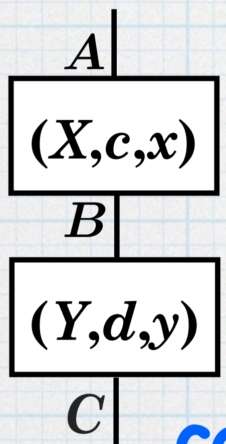
trace

Trans(T) by Coalgebraic Component Calculus

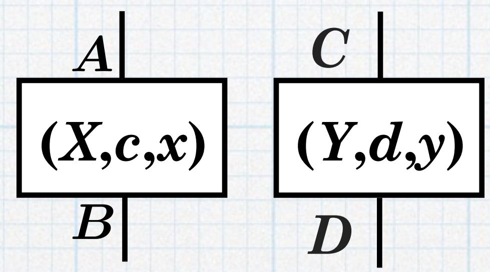
[Barbosa '03][IH & Jacobs '11]

Trans(T) Objects: sets $A, B, \ldots$

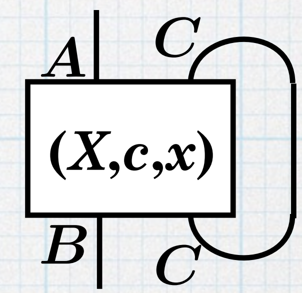
Arrows: $$\frac{A \longrightarrow B \text{ in } \mathbf{Trans}(T)}{\left(X,\ X \times A \xrightarrow{c} T(X \times B),\ x_0 \in X\right),\ T\text{-transducer}}$$



composition ∘



tensor ⊗



trace

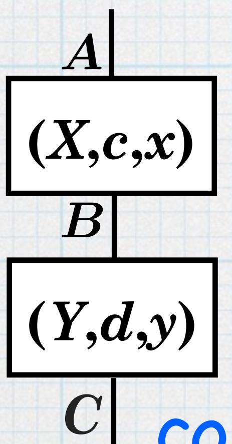
$$\left(X \times Y,\ \begin{array}{rl} (X \times Y) \times A & \xrightarrow{\cong} (X \times A) \times Y \\ & \xrightarrow{c \times Y} T(X \times B) \times Y \\ & \xrightarrow{\mathrm{str}'} T\big((X \times B) \times Y\big) \\ & \xrightarrow{\cong} T\big(X \times (Y \times B)\big) \\ & \xrightarrow{T(X \times d)} T\big(X \times T(Y \times C)\big) \\ & \xrightarrow{T\mathrm{str}} TT\big(X \times (Y \times C)\big) \\ & \xrightarrow{\mu^T} T\big(X \times (Y \times C)\big) \\ & \xrightarrow{\cong} T\big((X \times Y) \times C\big) \end{array},\ (x, y) \right)$$

Trans(T) by Coalgebraic Component Calculus

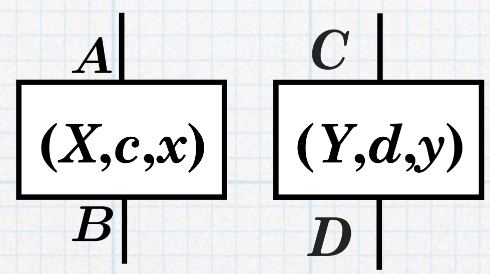
[Barbosa '03][IH & Jacobs '11]

Trans(T) Objects: sets $A, B, \ldots$

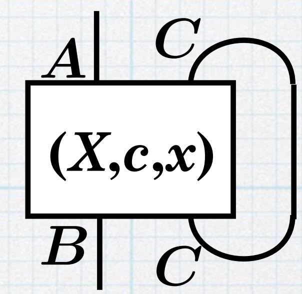
Arrows:
$$\frac{\color{red}{A} \longrightarrow \color{blue}{B} \text{ in } \mathbf{Trans}(T)}{(\, X,\ X \times \color{red}{A} \stackrel{c}{\to} T(X \times \color{blue}{B}),\ x_0 \in X\ ),\ T\text{-transducer}}$$



composition

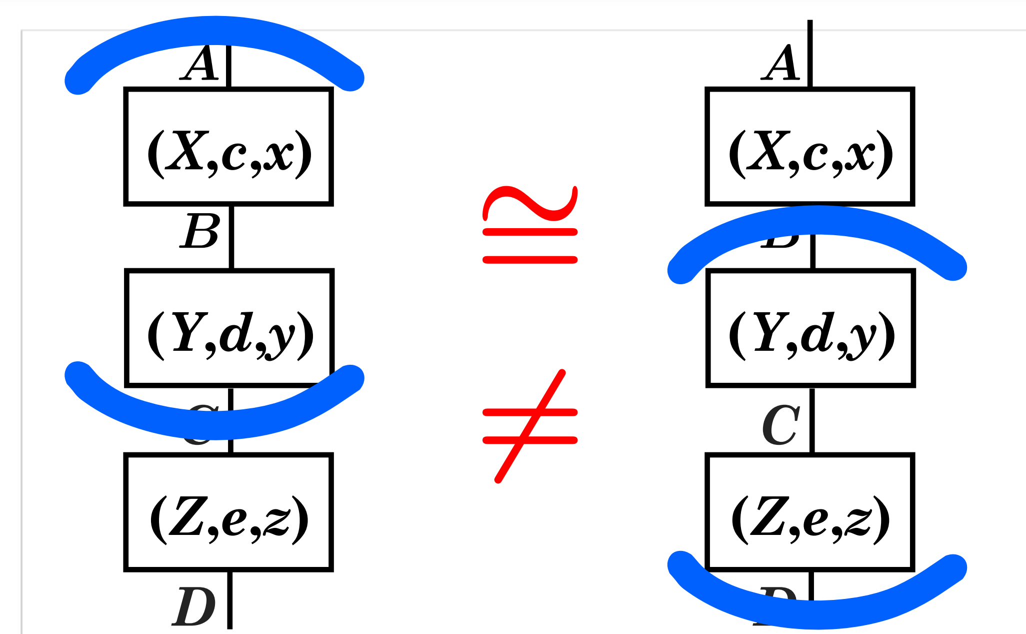


tensor



trace

- Trans(T) is a "category"...
- Fix: quotient modulo **behavioral equivalence** (homomorphisms of T-transducers) ➔ **resumptions** [Abramsky]



Hasuo (NII, JP)

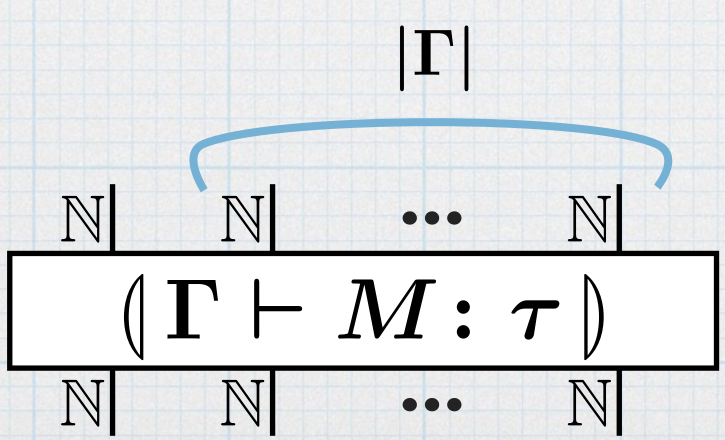
The Memoryful GoI Framework

- Given:
  - a **monad** T on $\mathbf{Sets}$, s.t. $\mathbf{Kl}(T)$ is $\mathbf{Cppo}$-enriched
  - an alg. signature Σ with **algebraic operations** on T [Plotkin & Power]

- *Exception* $\mathbf{1} + \boldsymbol{E} + (\_)$
  - with $\mathbf{0}$-ary opr. $\mathbf{raise}_{\boldsymbol{e}}$ ($\boldsymbol{e} \in \boldsymbol{E}$)
- *Nondeterminism* $\boldsymbol{\mathcal{P}}$
  - with binary opr. $\sqcup$
- *Probability* $\boldsymbol{\mathcal{D}}$, where $\boldsymbol{\mathcal{D}X} = \{\boldsymbol{d} \colon \boldsymbol{X} \to [\mathbf{0}, \mathbf{1}] \mid \sum_{\boldsymbol{x}} \boldsymbol{d}(\boldsymbol{x}) \le \mathbf{1}\}$
  - with binary opr. $\sqcup_{\boldsymbol{p}}$ ($\boldsymbol{p} \in [\mathbf{0}, \mathbf{1}]$)
- *Global state* $(\mathbf{1} + \boldsymbol{S} \times \_)^{\boldsymbol{S}}$
  - with $|\boldsymbol{V}|$-ary $\mathbf{lookup}_{\boldsymbol{l}}$ and unary $\mathbf{update}_{\boldsymbol{l},\boldsymbol{v}}$

- For the calculus: $\lambda_{\mathbf{c}}$ + (alg. opr. from Σ) + (co)products + arith.

