

Hypernet semantics and robust observational equivalence

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joint work with
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(University of Birmingham)

Overview

1. Motivation: robustness of observational equivalence
2. Hypernet semantics
3. Locality & step-wise reasoning
4. Discussion: complication of simulation notion

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1. Motivation: robustness of observational equivalence
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Observational equivalence on program fragments

“Do two program fragments behave the same?”

“Is it safe to replace a program fragment with another?”

```
let x = 100 in  
let y = 50 in  
y + y
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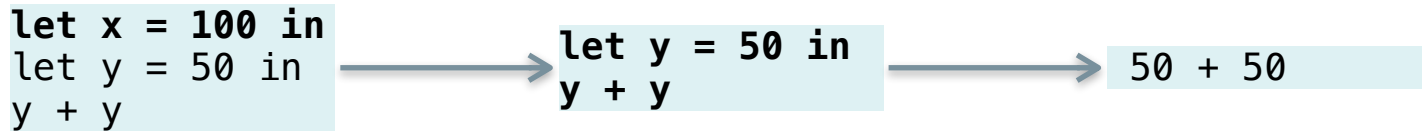


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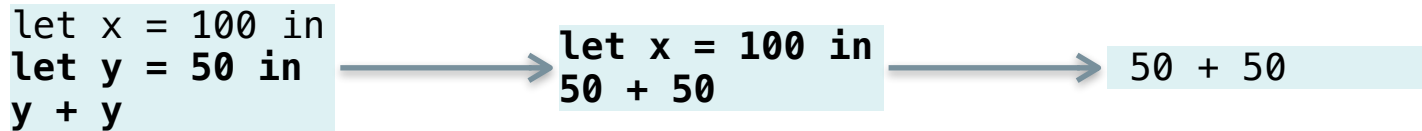
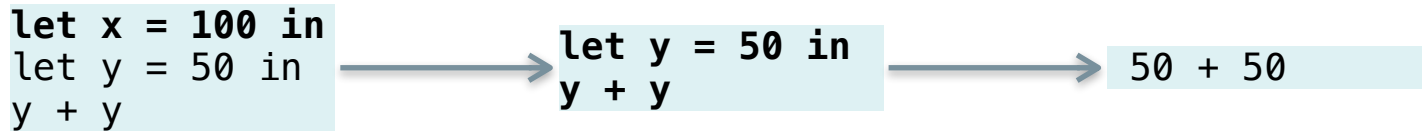
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If YES (“Two program fragments are observationally equal.”):

- justification of compiler optimisation
- program verification

Observational equivalence on program fragments

“Do two program fragments behave the same?”

Observational equivalence on program fragments

“Do two program fragments behave the same?”

*“**What** program fragments behave the same?”*

the beta-law

$$(\lambda x.M)N \simeq M[x := N]$$

a parametricity law

$$\text{let } a = \text{ref } 1 \text{ in } \lambda x.(a := 2; !a) \simeq \lambda x.2$$

Robustness of observational equivalence

“Do two program fragments behave the same?”

*“**When do** program fragments behave the same?”*

the beta-law

$$(\lambda x . M) N \simeq M[x := N]$$

Does the beta-law always hold?

Robustness of observational equivalence

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$$(\lambda x. M) N \simeq M[x := N]$$

Does the beta-law always hold?

No, it's violated if program contexts use OCaml's Gc module:

$$(\lambda x. 0) 100 \not\simeq 0$$

for memory
management

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How **robust** is the beta-law then?

Robustness of observational equivalence

~~“Do two program fragments behave the same?”~~

“What fragments, **in which contexts**, behave the same?”

Robustness of observational equivalence

~~“Do two program fragments behave the same?”~~

“What fragments, **in which contexts**, behave the same?”

... in the presence of (arbitrary) language features:

pure vs. effectful (e.g. `50 + 50` vs. `ref 1`)

encoded vs. native (e.g. `State` vs. `ref`)

extrinsics (e.g. `Gc.stat`)

foreign language calls

Robustness of observational equivalence

~~“Do two program fragments behave the same?”~~

“What fragments, **in which contexts**, behave the same?”

... in the presence of (arbitrary) language features

Our (big) goal:

analysing robustness/fragility of observational equivalence,
using a general framework

Robustness of observational equivalence

~~“Do two program fragments behave the same?”~~

“What fragments, **in which contexts**, behave the same?”

... in the presence of (arbitrary) language features

Our result:

analysing robustness/fragility of observational equivalence,
using a graphical framework

- hypernet semantics: a *graphical* abstract machine
- *local & step-wise* reasoning to prove observational equivalence, with the concept of *robustness*

Overview

1. Motivation: robustness of observational equivalence

2. Hypernet semantics

3. Locality & step-wise reasoning

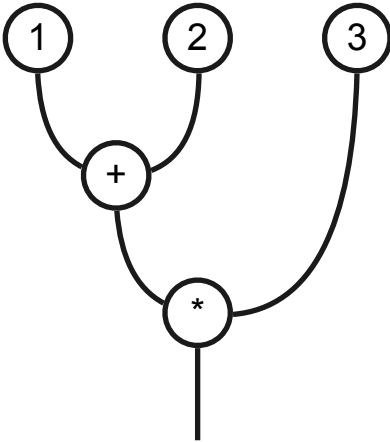
4. Discussion: complication of simulation notion

Hypernet semantics

- program execution by a *graphical* abstract machine
 - programs as
certain hierarchical hypergraphs (“*hypernets*”)
 - execution as
step-by-step strategical update of hypernets

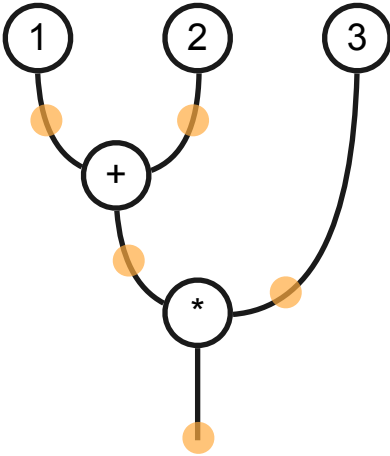
Programs, graphically as *hypernets*

Idea: abstracting away variable names, and more...

