Term Evaluation Systems with Refinements (to appear in Proc. FLOPS '24)

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Contributions

demonstrate the use

- of a **term-rewriting technique** (namely, critical pair analysis)
- for proving **observational equivalence** on programs (namely, Sands' contextual improvement)

by introducing *Term Evaluation and Refinement Systems (TERS)*

Critical pair analysis

- "When rewriting diverges, can it be joined?"
- a fundamental technique in term rewriting
  - enumerating patterns of interference between rewrite rules
- **automatable**

Sands' Contextual improvement

- a **quantitative** variant of observational equivalence
- *u improves t* $\iff$ for any context C, if $C[t] \xrightarrow{k} v$ then $C[u] \xrightarrow{m} v' \ \wedge \ k \geq m \ \wedge \ v =_{Val} v'$

$\lambda x\,.\,t =_{Val} \lambda x'\,.\,t'$
$\underline{n} =_{Val} \underline{n}$

Evaluation	Refinement
specification of operational semantics	model of compiler optimisation (to be **validated** wrt. evaluation)
run-time rewriting	compile-time rewriting
rule-based rewriting	rule-based rewriting
constrained by Felleisen's **evaluation contexts**	unrestricted
beta-reduction $E[(\lambda x\,.\,t)\ v] \to E[t\{v/x\}]$	beta-law $C[(\lambda x\,.\,t)\ v] \Rightarrow C[t\{v/x\}]$ eta-law $C[\lambda x\,.\,v\ x] \Rightarrow C[v]$

Term Evaluation and Refinement Systems (TERS)

Def. A TERS $(\Sigma, \mathscr{E}, \mathscr{R}, Ectx, SClass)$ is given by:

- a signature Σ
- a set $\mathscr{E}$ of evaluation rules $l \to r$
- a set $\mathscr{R}$ of refinement rules $l \Rightarrow r$
- a set $Ectx \subseteq Ctx$ of evaluation contexts
- a set $SClass$ of syntax classes including $Val \subseteq \mathrm{NF}(\to_{\mathscr{E}})$

$$\frac{(l \to r) \in \mathcal{E} \quad E \in Ectx}{E[l\theta] \to_{\mathcal{E}} E[r\theta]} \qquad \frac{(l \Rightarrow r) \in \mathcal{R} \quad C \in Ctx}{C[l\theta] \Rightarrow_{\mathcal{R}} C[r\theta]}$$

Ex. A TERS **Hndl** for effect handlers [Pretnar '15]

Syntax class *Sclass*
functions $F ::= x \mid \mathtt{fun}(x.P)$
values $V ::= \mathtt{true} \mid \mathtt{false} \mid F \mid H$
handlers $H ::= \mathtt{handler}_1(x.P, x.k.P_1) \mid \mathtt{handler}_0(x.P)$
computations $P, P_1, P_2 ::= \mathtt{return}(V) \mid \mathtt{op}(V, y.P) \mid \mathtt{do}(P_1, x.P_2) \mid \mathtt{if}(V, P_1, P_2) \mid F\ V \mid \mathtt{with\_handle}(H, P)$
Evaluation contexts *Ectx* $E ::= \Box \mid \mathtt{do}(E, x.P) \mid \mathtt{with\_handle}(H, E)$
Evaluation rules $\mathcal{E}$ where $i \in [2]$