- We give



in $\mathbf{Trans}(T)$

Recursion: impl

Theorem (Adequacy)

Let $\vdash M : \mathtt{nat}$. Then, as elem. of $T(\mathbb{N})$,

$$\left(\begin{array}{c} \mathbb{N} \\ \boxed{(\!| \vdash M : \mathtt{nat} |\!)} \\ \mathbb{N} \end{array} \right)^{\dagger} = [\![\, |M| \,]\!] \ .$$

Interpretation

$[\![\_]\!] : \mathbf{EffVal}^{\Sigma}_{\mathbb{N}} \longrightarrow T(\mathbb{N})$

(exploiting free conti. Σ-alg.)

feeding a query and observing the outcome

Opr. sem.: Plotkin-Power **effect-value**. E.g.

$$|\, 3 \sqcup (5 \sqcup \mathtt{div}) \,| = \begin{array}{c} \sqcup \\ 3 \quad \sqcup \\ \quad 5 \quad \bot \end{array}$$

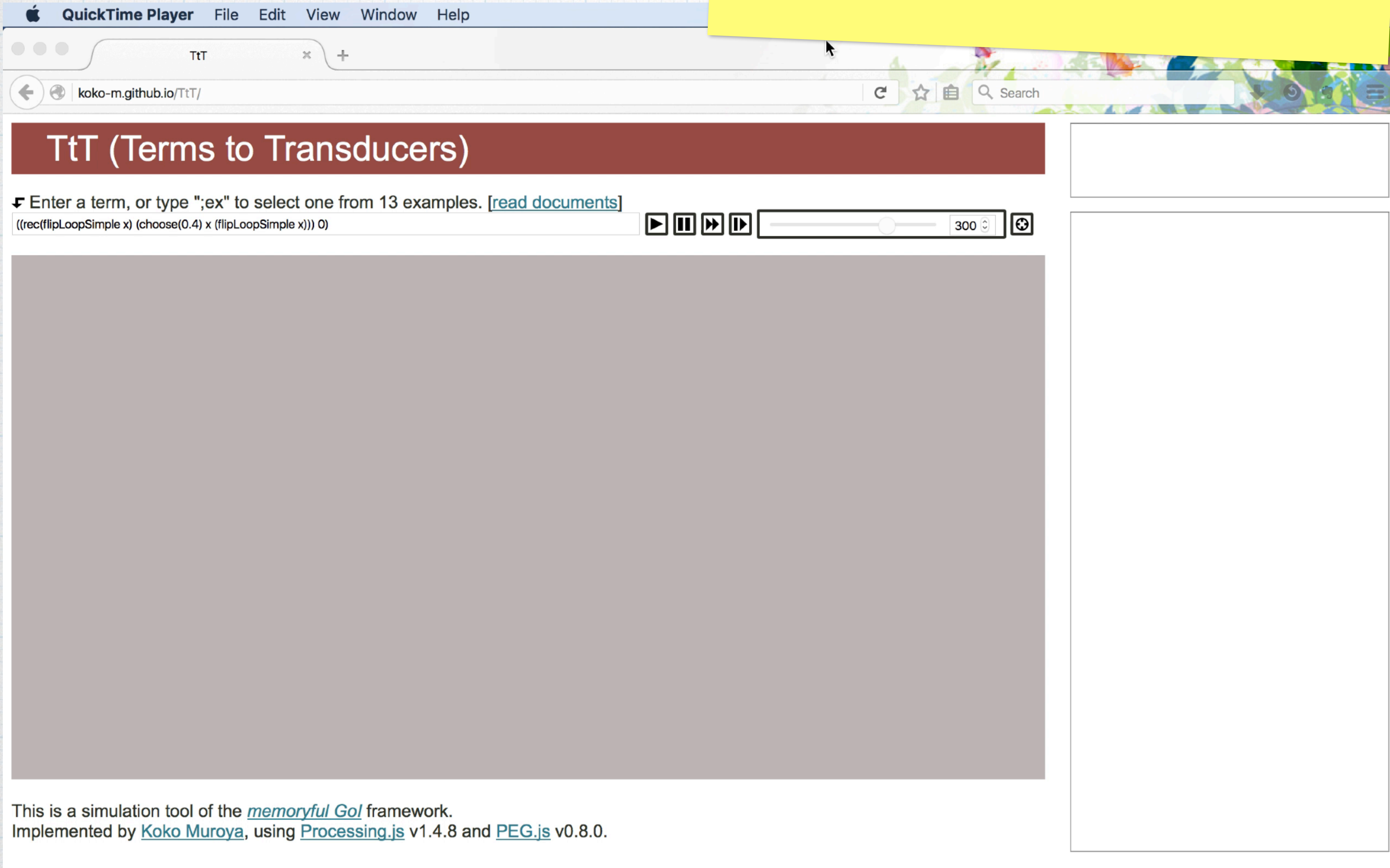
- Obviously a
- Fixed-point

Fi

Theorem The two coincide. (for

Our Tool TtT

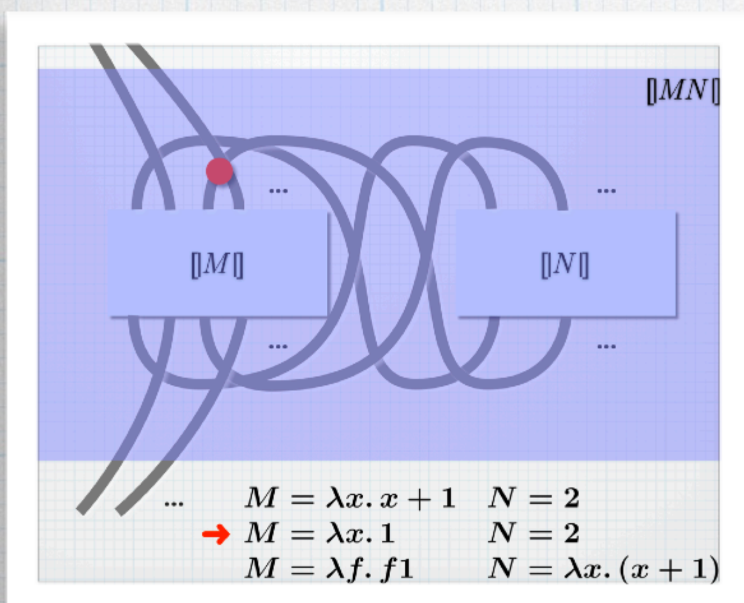
Developed by Koko Muroya
http://koko-m.github.io/TtT/

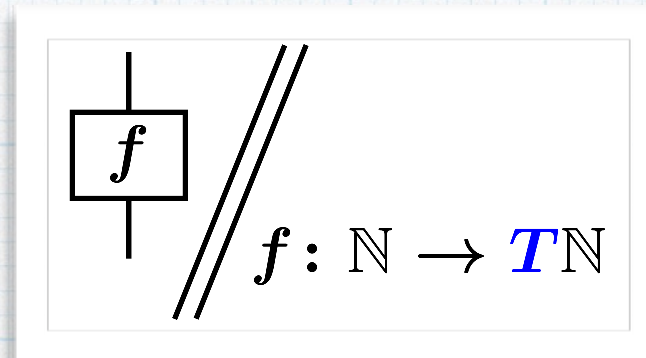


Summary

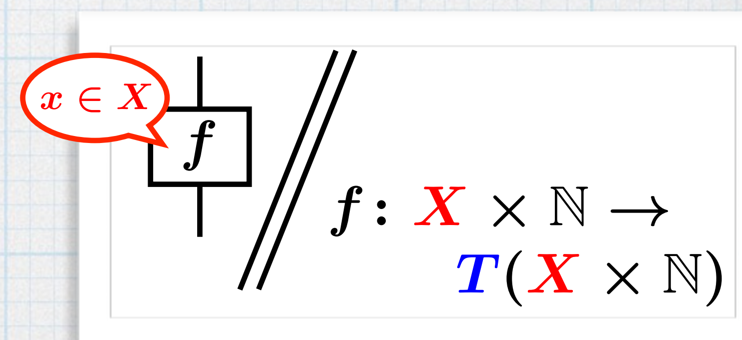
Coalgebra meets **higher-order computation**
in **Geometry of Interaction** [Girard, LC'88]

"GoI Animation"

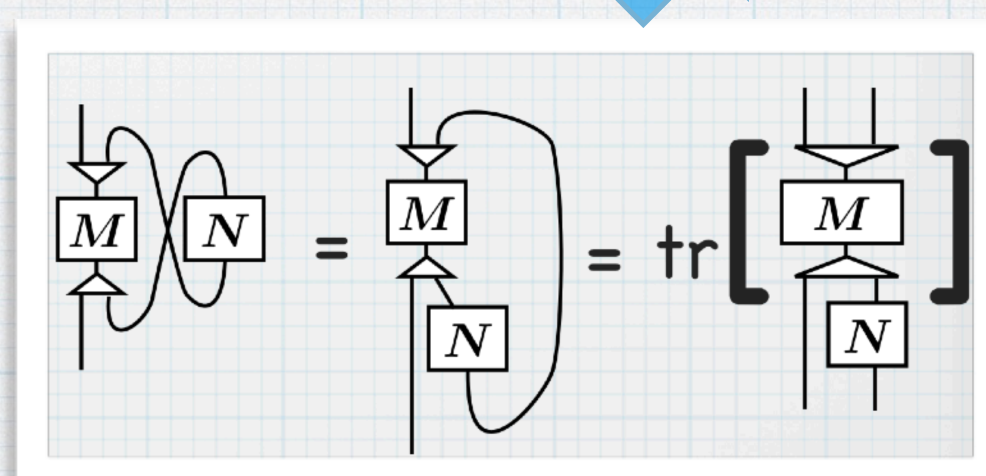




GoI w/ T-branching
[IH & Hoshino, LICS'11]

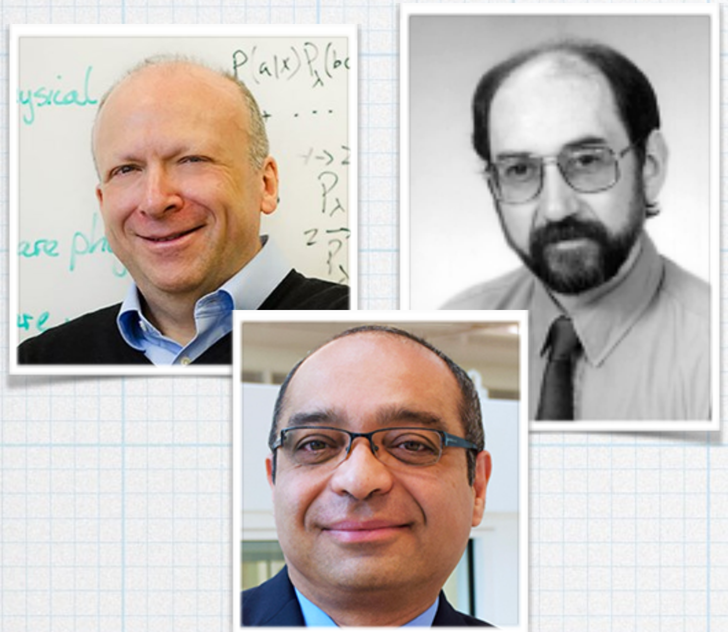


Memoryful GoI
[Hoshino, Muroya & IH, CSL-LICS'14 & POPL'16]