program	hypernet (hierarchical hypergraph)
<p data-bbox="363 801 691 843">(1 + 2) * 3</p>	

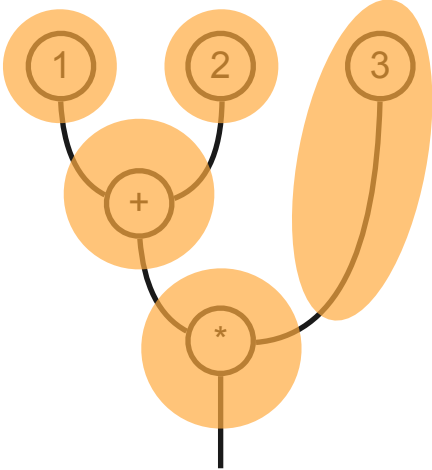
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<p data-bbox="363 796 691 845">(1 + 2) * 3</p>	<p data-bbox="1367 472 1746 605">nodes</p> 

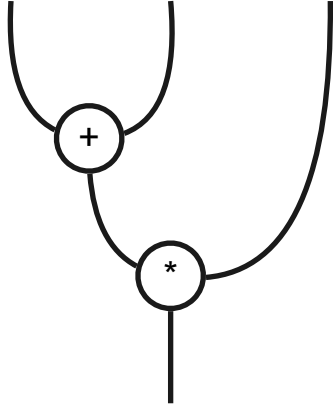
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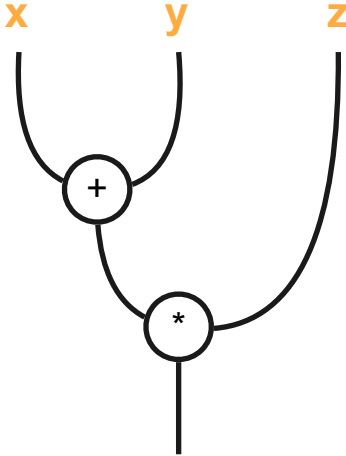
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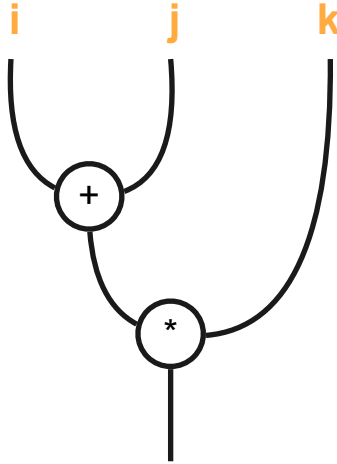
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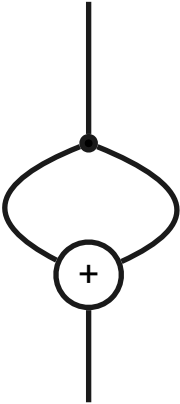
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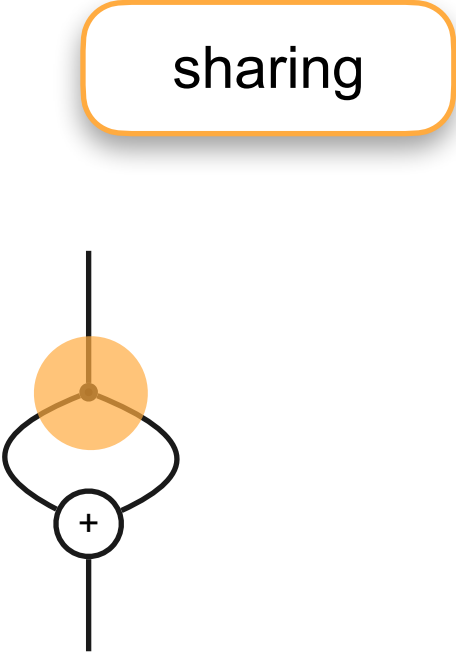
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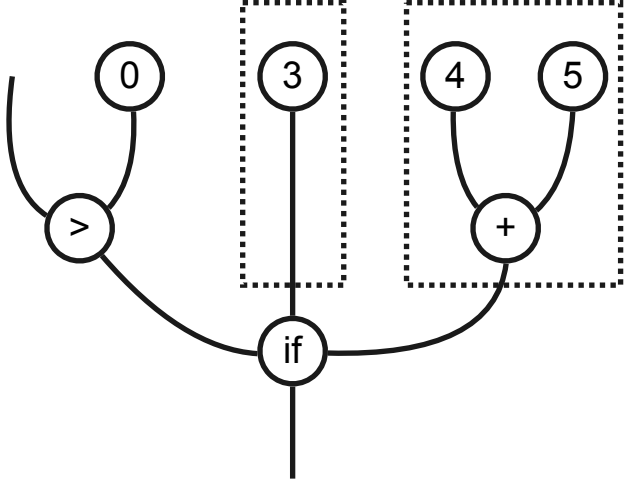
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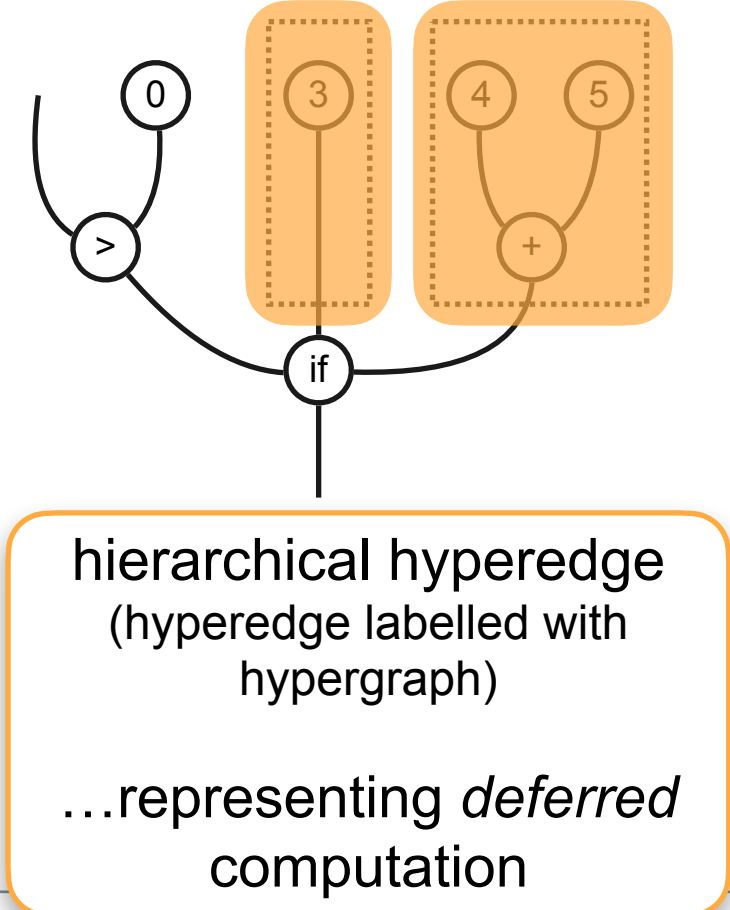
Programs, graphically as *hypernets*

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program	hypernet (hierarchical hypergraph)
<pre data-bbox="363 802 691 929">if x > 0 then 3 else 4 + 5</pre>	

Programs, graphically as *hypernets*

Idea: abstracting away variable names, and more...

