

# Provable Anonymity

## *Epistemic Logic for Anonymizing Protocols*

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# Introduction

Concern about **on-line privacy** is growing...

ISPs in EU might soon start logging all the URLs you browse

A number of **anonymizing protocols** have been introduced  
Chaum Mix, Onion Routing, Crowds,...

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# Introduction

A lot of work in formal verification of **authentication** protocols (e.g. Needham-Schroeder, Otway-Rees, ...) but

**Formulation and verification of anonymity**  
is still quite immature

Our work is first to

- comprehensively formulate competing notions for “anonymity”, and
- actually verify real protocols, using **crypto-conscious epistemic logic**

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# Coauthors

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Peter van Rossum



Wouter Pieters



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Peter van Rossum



Wouter Pieters



Flavio D. Garcia

Full paper available:

**Provable Anonymity.**

F. Garcia, I. Hasuo, W. Pieters, and P. van Rossum.

To appear in FMSE 2005.



# Motivating example: onion routing

Introduced by [Chaum, '81] and  
[Goldschlag, Reed, Syverson, '96]

Practical implementation available as  
TOR (The Onion Router), <http://tor.eff.org>

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# Motivating example: onion routing

$A$  tries to send a message  $m$  to  $B$  anonymously

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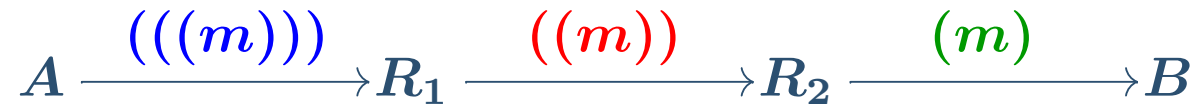
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$\{-\}_X$ : public-key encryption       $n_i$ : nonce



$$(m) = \{m\}_B$$

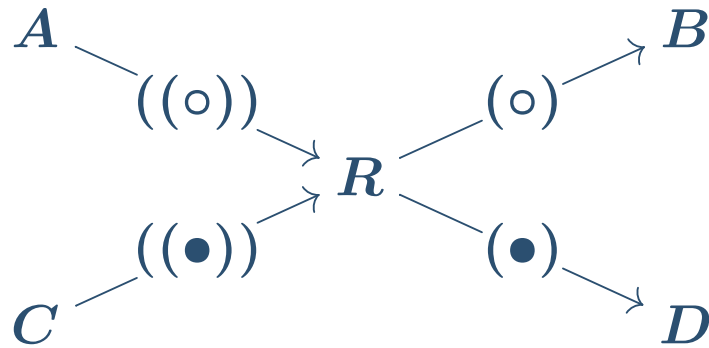
$$((m)) = \{n_1, B, (m)\}_{R_2}$$

$$(((m))) = \{n_0, R_2, ((m))\}_{R_1}$$



# Onion routing

actual run



where

$$((o)) = \{n, X, (o)\}_R$$

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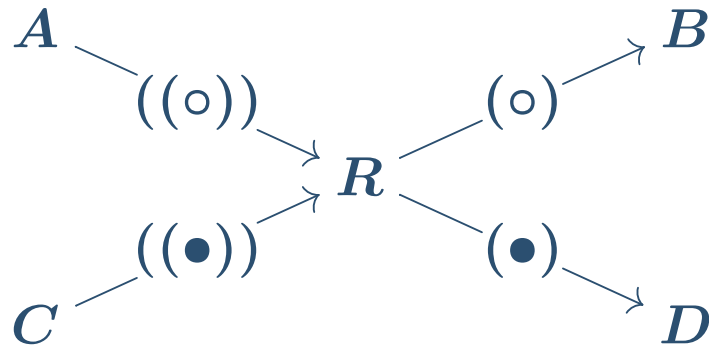
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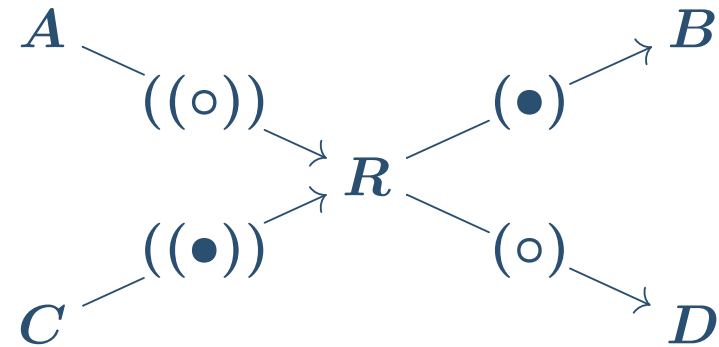