Categorical GoI
[Abramsky, Haghverdi & Scott, MSCS'02]

Abstraction in a Theory: "Categorical Transfer"

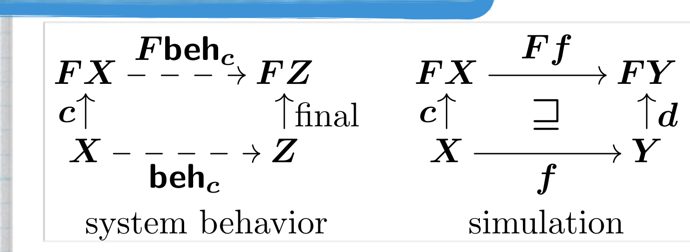


Categorical Geometry of Interaction [Abramsky, Haghverdi, Scott]

Abstract Technique

$T[\_]$

Coalgebra [Jacobs, Rutten, ⋯]



Identify "mathematical essense"

Choose parameter e_1

Choose parameter e_2

Existing Technique

$T_1 = T[e_1]$

Automata, Mealy machines, ⋯

Geometry of Interaction [Girard]

Novel Technique

$T[e_2]$

GoI with nondeterministic, probabilistic, quantum, and other effects

⋯ ➜ Memoryful Geometry of Interaction [Hoshino, Muroya, Hasuo, ⋯]

Retracing some paths in Process Algebra

Samson Abramsky
Laboratory for the Foundations of Computer Science
University of Edinburgh

1 Introduction

The very existence of the CONCUR conference bears witness to the fact that "concurrency theory" has developed into a subject unto itself, with substantially different emphases and techniques to those prominent elsewhere in the semantics of computation.

Whatever the past merits of this separate development, it seems timely to look for some convergence and unification. In addressing these issues, I have found it instructive to trace some of the received ideas in concurrency back to their origins in the early 1970's. In particular, I want to focus on a seminal paper by Robin Milner [Mil75][1], which led in a fairly direct line to his enormously influential work on CCS [Mil80, Mil89]. I will take (to the extreme) the liberty of of applying hindsight, and show how some different paths could have been taken, which, it can be argued, lead to a more unified approach to the semantics of computation, and moreover one which may be better suited to modelling today's concurrent, object-oriented languages, and the type systems and logics required to support such languages.

2 The semantic universe: transducers

Milner's starting point was the classical automata-theoretic notion of *transducers*, *i.e.* structures

$$(Q, X, Y, q_0, \delta)$$

Such unification had been already suggested by Samson!

CONCUR'96

Outline

* **Abstraction in a Theory I**
  Categorical GoI (geometry of interaction)
  [Abramsky, Haghverdi & Scott, MSCS'02] [Hasuo & Hoshino, LICS'11] [Hoshino, Muroya & Hasuo, CSL-LICS'14]
  [Muroya, Hoshino & Hasuo, POPL'16] [Hasuo & Hoshino, APAL'17]

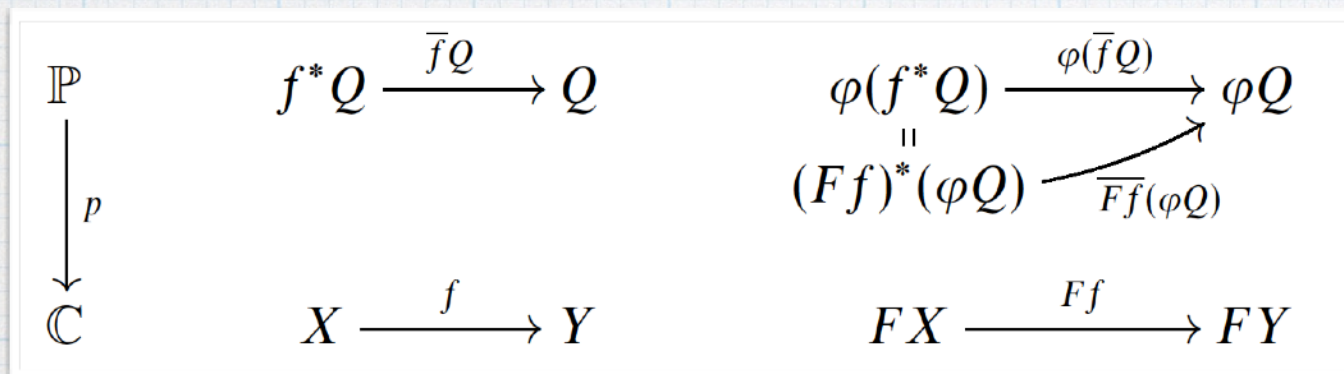
* **Abstraction in a Theory II**
  Codensity Bisimulation Games
  [Katsumata & Sato, CALCO'15] [Sprunger, Katsumata, Dubut & Hasuo, CMCS'18]
  [Komorida, Katsumata, Hu, Klin & Hasuo, submitted (soon on arXiv)]

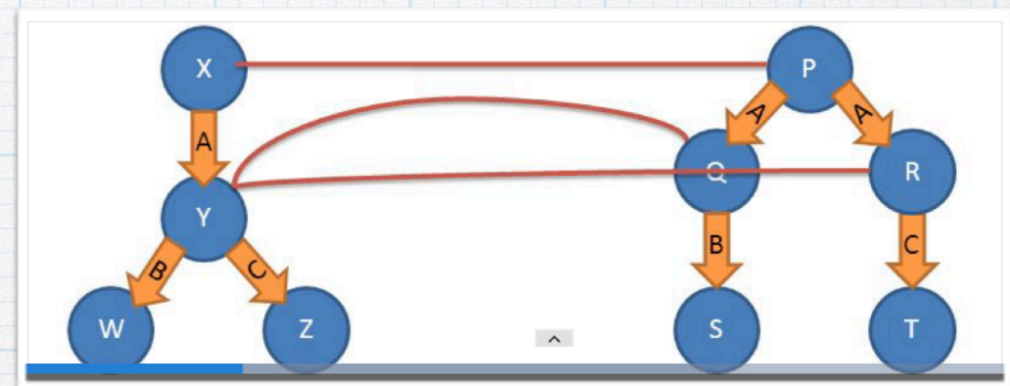
* **Abstraction in a Project**
  Introducing ERATO MMSD Project:
  from categorical foundations to automated driving

* **Abstraction in an Inspiring Mind**

Abstraction in a Theory II

Codensity Bisimulation Games





position	player	possible moves
$(x, y) \in X^2$	S	$Z \subseteq X$ s.t. $c(x)(Z) \neq c(y)(Z)$
$Z \subseteq X$	D	$(x', y') \in X^2$ s.t. $x' \in Z \wedge y' \notin Z$

[Katsumata & Sato, CALCO'15] [Sprunger, Katsumata, Dubut & Hasuo, CMCS'18]
[Komorida, Katsumata, Hu, Klin & Hasuo, submitted (soon on arXiv)]

"Categorical Transfer"

Coalgebra
[Jacobs, Rutten, …]

Codensity bisimilarity
in a fibration

Codensity
games

Abstract Technique

$$T[\_]$$

$$A \;::=\; \mathtt{true} \mid \mathtt{false} \mid A_1 \wedge A_2 \mid \neg A \mid a_1 < a_2 \mid \forall x \in {}^*\mathbb{N}.\, A \mid \forall x \in {}^*\mathbb{R}.\, A$$

$$\begin{array}{ccc} FX & \xrightarrow{F\mathsf{beh}_c} & FZ \\ c\uparrow & & \uparrow\text{final} \\ X & \xrightarrow[\mathsf{beh}_c]{} & Z \end{array} \qquad \begin{array}{ccc} FX & \xrightarrow{Ff} & FY \\ c\uparrow & \sqsupseteq & \uparrow d \\ X & \xrightarrow[f]{} & Y \end{array}$$

system behavior — simulation

Identify
"mathematical
essense"

Choose
parameter e_1

Choose
parameter e_2

Automata,
Mealy machines, …

Existing Technique

$$T_1 = T[e_1]$$

Bisimulation
games

Novel Technique

$$T[e_2]$$

Games for bisimulation
metric, topology, …

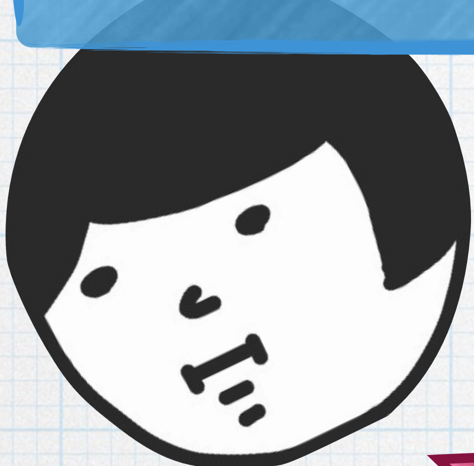
* LaTeX slides

"Categorical Transfer"

Coalgebra
[Jacobs, Rutten, …]

Codensity bisimilarity
in a fibration

Codensity
games



Abstract Technique

$T[\_]$

$$A ::= \mathtt{true} \mid \mathtt{false} \mid A_1 \wedge A_2 \mid \neg A \mid a_1 < a_2 \mid \forall x \in {}^*\mathbb{N}.\, A \mid \forall x \in {}^*\mathbb{R}.\, A$$

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Choose
parameter e_1

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Automata,
Mealy machines, …

Existing Technique

$T_1 = T[e_1]$

Novel Technique

$T[e_2]$

Bisimulation
games

Games for bisimulation
metric, topology, …

Perspectives

* Games ➜ algorithms
  * Infinite state?
    ➜ CEGAR, template-based symbolic presentation, …

* The roles of **observations** and **indistinguishability** in bisimulation notions, formalized
  * **relational**
  * **metric**
  * **topological** ➜ domain theory!
    * open = observable
      continuous = computable