program	hypernet (hierarchical hypergraph)
<pre data-bbox="363 803 691 929">if x > 0 then 3 else 4 + 5</pre>	 <p data-bbox="1126 996 1744 1168">hierarchical hyperedge (hyperedge labelled with hypergraph)</p> <p data-bbox="1116 1232 1744 1350">...representing <i>deferred</i> computation</p>

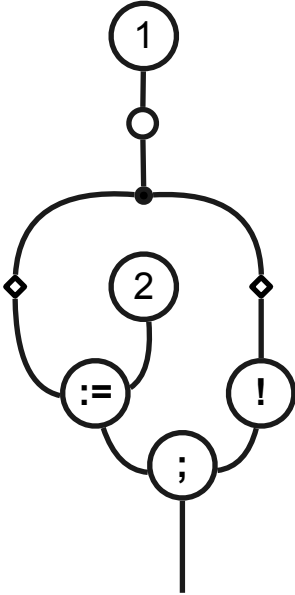
Programs, graphically as *hypernets*

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program	hypernet (hierarchical hypergraph)
$(\lambda x. x + x) 3$	

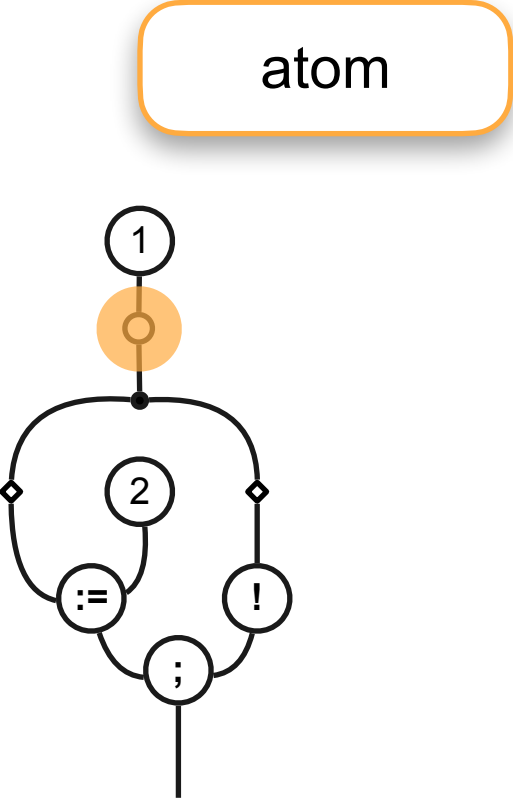
Programs, graphically as *hypernets*

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program	hypernet (hierarchical hypergraph)
<pre data-bbox="363 801 691 886">new a = 1 in a := 2; !a</pre>	

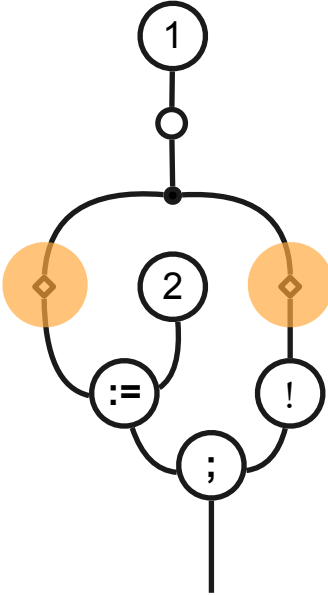
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program	hypernet (hierarchical hypergraph)
<pre data-bbox="363 801 691 886">new a = 1 in a := 2; !a</pre>	<p data-bbox="1367 472 1746 605">atom occurrences</p> 

Programs, graphically as *hypernets*

Idea: abstracting away variable names, and more...