# Onion routing

actual run



“counter” run



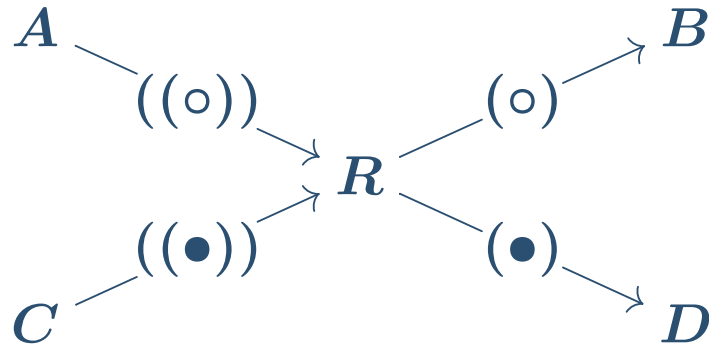
where

$$((o)) = \{n, X, (o)\}_R$$

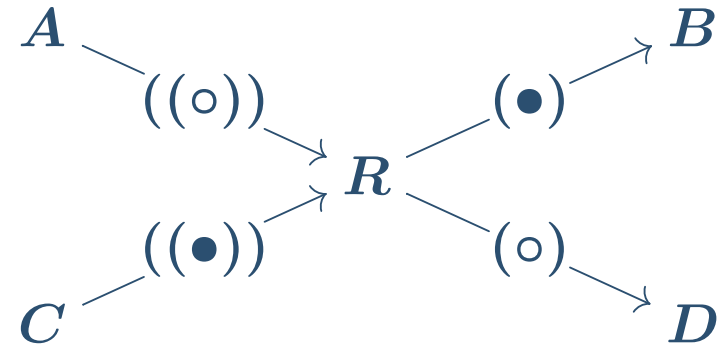
This is “anonymous” because the counter run is equally possible, so adversary is not sure whether  $A$  sent something to  $B$  or  $C$

# Onion routing

actual run



“counter” run



where

$$((o)) = \{n, X, (o)\}_R$$

Anonymity fails when:

- private key of  $R$  is compromised
- we omit nonces,  $((o)) = \{X, (o)\}_R$
- not enough padding, e.g.  $C$  is absent



# Various “anonymity”

A number of proposals and objections...

*Anonymity, Unobservability, Pseudonymity, and Identity Management – A Proposal for Terminology*  
(Ongoing draft from July 2000)

[http://dud.inf.tu-dresden.de/Literatur\\_V1.shtml](http://dud.inf.tu-dresden.de/Literatur_V1.shtml)

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With **epistemic language** we can formulate and verify competing notions in a uniform manner! [Halpern, O’Neill]

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# Epistemic logic

 $\Box_A \varphi$ 

**A knows  $\varphi$**

 $\Diamond_A \varphi \quad (:= \neg \Box_A \neg \varphi)$ 

**A suspects  $\varphi$**

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**Semantics**  $(W, \{\cong_A \mid A : \text{agent}\})$

- $W$ : set of possible worlds
- $\cong_A$ : observational equivalence for  $A$

$$x \models \Box_A \varphi \quad \stackrel{\text{def}}{\iff} \quad \forall y \cong_A x. \quad y \models \varphi$$

$$x \models \Diamond_A \varphi \quad \stackrel{\text{def}}{\iff} \quad \exists y \cong_A x. \quad y \models \varphi$$



# “Anonymity” expressed with epistemic logic

## Sender anonymity

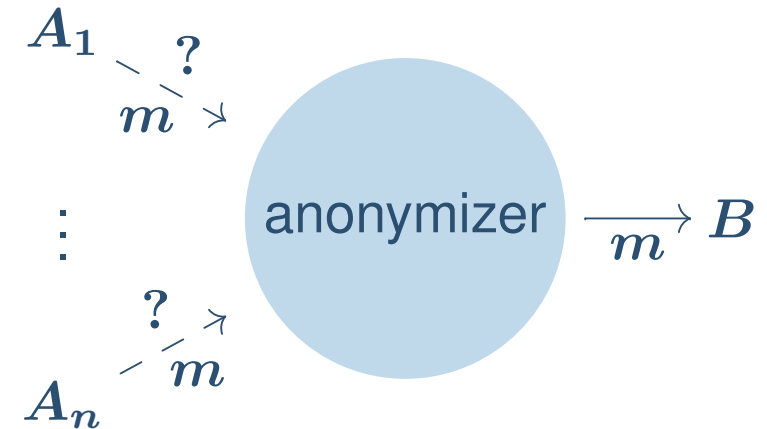
Given:  $B$  receives message (containing)  $m$ .