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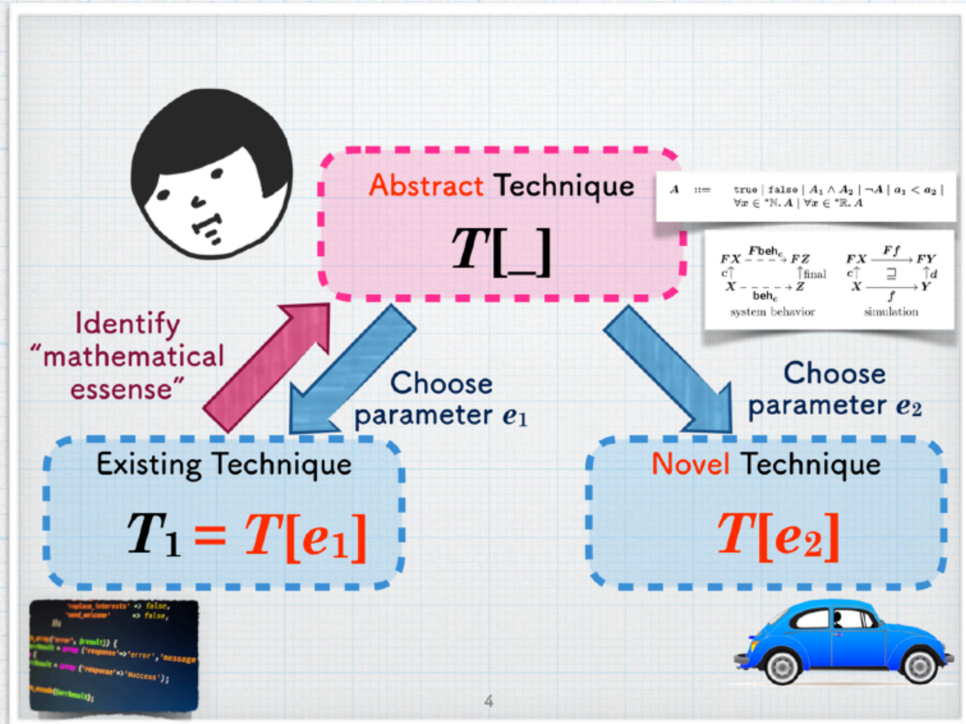
  Introducing ERATO MMSD Project:
  from categorical foundations to automated driving

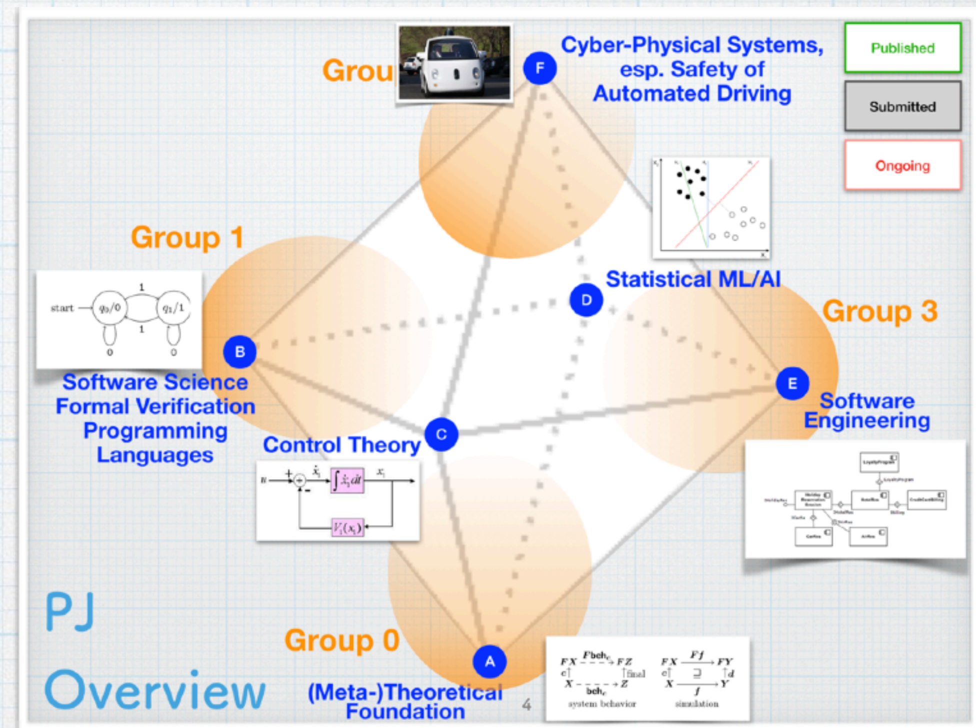
* **Abstraction in an Inspiring Mind**

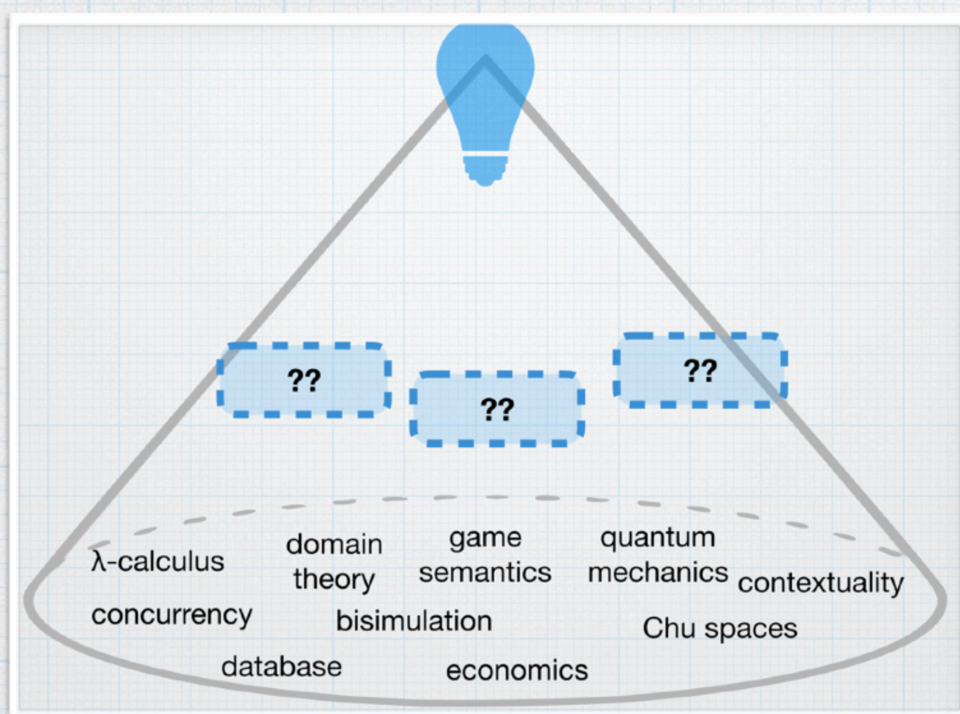
The Power of Abstraction



in a **theory**

in a **project**





… and in an **inspiring mind**

Outline

- **Abstraction in a Theory I**
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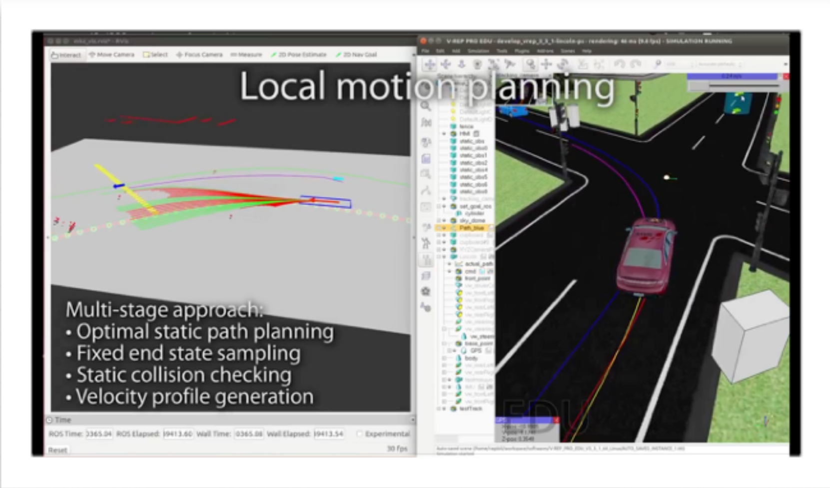
  from categorical foundations to automated driving

- **Abstraction in an Inspiring Mind**

On ERATO MMSD

- JST ERATO Project, 2016/10-2022/03.
  Several faculty members,
  15+ researchers, 20+ students, 6 sites

- Our goal:
  formal methods for cyber-physical systems (CPS)
  - Extend formal methods, from software to CPS
  - Safety, reliability, V&V (Verification & Validation).
    "Check if a system behaves as expected"
  - Automated driving as a strategic target domain.
    Collaboration with U Waterloo: www.autonomoose.net

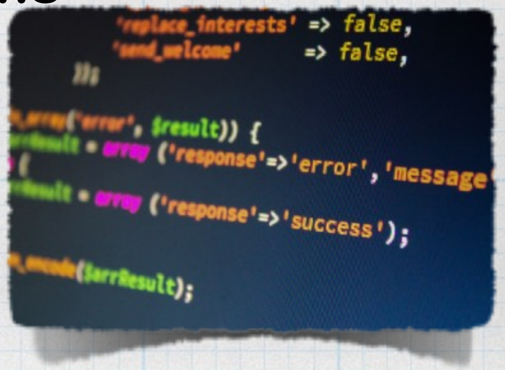


The Autonomoose Project, U Waterloo

- Our principle: broaden the realm of CPS research
  - Theory:
    abstract mathematical metatheory
    ➜ scale out to diverse applications
  - Practice: real-world systems (not only toy examples)

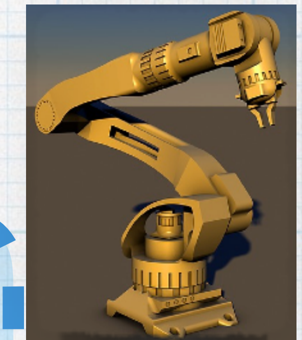
Cyber-Physical Systems: Control Theory and Formal Methods/Software Science

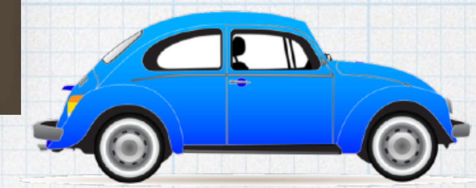
* Cyber-Physical System (CPS)
  * "A mechanism that is controlled or monitored by computer-based algorithms, tightly integrated with the Internet and its users" (Wikipedia)
  * Physical plant (continuous) + Digital control (discrete)
  * In US: NSF Key Area of Research (2006-)
* Formal methods: Logical proofs for "correctness" of (discrete) programs
  * Model checking [Pnueli, Clarke, Emerson, Sifakis, ...]
  * Theorem Proving (Coq, Agda, ...) [Milner, Coquand, Leroy, Voevodsky, ...]