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Idea: abstracting away variable names, and ~~more...~~

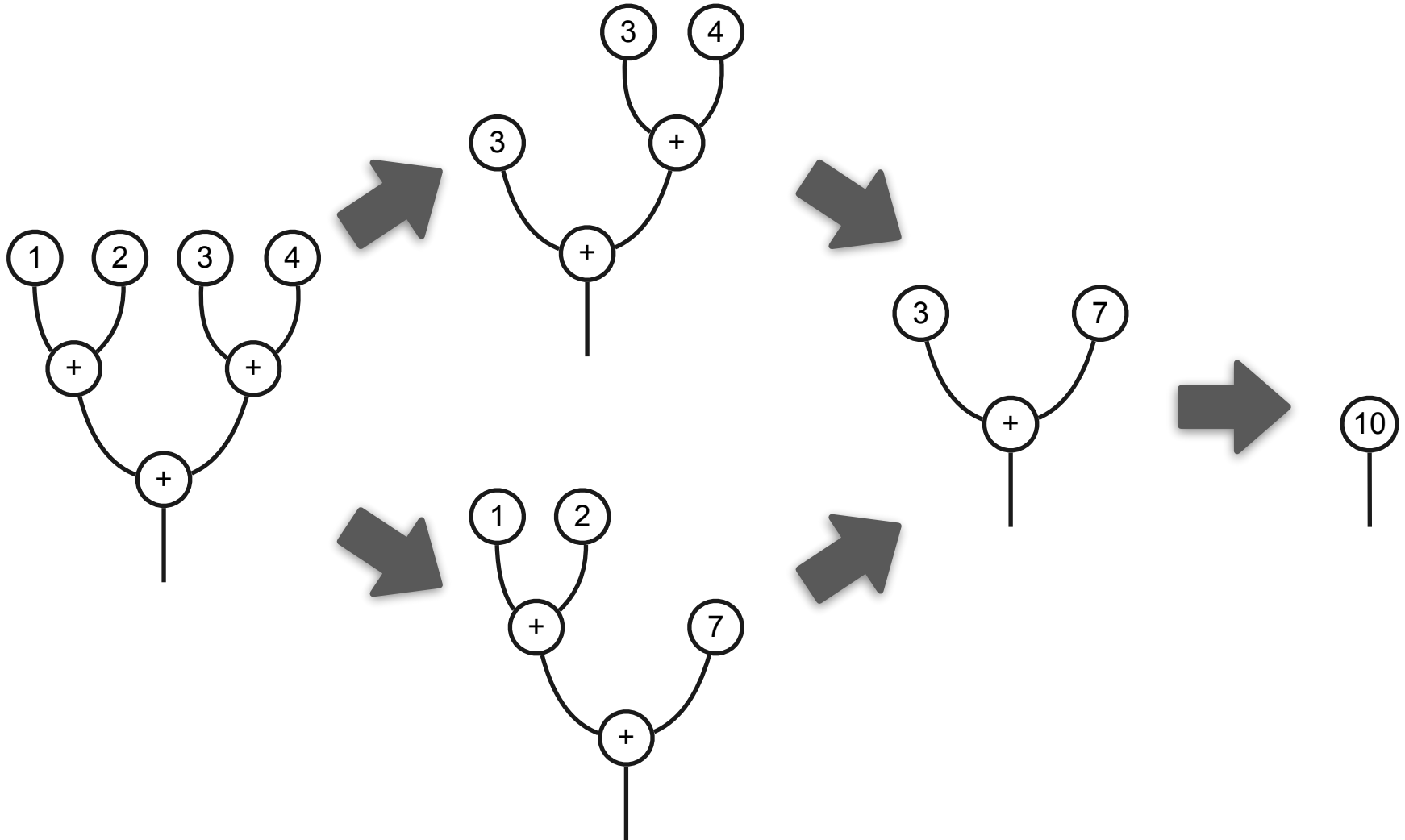
- making blocks of deferred computation explicit
- accommodating atoms (reference names/locations)

Program execution, graphically

Idea: updating hypernets step-by-step

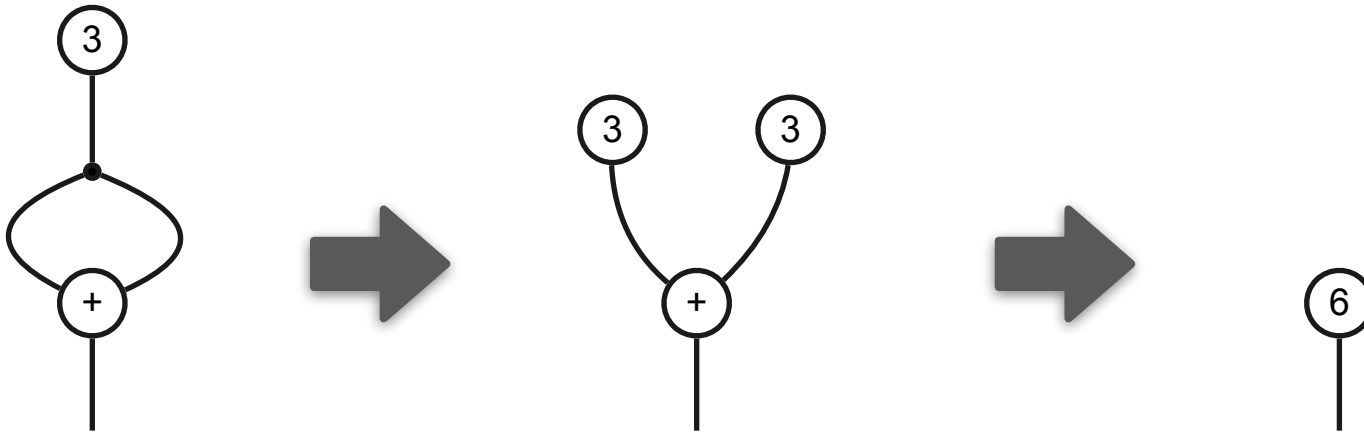
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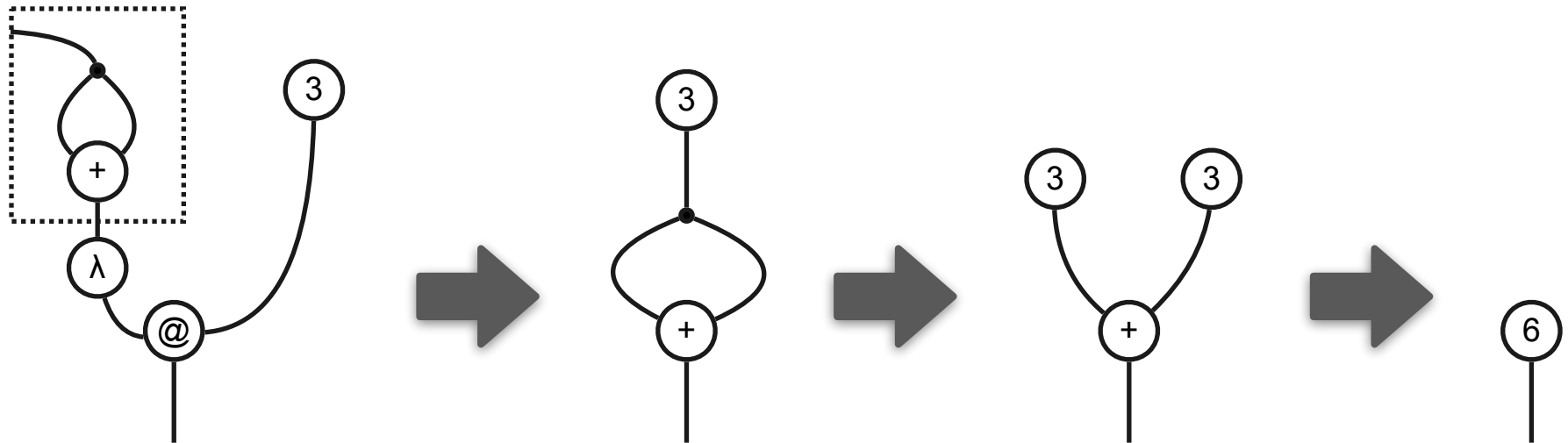


```
let x = 3 in  
x + x
```

```
3 + 3
```


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$(\lambda x. x + x) 3$

let $x = 3$ in
 $x + x$

$3 + 3$

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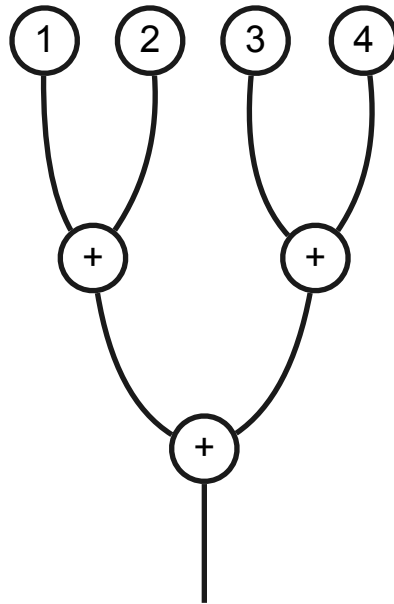
... and strategically, using *focus* with three modes:

- ① depth-first redex search
- ✓ backtracking
- ⚡ triggering update of hypernet

Program execution, graphically

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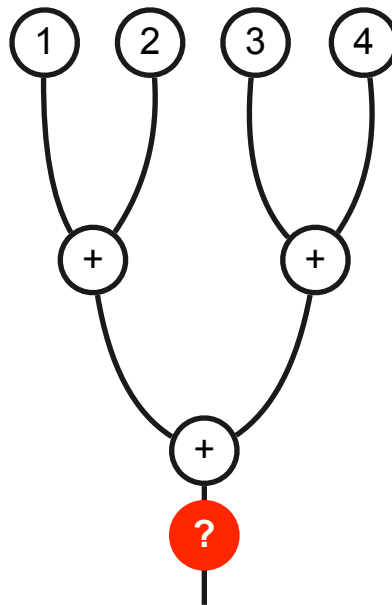
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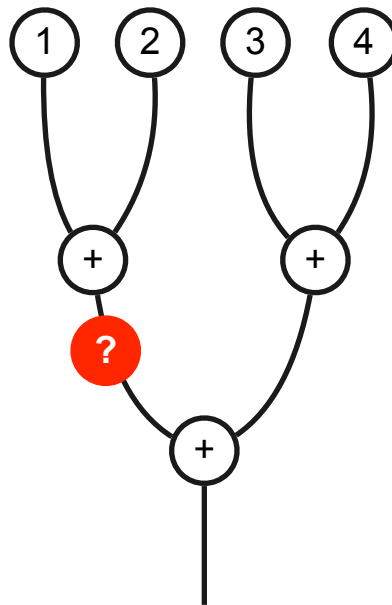


depth-first redex search