### Weak ver.



Not sure if  $A$  sent  $m$

### Anonymity set $\{A_1, \dots, A_n\}$



Each  $A_i$  is suspected as sender

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# “Anonymity” expressed with epistemic logic

## Sender anonymity

Given:  $B$  receives message (containing)  $m$ .

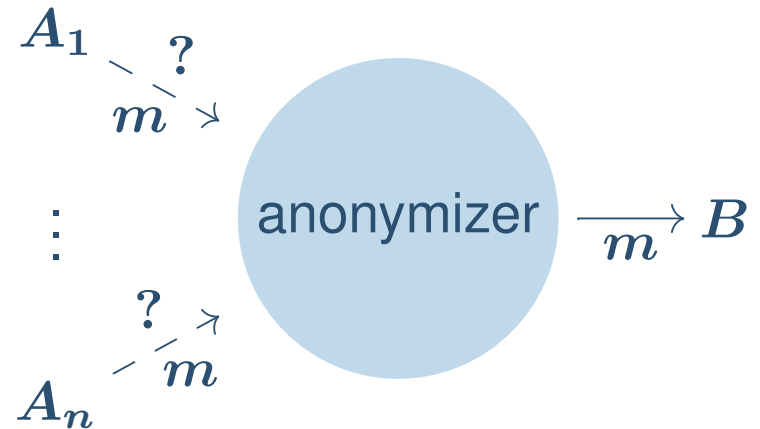
### Weak ver.



Not sure if  $A$  sent  $m$

$$\neg \Box_B A \text{ Sends } m$$

### Anonymity set $\{A_1, \dots, A_n\}$



Each  $A_i$  is suspected as sender

$$\begin{aligned} & \Diamond_B A_1 \text{ Sends } m \\ \wedge & \Diamond_B A_2 \text{ Sends } m \\ \wedge & \dots \\ \wedge & \Diamond_B A_n \text{ Sends } m \end{aligned}$$

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# “Anonymity” expressed with epistemic logic

## Unlinkability



Adversary is not sure if *A* sent something to *B*.

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# “Anonymity” expressed with epistemic logic

## Unlinkability



Adversary is not sure if  $A$  sent something to  $B$ .

$$\neg \Box_{\text{spy}} \exists m. (A \text{ Sends } m \wedge B \text{ Receives } m)$$

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# “Anonymity” expressed with epistemic logic

## Plausible deniability

$R$  can claim it is not aware of content  $m$

“I relayed something, but don’t know what it was!”



$$(m) = \{m\}_B$$

$$((m)) = \{n_1, B, (m)\}_R$$

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# “Anonymity” expressed with epistemic logic

## Plausible deniability

$R$  can claim it is not aware of content  $m$

“I relayed something, but don’t know what it was!”



$$(m) = \{m\}_B$$

$$((m)) = \{n_1, B, (m)\}_R$$

$$\forall m. \neg \Box_R (R \text{ Sends } m)$$

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# Semantics of epistemic operators

Possible world = a **run**, or **trace** of protocol

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● Reinterpretation of messages

● Observational equivalence

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Two aspects of observational equivalence  $\cong_A$ :

- Not every event is observed by an agent  
(However we assume global eavesdropper as adversary)

- **Use of cryptographic operation**

Encryptions/hashes makes messages look random junk!  
(Mauw, Verschuren, de Vink)



# Semantics of epistemic operators

However, two random junks

$$\{m\}_A \quad \text{and} \quad \{\{m\}_A\}_B$$

should be related.

That is, mapping all undecryptable messages to single  $\perp$  is not fine enough.

Our approach is finer than preceding work, taking care of this point.

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# Reinterpretation of messages

Our approach is finer, using **reinterpretation**

We cheat adversary, by reinterpreting

message which looks  
junk for adversary

into

another message

in the way adversary cannot detect a lie.

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# Reinterpretation of messages

Our approach is finer, using **reinterpretation**

We cheat adversary, by reinterpreting

message which looks  
junk for adversary

into

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in the way adversary cannot detect a lie.