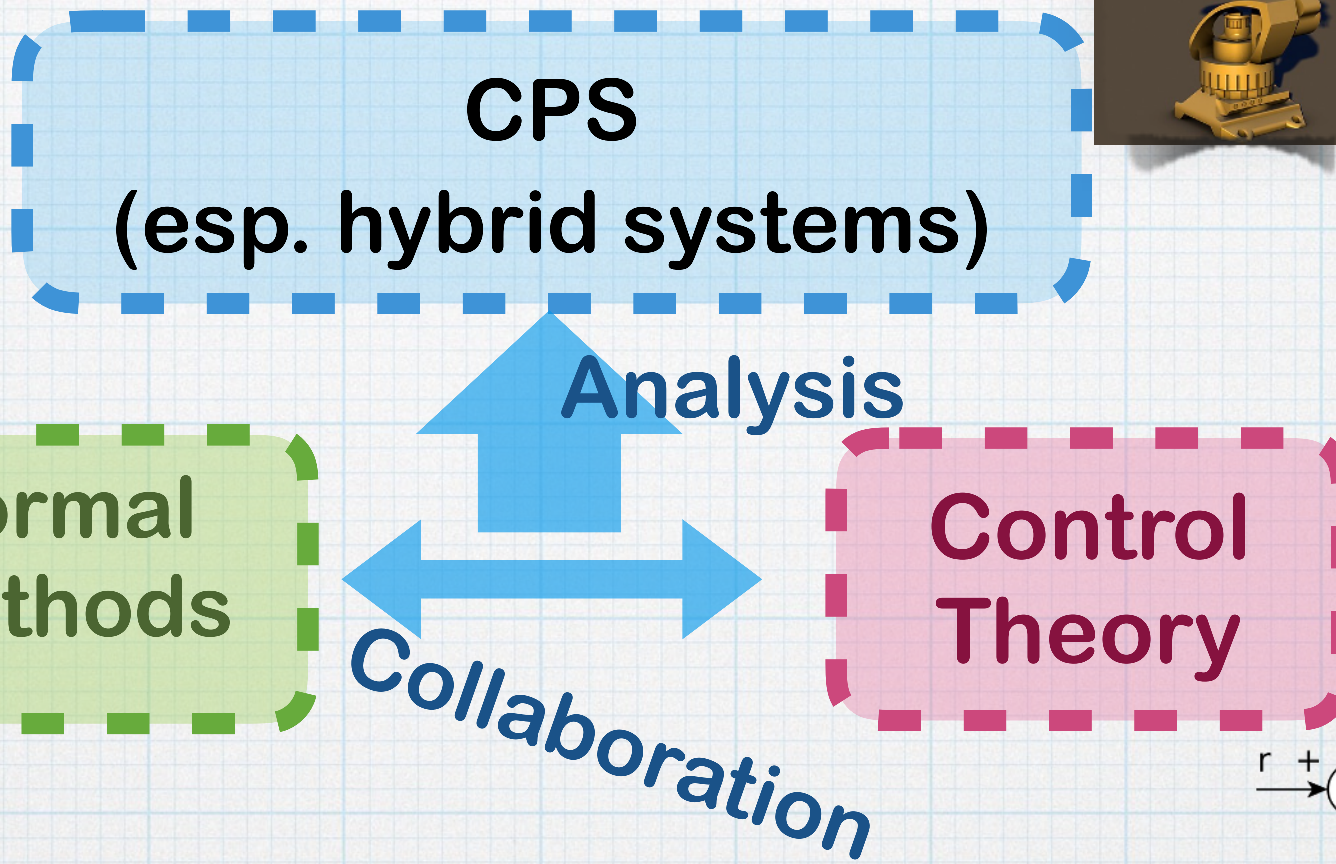


* Control Theory: Analysis of continuous dynamics
  * Stability, Lyapunov function, ...
* Their similarity is widely recognized
  * e.g. HSCC, one of the main conferences of annual CPS Week

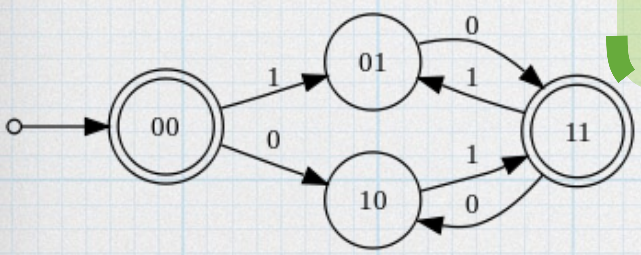
CPS Research, So Far (the V&V Aspect)



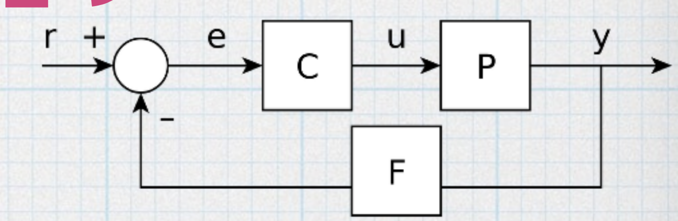




$\Box(p \Rightarrow \Diamond q)$

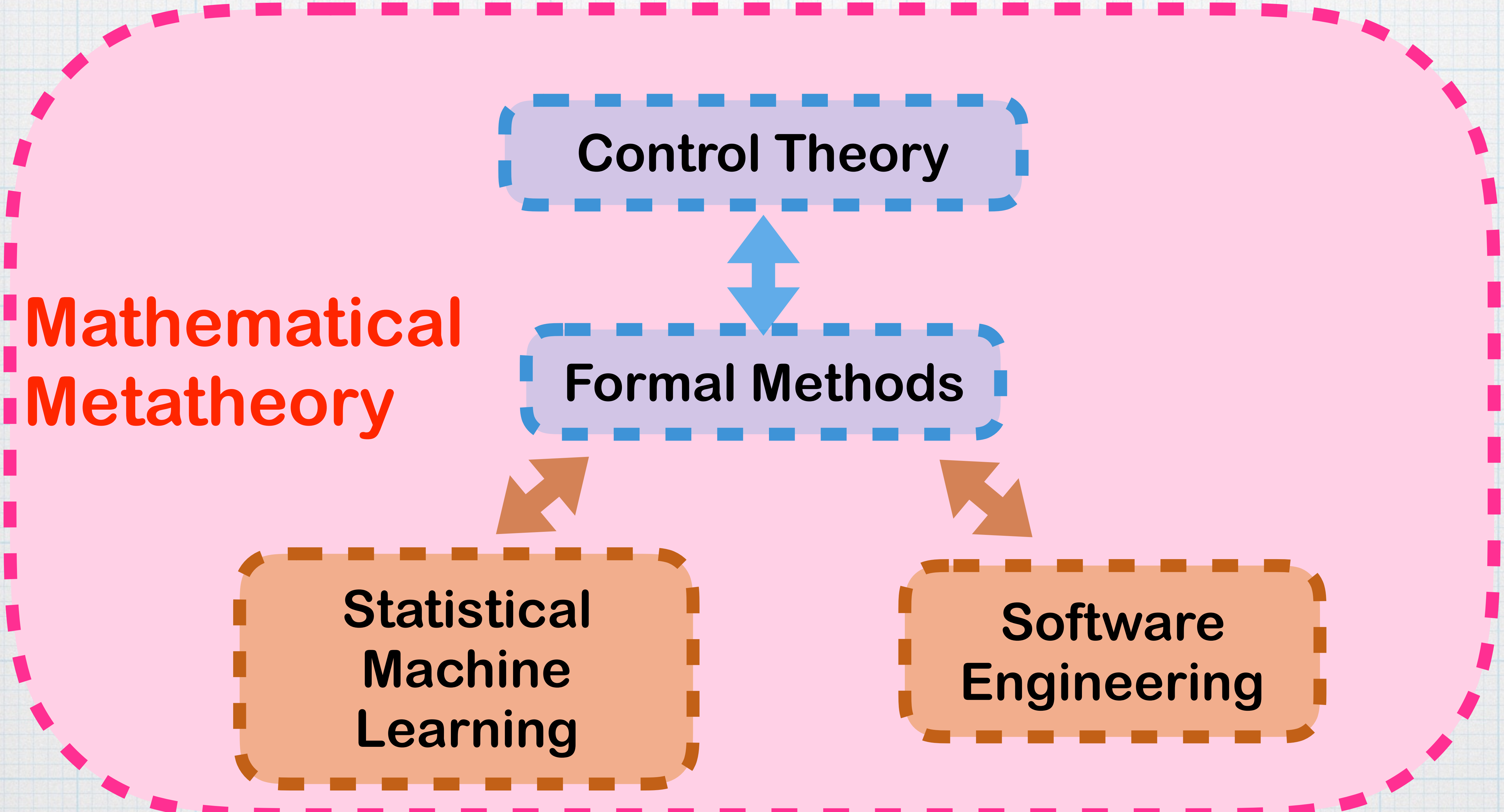


$x' = f(x, u)$



- Challenge: scalability, esp. for real-world CPSs
  - Require complete understanding of a white-box model
  - Insist on being absolutely sound and correct
  - Little tolerance to uncertainty and noise
    ➜ don’t get along with statistical machine learning

CPS Research: Our Comprehensive Approach



ERATO 蓮尾メタ数理システムデザインプロジェクト

ERATO Metamathematics for Systems Design Project

国立情報学研究所 & 科学技術振興機構 National Institute of Informatics & Japan Science and Technology Agency