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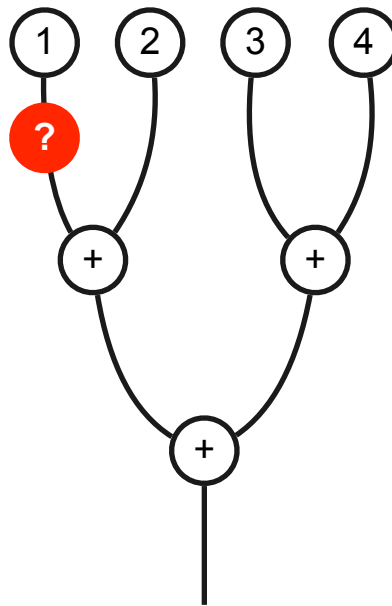


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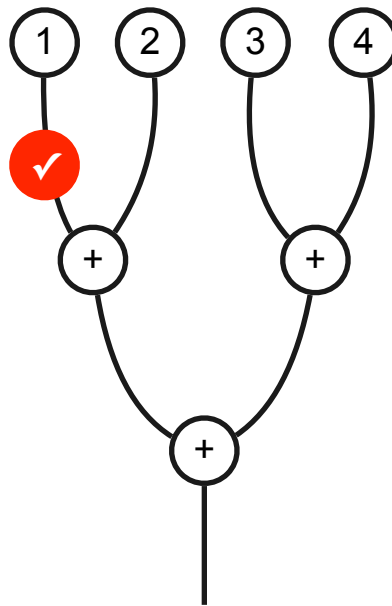


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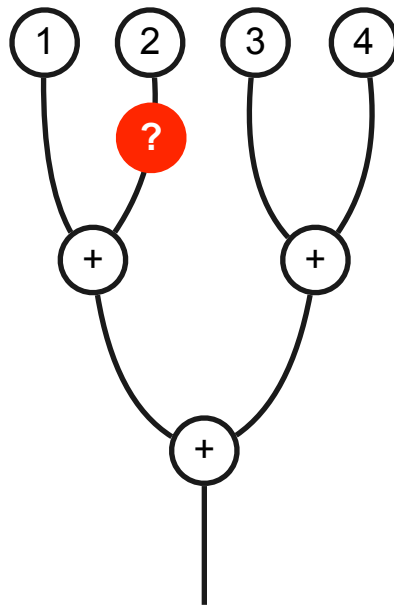


backtracking

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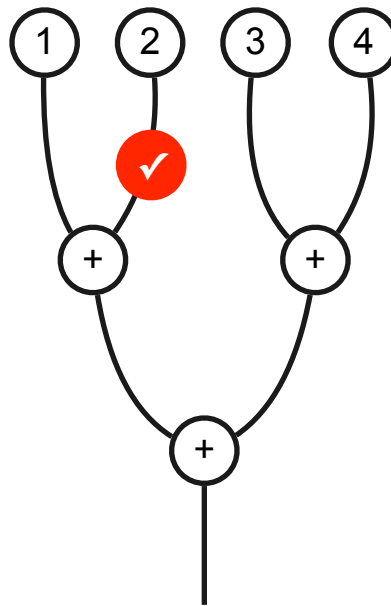


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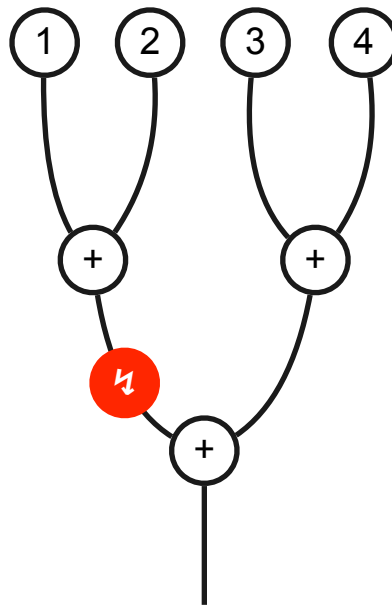


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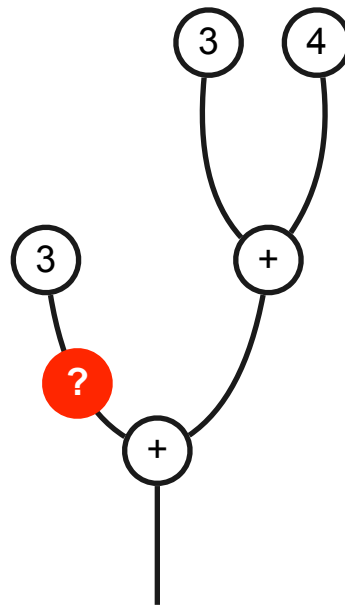


triggering update of hypernet

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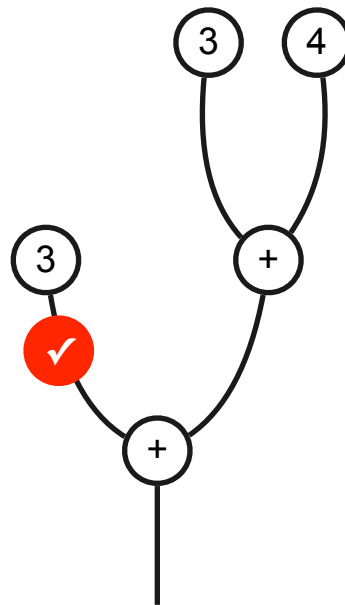


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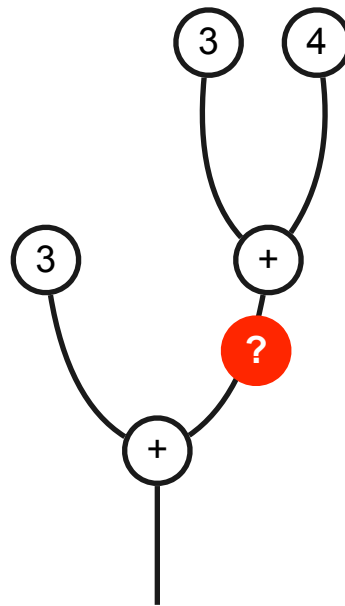


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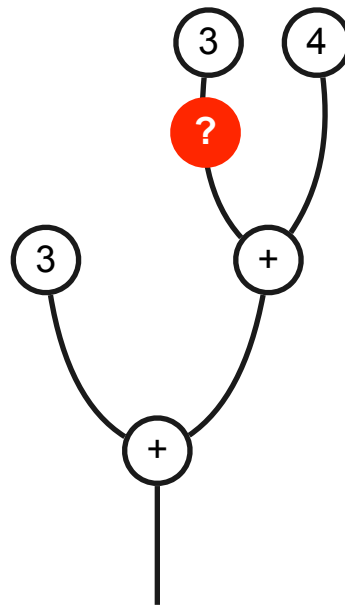


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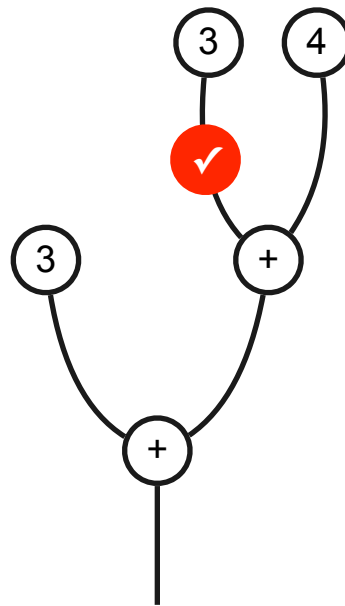


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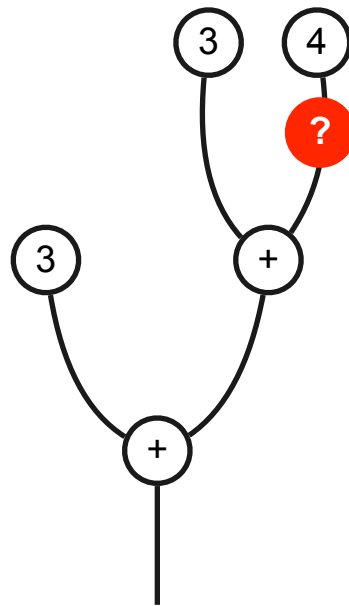