**Definition**  $U$ : a set of messages (e.g. spy's possession)  
Permutation  $\pi$  of messages is **reinterpretation under  $U$**  if:

$$\pi(p) = p$$

for a primitive term  $p$

$$\pi(\{m\}_K) = \{\pi(m)\}_K$$

if  $\begin{cases} m, K \in U, \text{ or} \\ \{m\}_K, K^{-1} \in U \end{cases}$

$$\pi(\text{hash}(m)) = \text{hash}(\pi(m))$$

if  $m \in U$

$$\pi(\langle m_1, m_2 \rangle) = \langle \pi(m_1), \pi(m_2) \rangle$$

In short,  $\pi$  preserves term structures available in  $U$ .

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# Observational equivalence

## Definition

$$r \cong_A r'$$

def  
 $\longleftrightarrow$

$\exists \pi$ , reinterpretation under  $A$ 's possession, s.t.

$$\pi(r|_A) = r'|_A$$

where  $r|_A$ :  **$A$ -visible part** of  $r$

- For  $A \neq \text{spy}$ ,  $r|_A$  consists of events where  $A$  is sender or receiver.
- $r|_{\text{spy}} = r$ , i.e., **spy** is a global eavesdropper.

$\cong_A$  is in fact an equivalence relation.  
Hence  $\Box_A$  is S5-modality.

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# Observational equivalence

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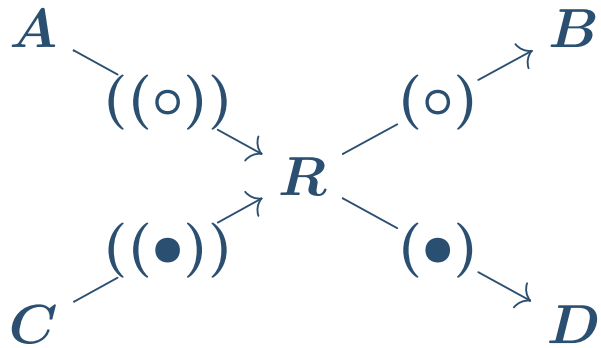
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- Semantics of epistemic operators
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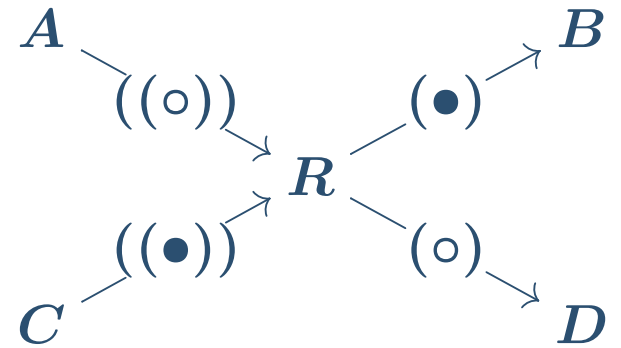
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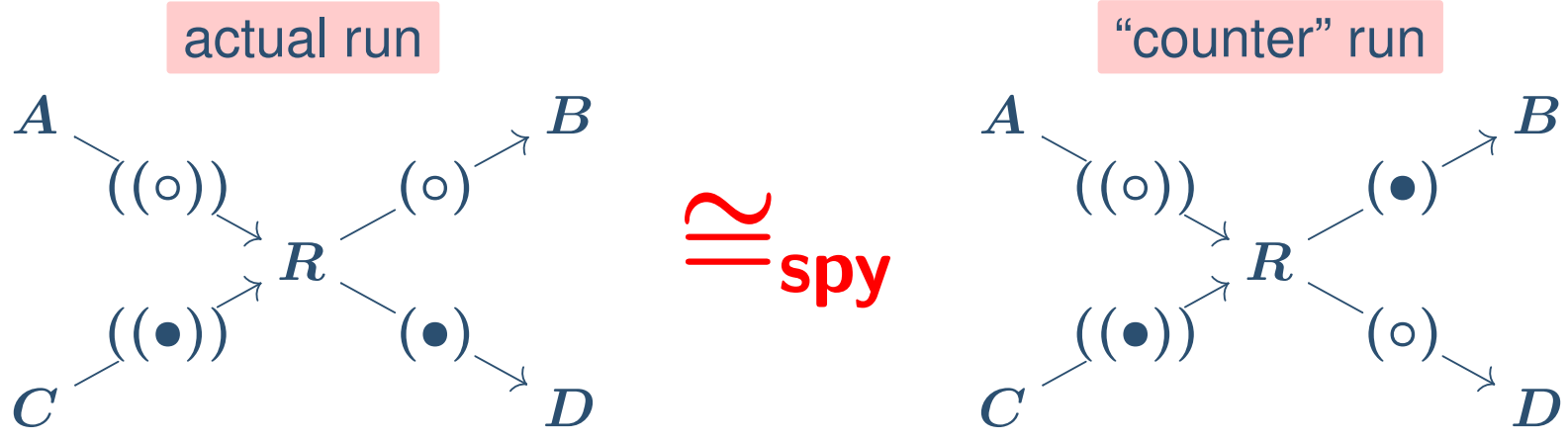
“counter” run