ERATO

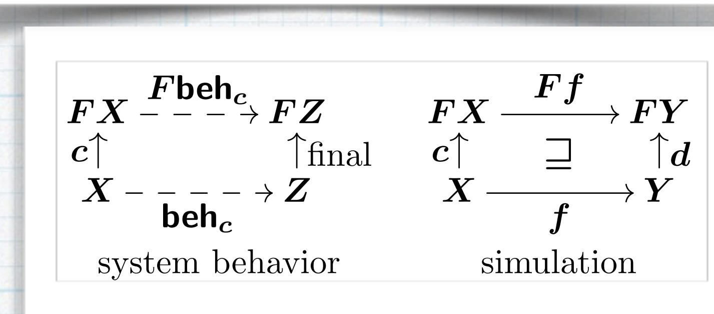
MMSD

Logic, esp. model theory

Abstract Technique

$T[\_]$

$A ::= \mathtt{true} \mid \mathtt{false} \mid A_1 \wedge A_2 \mid \neg A \mid a_1 < a_2 \mid \forall x \in {}^*\mathbb{N}.\, A \mid \forall x \in {}^*\mathbb{R}.\, A$



Categorical simulation

Identify "mathematical essense"

Choose parameter e_1

Choose parameter e_2

Discrete dynamics

Existing Technique

$T_1 = T[e_1]$

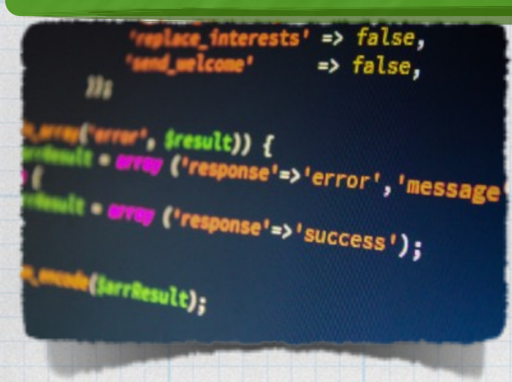
Boolean simulation

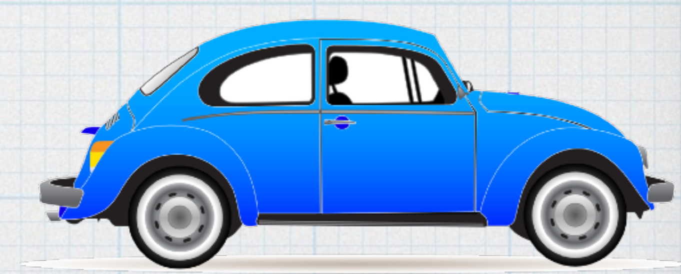
Continuous dynamics

Novel Technique

$T[e_2]$

Probabilistic simulation





Example 1: Coalgebraic Unfolding
[Hasuo, Urabe, Shimizu et al.]

Example 2: Nonstandard Transfer
[Suenaga, Hasuo, Sekine, Kido et al.]

Our Organization

International and multi-disciplinary. "creative chaos"

ERATO MMSD

Group 0 @ NII:
Metatheoretical Integration
Leader: Shin-ya Katsumata

Topics:
Programming Languages, Formal Semantics, Categorical Models, Mathematical Logic, ...

Group 3 @ NII:
Formal Methods and Intelligence
Leader: Fuyuki Ishikawa

Topics:
Software Engineering, Formal Modeling, Testing, Safe & Explainable AI

Kyoto U RIMS Site:
Categorical Infrastructure
Leader:
Masahito Hasegawa

Kyushu U Site:
Optimization for CPS V&V
Leader:
Hayato Waki

Osaka U Site:
Control Theory for CPS
Leader:
Toshimitsu Ushio

Group 1 @ NII:
Heterogeneous Formal Methods
Leader: Ichiro Hasuo

Topics:
Automata Theory, Formal Verification, Proof Assistants, Automated Deduction, Runtime Verification

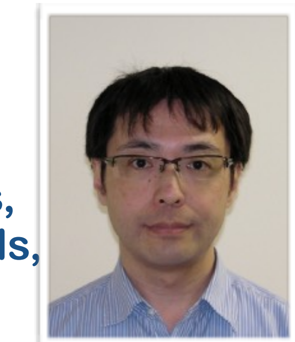
Group 2 @ U Waterloo:
Formal Methods in Industry
Leader: Krzysztof Czarnecki

Topics:
Automated Driving, Software Engineering, Machine Learning

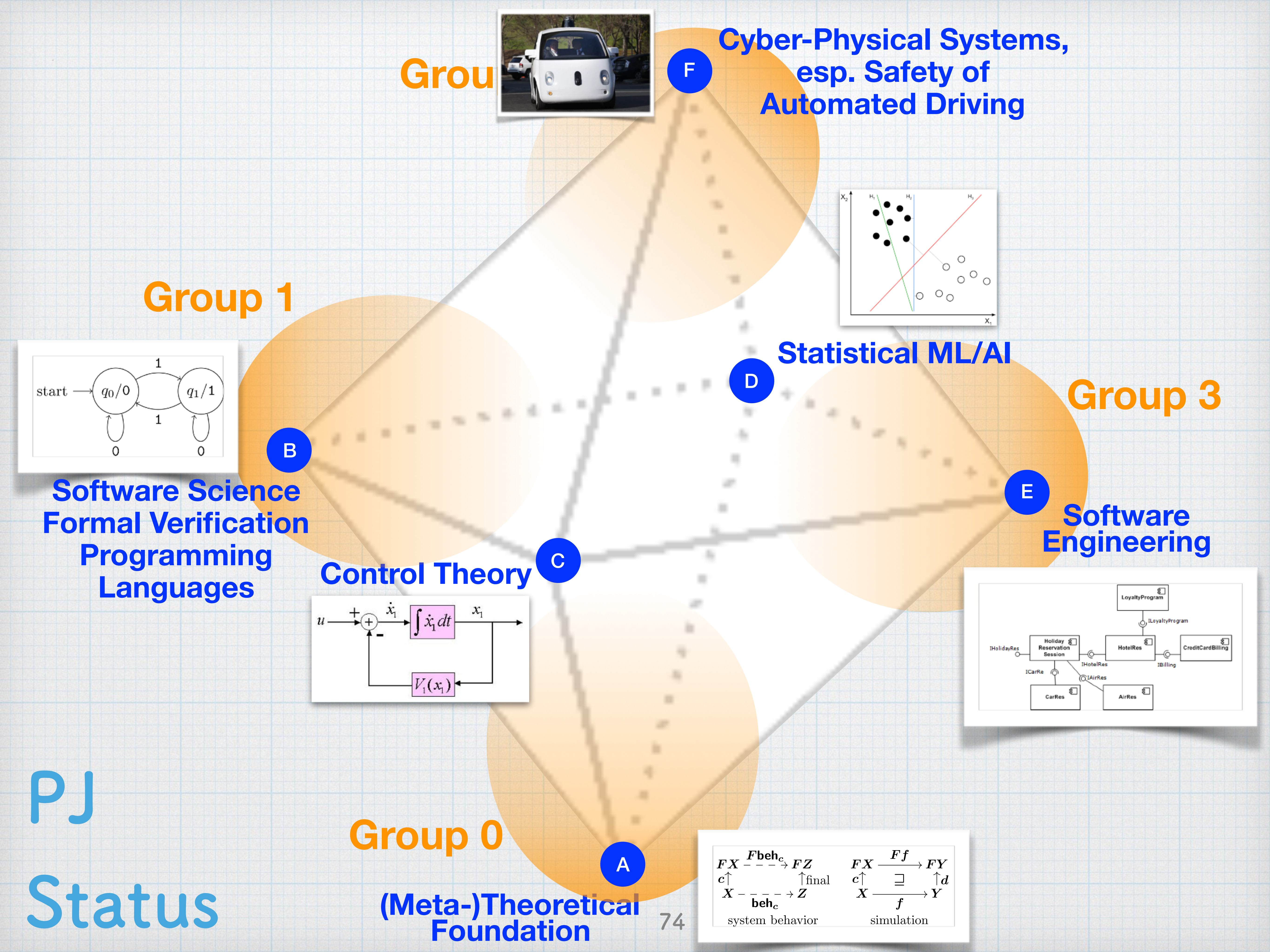
Our Categorical Team: ERATO MMSD Group 0

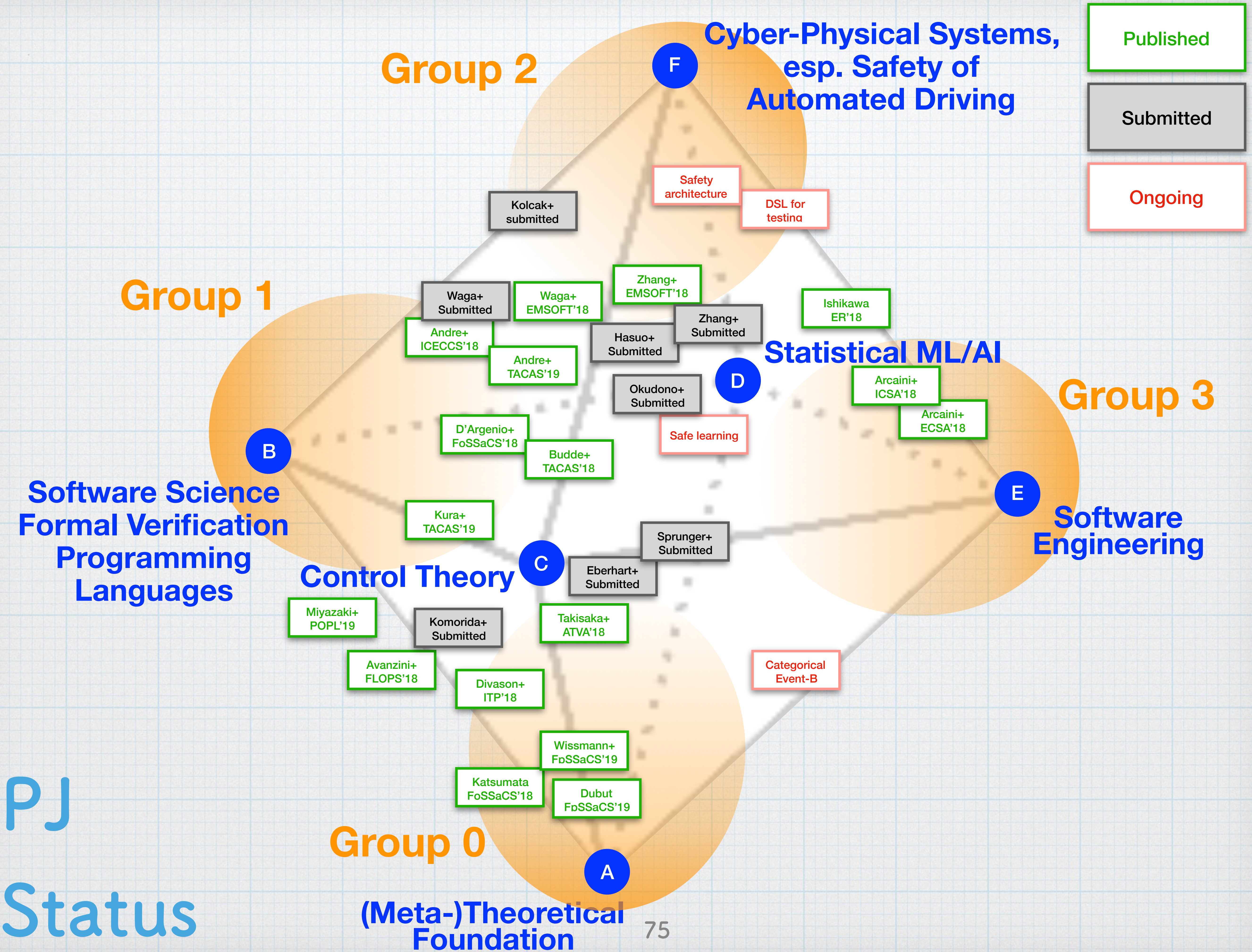
Group 0 @ NII:
Metatheoretical Integration
Leader: Shin-ya Katsumata