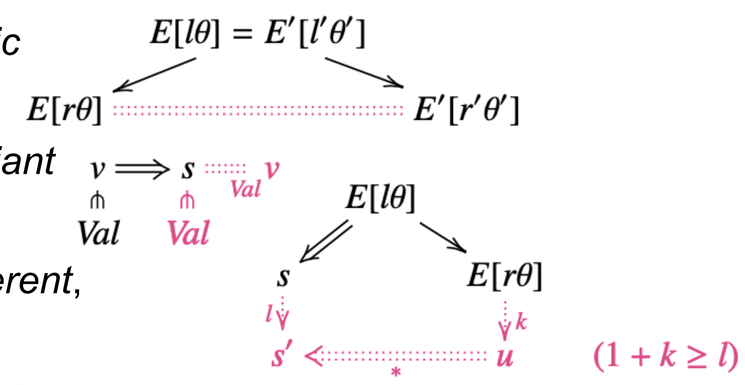
$\mathtt{do}(\mathtt{return}(V), x.P[x]) \to P[V]$ (1)
$\mathtt{do}(\mathtt{op}_i(V, y.P_1[y]), x.P_2[x]) \to \mathtt{op}_i(V, y.\mathtt{do}(P_1[y], x.P_2[x]))$ (2)
$\mathtt{if}(\mathtt{true}, P_1, P_2) \to P_1$ (3)
$\mathtt{if}(\mathtt{false}, P_1, P_2) \to P_2$ (4)
$\mathtt{fun}(x.P[x])\ V \to P[V]$ (5)
In the following three rules, $h_1 \equiv \mathtt{handler}_1(x.P[x], x.k.P_1[x,k])$.
$\mathtt{with\_handle}(h_1, \mathtt{return}(V)) \to P[V]$ (6)
$\mathtt{with\_handle}(h_1, \mathtt{op}_1(V, y.P'[y])) \to P_1[V, \mathtt{fun}(y.P'[y])]$ (7)
$\mathtt{with\_handle}(h_1, \mathtt{op}_1(V, y.P'[y])) \to P_1[V, \mathtt{fun}(y.\mathtt{with\_handle}(h_1, P'[y]))]$ (7')
$\mathtt{with\_handle}(h_1, \mathtt{op}_2(V, y.P'[y])) \to \mathtt{op}_2(V, y.\mathtt{with\_handle}(h_1, P'[y]))$ (8)
In the following two rules, $h_0 \equiv \mathtt{handler}_0(x.P[x])$.
$\mathtt{with\_handle}(h_0, \mathtt{return}(V)) \to P[V]$ (9)
$\mathtt{with\_handle}(h_0, \mathtt{op}_i(V, y.P'[y])) \to \mathtt{op}_i(V, y.\mathtt{with\_handle}(h_0, P'[y]))$ (10)
Refinement rules $\mathcal{R}$
$\mathtt{do}(P, x.\mathtt{return}(x)) \Rightarrow P$ (r3)
$\mathtt{do}(\mathtt{do}(P_1, x_1.P_2[x_1]), x_2.P_3[x_2]) \Rightarrow \mathtt{do}(P_1, x_1.\mathtt{do}(P_2[x_1], x_2.P_3[x_2]))$ (r4)
$\mathtt{if}(V, P[\mathtt{true}], P[\mathtt{false}]) \Rightarrow P[V]$ (r7)
$\mathtt{fun}(x.F\ x) \Rightarrow F$ (r9)
$\mathtt{with\_handle}(\mathtt{handler}_0(x.P[x]), P') \Rightarrow \mathtt{do}(P', x.P[x])$ (r13)

Local coherence and contextual improvement

Thm. (sufficient conditions for improvement)

If a TERS is

- *deterministic*
- *value-invariant*
- *locally coherent*,



then refinement $\mathscr{R}$ is *improvement* w.r.t. evaluation $\mathscr{E}$

(i.e. s improves t, for each $t \Rightarrow_{\mathscr{R}} s$).

Thm. (critical pair theorem)

A *well-behaved* TERS

- refinement respecting evaluation contexts $E \Longrightarrow C$ (Ectx, Ectx)
- linearity for $(l \Rightarrow r) \in \mathscr{R}$
- left-linearity for $(l \to r) \in \mathscr{E}$
- lhs l being Miller's HO pattern for $(l \multimap r) \in \mathscr{R} \cup \mathscr{E}$
- …

is *locally coherent* iff every critical pair is *joinable*.

Prop. **Hndl** is deterministic, value-invariant and locally coherent.

10 critical pairs **automatically** enumerated & checked for joinability