where  $((o)) = \{n, (o)\}_R$        $n$  : random nonce

# Observational equivalence

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  - Semantics of epistemic operators
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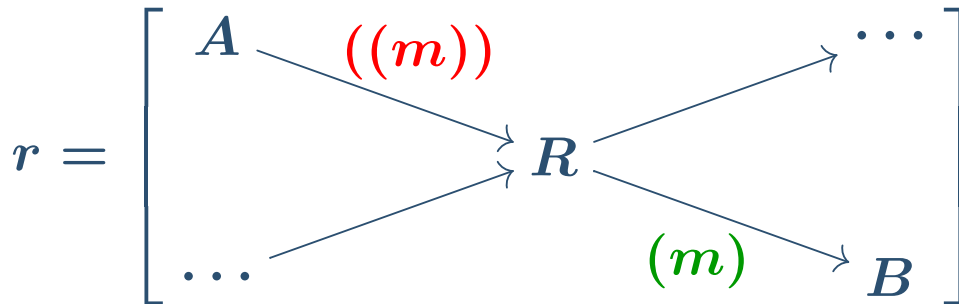


where  $((o)) = \{n, (o)\}_R$        $n$  : random nonce

Reinterpretation  $\pi$ :

$((o)) \mapsto ((o))$	$((bullet)) \mapsto ((bullet))$
$(o) \mapsto (bullet)$	$(bullet) \mapsto (o)$

# Onion routing: unlinkability



where

$$((m)) = \{n, B, (m)\}_R$$

## Theorem

$$r \models \neg \Box_{\text{spy}} \exists m. (A \text{ Sends } m \wedge B \text{ Receives } m)$$

( $A$  and  $B$  are unlinkable)



some  $C \neq A$  sends  $((m'))$  to  $R$ , before  $R$  relays  $(m)$ .  
(there is enough padding)

**Proof**  $[\Rightarrow]$  By contradiction. In  $\forall r' \cong_{\text{spy}} r$ ,  $\pi$  of  $(m)$  must result from  $\pi$  of  $((m))$ . Hence they have same core of onion.



# Onion routing: plausible deniability

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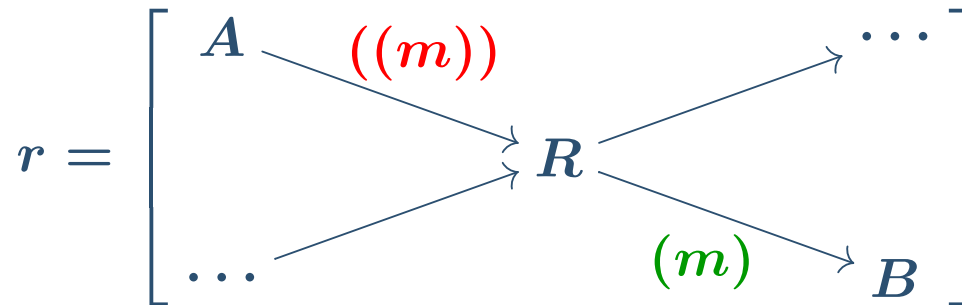
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- Onion routing: unlinkability
- Onion routing: plausible deniability
- Forgotten nonces
- Private-key compromised
- Other examples

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where

$$((m)) = \{n, B, (m)\}_R$$

**Theorem** For any  $m$ ,

$$r \models \neg \Box_R (R \text{ Sends } m)$$

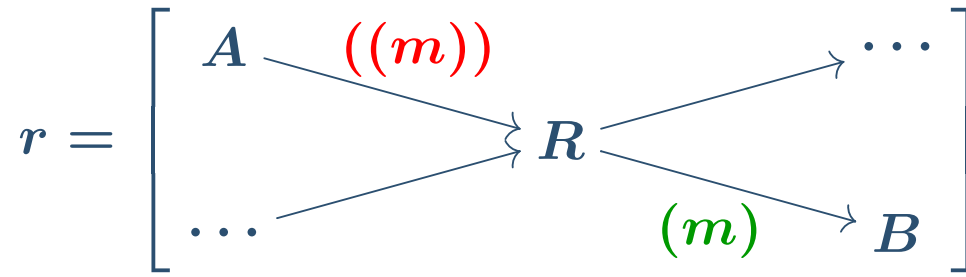
**Proof**  $R$  doesn't possess private-key of  $B$ , hence for  $\forall m'$ ,  $\exists \pi$ : reinterpretation under  $R$ , which gives