Topics:
Programming Languages, Formal Semantics, Categorical Models, Mathematical Logic, ...



- **Shin-ya Katsumata (Group Leader, PhD (Edinburgh)):**
  fibration, lambda-calculus, categorical semantics, monoidal category, programming language, program verification
- **Jérémy Dubut (PhD (ENS Paris Saclay, 2017)):**
  concurrency, directed topology, topos theory, coalgebra, verification, hybrid system
- **David Sprunger (PhD (Indiana, 2017)):**
  coalgebra, monoidal category, categorical logic, fibration, machine learning
- **Clovis Eberhart (PhD (U Savoie Mont Blanc, 2018)):**
  game semantics, nominal set, verification, machine learning
- **Kenta Cho (PhD expected (Nijmegen)):**
  categorical logic, quantum mechanics & logic, verification, machine learning
- **Soichiro Fujii (PhD confirmed (Tokyo)):**
  Lawvery theory, higher-dimensional category, algebraic effect
- **Natsuki Urabe (PhD confirmed (Tokyo)):**
  coalgebra, concurrency, model checking, game, verification, probabilistic systems
- **Yuichi Komorida (PhD student (NII/Sokendai)):**
  fibration, coalgebra, game
- … and at **Kyoto Site** (**Masahito “Hassei” Hasegawa**, **Naohiko Hoshino**, **Koko Muroya**, and more)

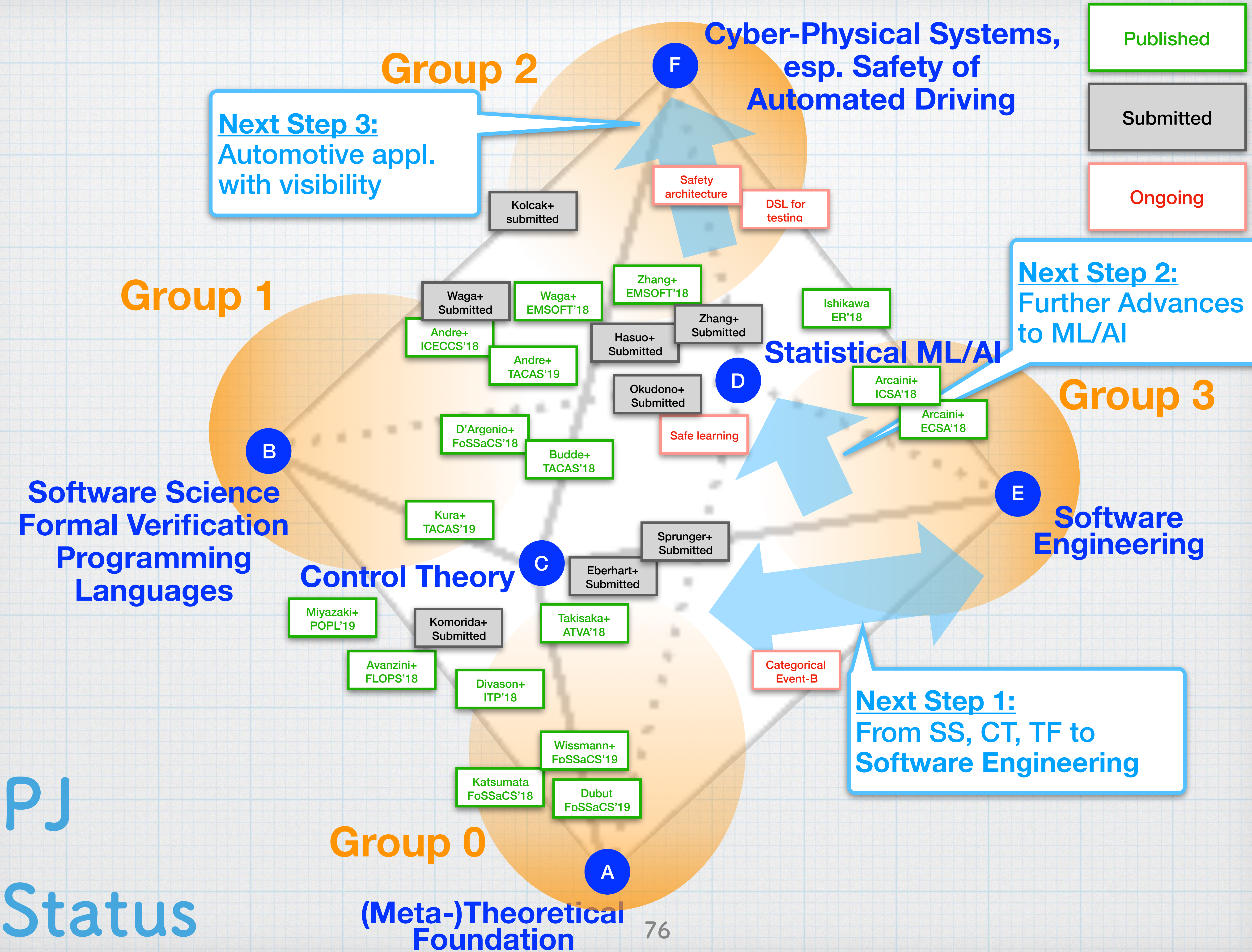

PJ Status
Group 1
Group 3
Group 0
Grou
Cyber-Physical Systems, esp. Safety of Automated Driving
F
Statistical ML/AI
D
Software Science
Formal Verification
Programming Languages
B
Control Theory
C
Software Engineering
E
(Meta-)Theoretical Foundation
A
start
q0/0
q1/1
1
1
0
0
u
+
-
V1(x1)
LoyaltyProgram
ILoyaltyProgram
IHolidayRes
Holiday Reservation Session
HotelRes
CreditCardBilling
IHotelRes
IBilling
ICarRe
IAirRes
CarRes
AirRes
system behavior
simulation



Published
Submitted
Ongoing
Group 2
F
Cyber-Physical Systems, esp. Safety of Automated Driving
Safety architecture
DSL for testing
Kolcak+ submitted
Group 1
Waga+ Submitted
Waga+ EMSOFT'18
Zhang+ EMSOFT'18
Ishikawa ER'18
Andre+ ICECCS'18
Zhang+ Submitted
Hasuo+ Submitted
Andre+ TACAS'19
Statistical ML/AI
D
Okudono+ Submitted
Arcaini+ ICSA'18
Group 3
Arcaini+ ECSA'18
Safe learning
D'Argenio+ FoSSaCS'18
B
Budde+ TACAS'18
E
Software Science Formal Verification Programming Languages
Software Engineering
Kura+ TACAS'19
Sprunger+ Submitted
C
Control Theory
Eberhart+ Submitted
Miyazaki+ POPL'19
Komorida+ Submitted
Takisaka+ ATVA'18
Avanzini+ FLOPS'18
Categorical Event-B
Divason+ ITP'18
Wissmann+ FoSSaCS'19
Katsumata FoSSaCS'18
Dubut FoSSaCS'19
Group 0
A
(Meta-)Theoretical Foundation

PJ Status

PJ Status



Published
Submitted
Ongoing
Group 2
F
Cyber-Physical Systems, esp. Safety of Automated Driving
Next Step 3:
Automotive appl. with visibility
Kolcak+ submitted
Safety architecture
DSL for testing
Group 1
Waga+ Submitted
Waga+ EMSOFT'18
Zhang+ EMSOFT'18
Zhang+ Submitted
Hasuo+ Submitted
Ishikawa ER'18
Andre+ ICECCS'18
Andre+ TACAS'19
Next Step 2:
Further Advances to ML/AI
Statistical ML/AI
D
Okudono+ Submitted
Safe learning
Arcaini+ ICSA'18
Arcaini+ ECSA'18
Group 3
B
D'Argenio+ FoSSaCS'18
Budde+ TACAS'18
E
Software Science
Formal Verification
Programming Languages
Kura+ TACAS'19
Software Engineering
Sprunger+ Submitted
Control Theory
C
Eberhart+ Submitted
Miyazaki+ POPL'19
Komorida+ Submitted
Takisaka+ ATVA'18
Avanzini+ FLOPS'18
Divason+ ITP'18
Categorical Event-B
Next Step 1:
From SS, CT, TF to Software Engineering
Wissmann+ FoSSaCS'19
Katsumata FoSSaCS'18
Dubut FoSSaCS'19
Group 0
A
(Meta-)Theoretical Foundation