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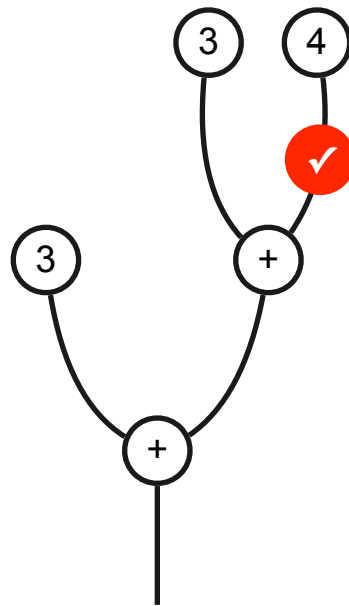


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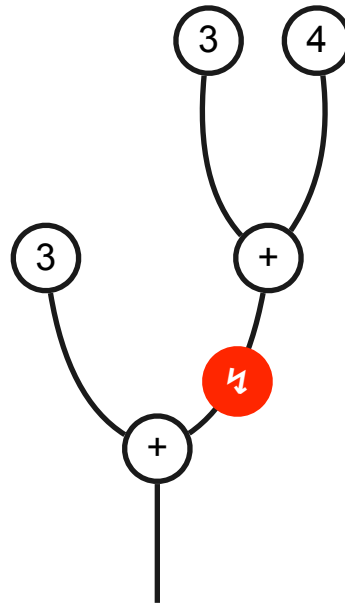


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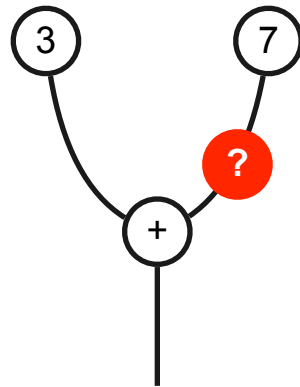
triggering update of hypernet

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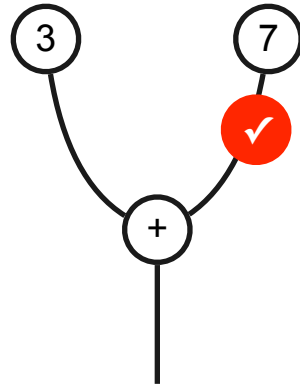


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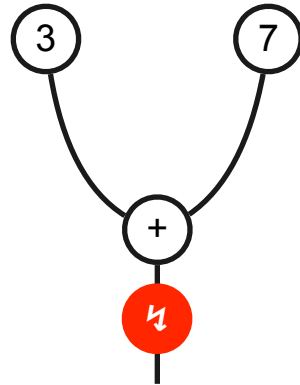
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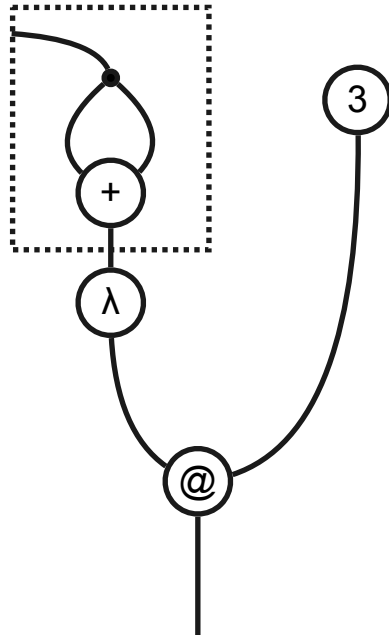
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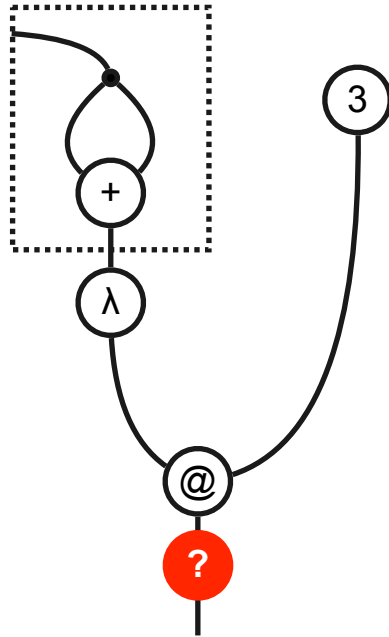
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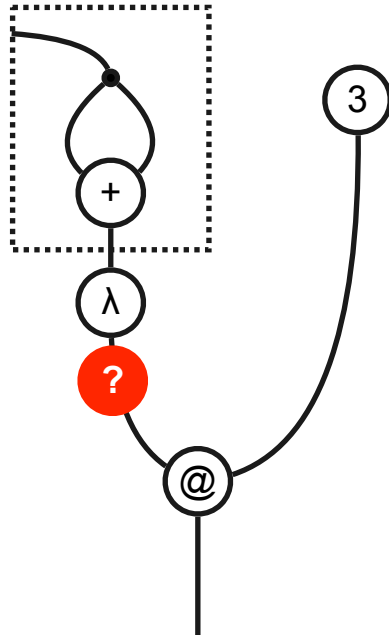


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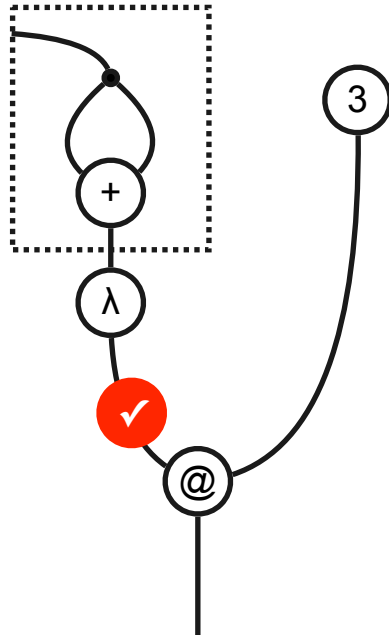


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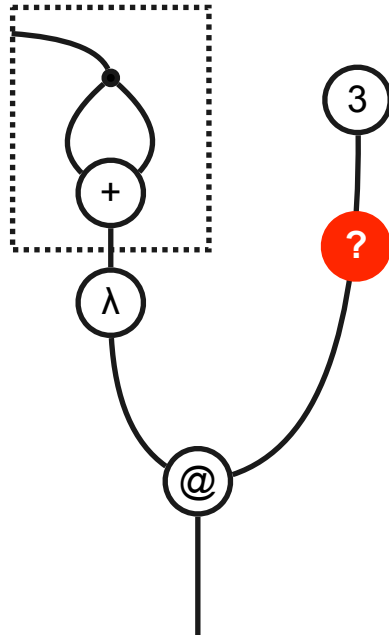


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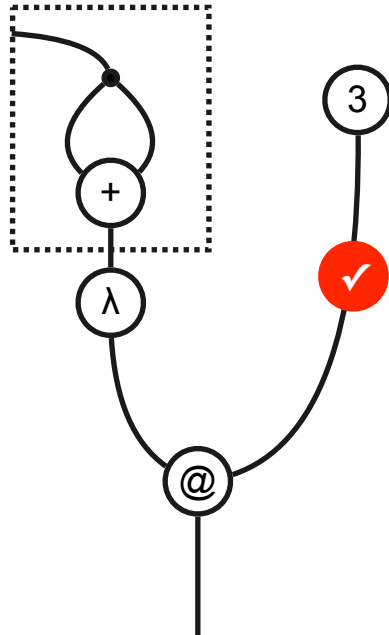


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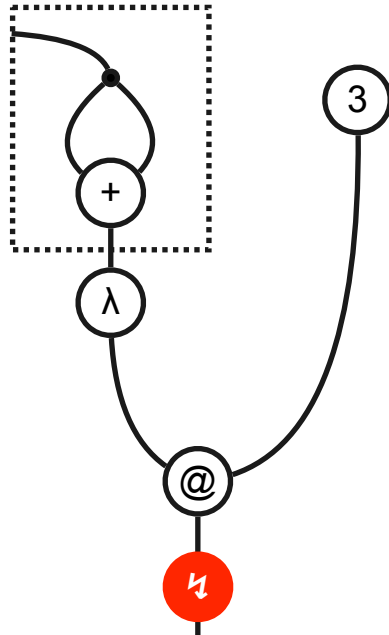