$$(\text{actual run}) \cong_R A \xrightarrow{((m'))} R \xrightarrow{(m')} B$$



# Flawed onion routing 1: forgotten nonces

We forget nonces beneath skin of onion.



where

$$((m)) = \{B, (m)\}_R$$

**Theorem** Unlinkability fails, i.e.

$$r \models \square_{\text{spy}} \exists m. (A \text{ Sends } m \wedge B \text{ Receives } m)$$

**Proof** Any reinterpretation  $\pi$  must be like

$$(m) \mapsto m_1 \quad ((m)) \mapsto \{B, m_1\}_R$$

since  $\text{spy}$  possesses public-key of  $R$ .

Hence any  $r' \cong_{\text{spy}} r$  is like  $A \xrightarrow{\{B, m_1\}_R} R \xrightarrow{m_1} B$ ,  
therefore  $r' \models \exists m. (A \text{ Sends } m \wedge B \text{ Receives } m)$ .

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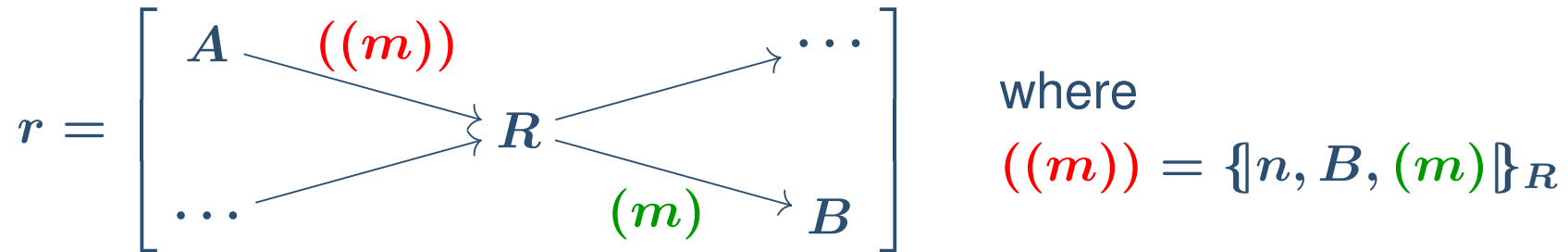
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- Onion routing: unlinkability
- Onion routing: plausible deniability
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# Flawed OR 2: private-key compromised

Private-key of  $R$  possessed by spy.



**Theorem** Unlinkability fails, i.e.

$$r \models \Box_{\text{spy}} \exists m. (A \text{ Sends } m \wedge B \text{ Receives } m)$$

**Proof** Any reinterpretation  $\pi$  must be like

$$(m) \mapsto m_1 \quad ((m)) \mapsto \{n, B, m_1\}_R$$

Hence any  $r' \cong_{\text{spy}} r$  is like

$$A \xrightarrow{\{n, B, m_1\}_R} R \xrightarrow{m_1} B, \text{ therefore}$$

$$r' \models \exists m. (A \text{ Sends } m \wedge B \text{ Receives } m).$$

- Onion routing: unlinkability
- Onion routing: plausible deniability
- Forgotten nonces
- **Private-key compromised**
- Other examples



# Other examples

- Can detect even more subtle (artificial) flaw in Onion Routing: see full paper
- Crowds, for sender anonymity
- Internet voting protocol RIES  
In real use in the Netherlands (ongoing analysis)

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- Onion routing: unlinkability
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# Conclusion

- **Anonymity** is important, hard to define, hard to verify
- Competing notions are straightforwardly expressed with **epistemic** language
- First to consider use of **cryptographic operations** in semantics of epistemic logic
- Finer treatment of cryptographic operations using **reinterpretation**
- Able to **uniformly** verify/falsify wide variety of anonymizing systems

## Future work

- Justification of reinterpretation (cf. Abadi, Rogaway)
- Tool support    ■ Quantitative analysis

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# Conclusion

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**Thank you for your attention!**

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