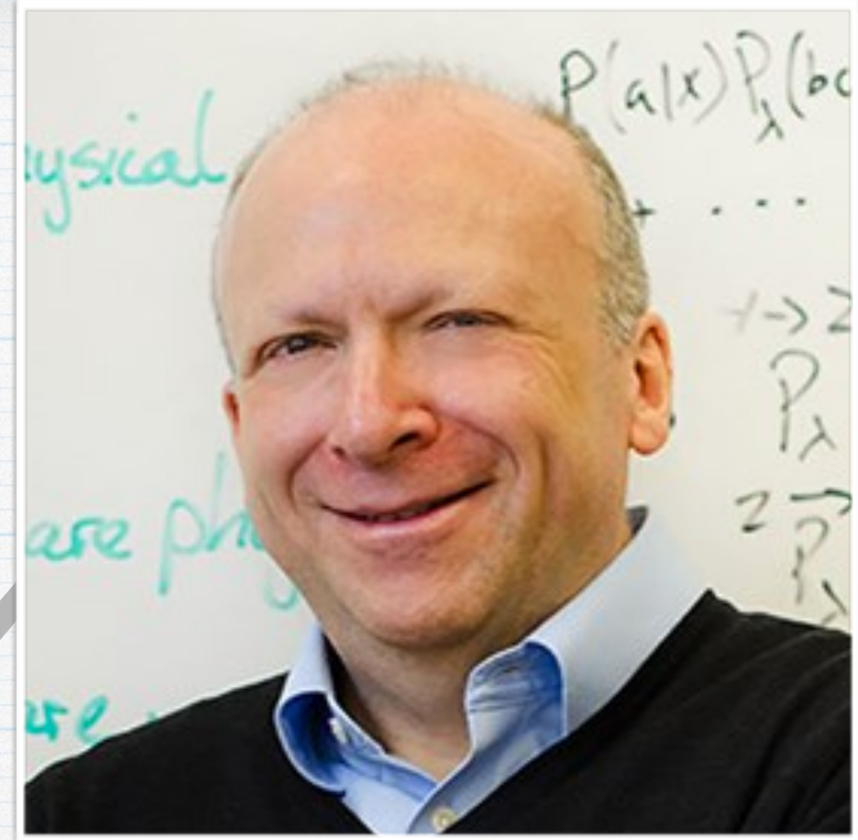
Outline

* **Abstraction in a Theory I**
  Categorical GoI (geometry of interaction)
  [Abramsky, Haghverdi & Scott, MSCS'02] [Hasuo & Hoshino, LICS'11] [Hoshino, Muroya & Hasuo, CSL-LICS'14] [Muroya, Hoshino & Hasuo, POPL'16] [Hasuo & Hoshino, APAL'17]

* **Abstraction in a Theory II**
  Codensity Bisimulation Games
  [Katsumata & Sato, CALCO'15] [Sprunger, Katsumata, Dubut & Hasuo, CMCS'18] [Komorida, Katsumata, Hu, Klin & Hasuo, submitted (soon on arXiv)]

* **Abstraction in a Project**
  Introducing ERATO MMSD Project:
  from categorical foundations to automated driving

* **Abstraction in an Inspiring Mind**



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λ-calculus

domain theory

game semantics

quantum mechanics

contextuality

concurrency

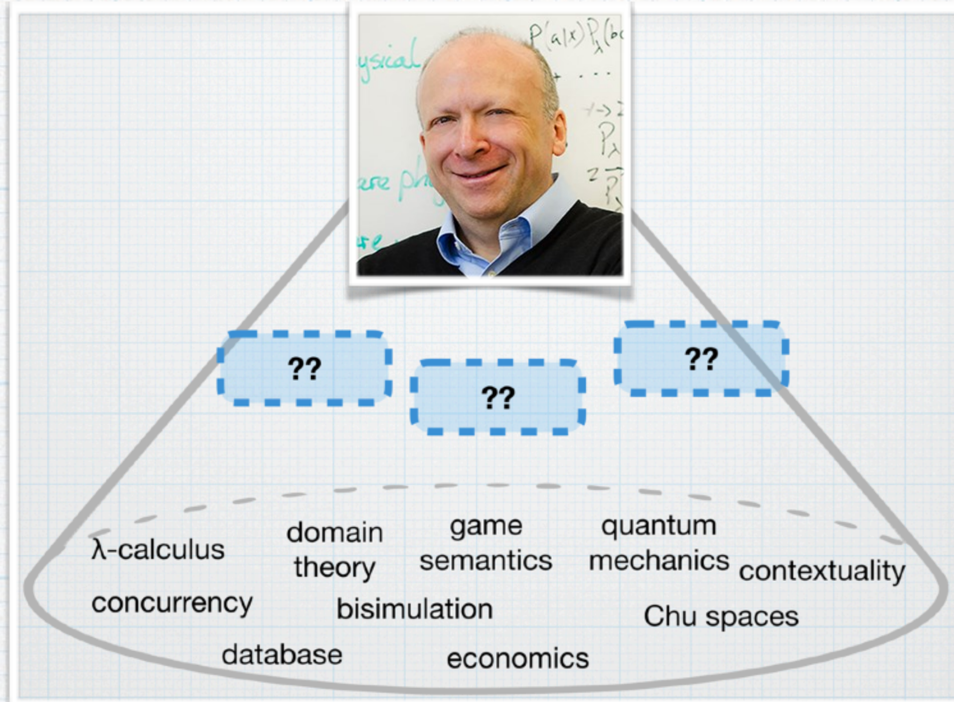
bisimulation

Chu spaces

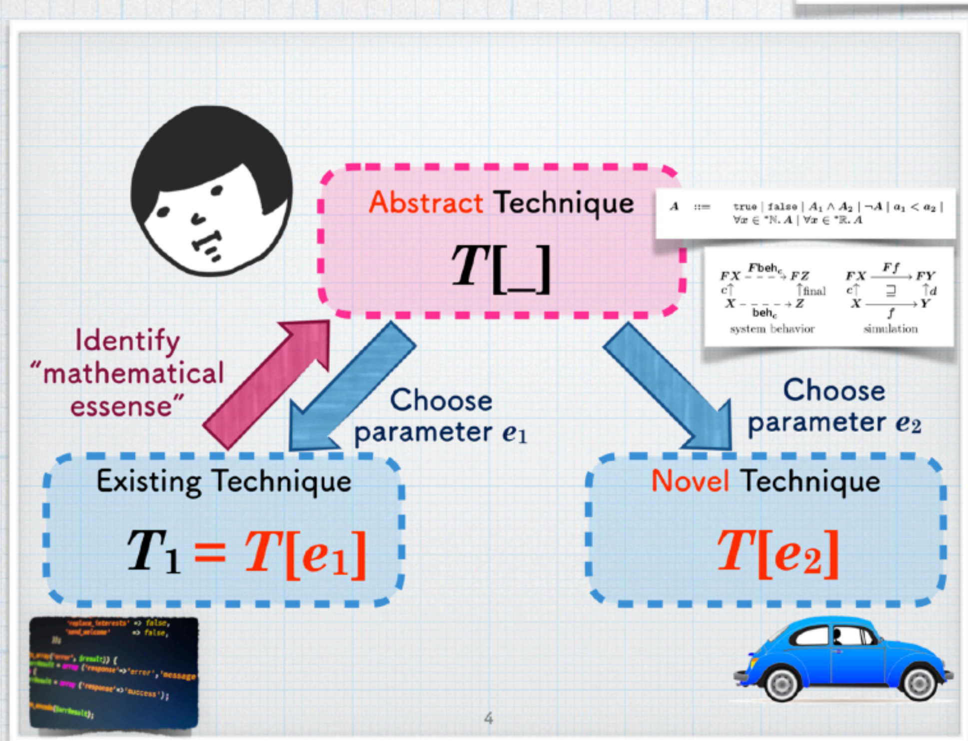
database

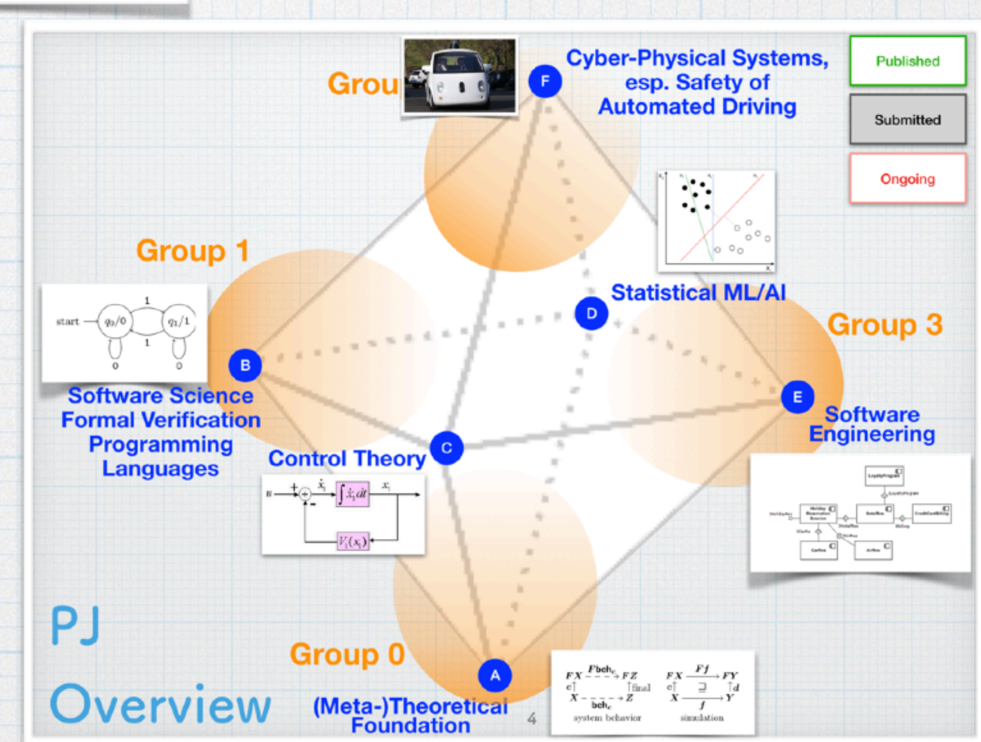
economics

The Power of Abstraction



To come any close to an **inspiring mind...**





individual efforts

… and our **collective effort**