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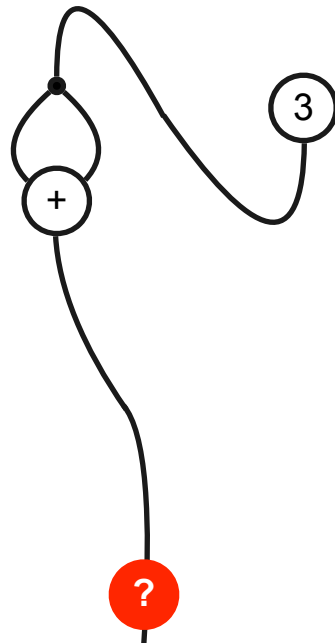


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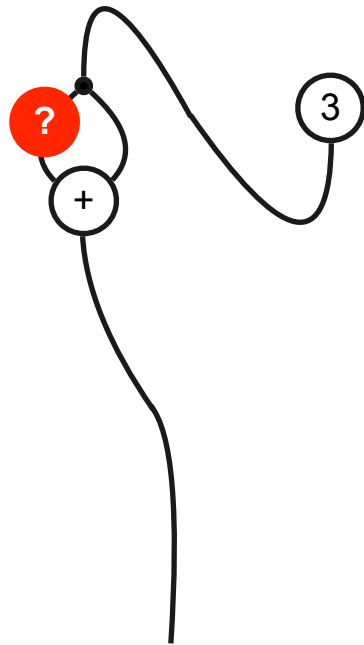


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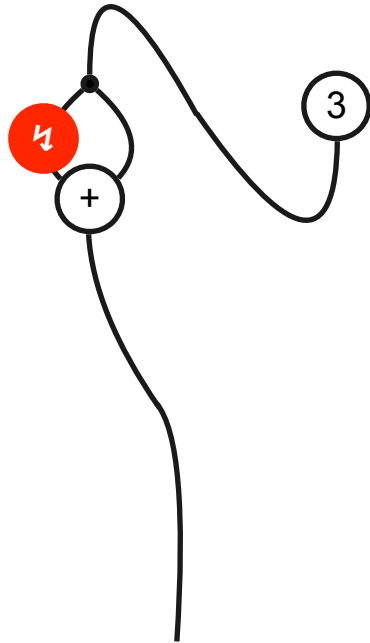


depth-first redex search

Program execution, graphically

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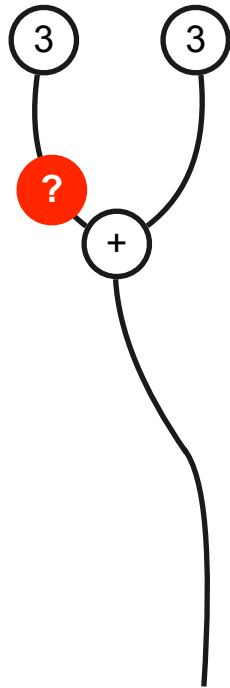


triggering update of hypernet

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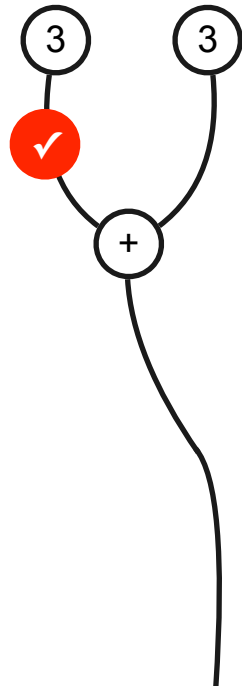


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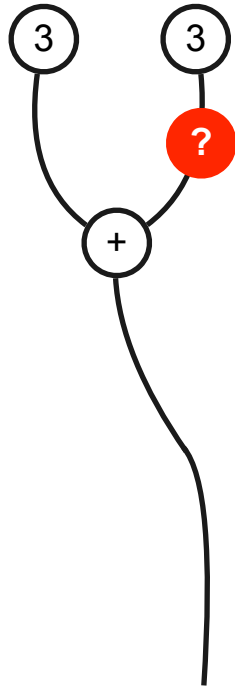


backtracking

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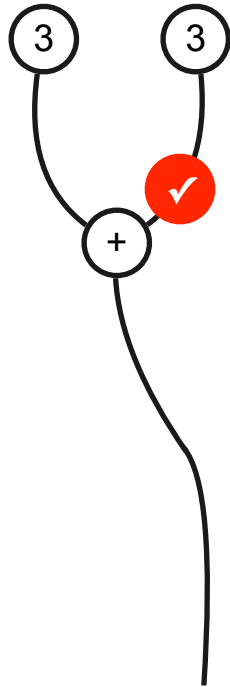


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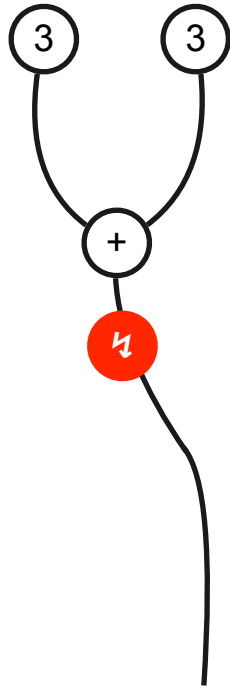


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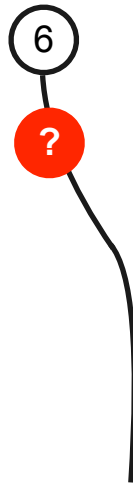
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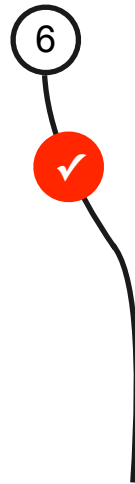


Program execution, graphically

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Hypernet semantics

- program execution by a *graphical* abstract machine
 - programs as
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- program execution by a *graphical* abstract machine
 - programs as
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 - execution as
step-by-step strategical update of hypernets
- state = hypernet with focus 
- transition = move of focus, or update of hypernet

Overview

1. Motivation: robustness of observational equivalence
2. Hypernet semantics
3. Locality & step-wise reasoning
4. Discussion: complication of simulation notion

Proof of observational equivalence, using *locality*

“Do two program fragments behave the same?”

Proof of observational equivalence, using *locality*

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“Do two sub-graphs behave the same in hypernet semantics?”

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- ★ Sub-graphs can represent parts of a program that are not necessarily well-formed,
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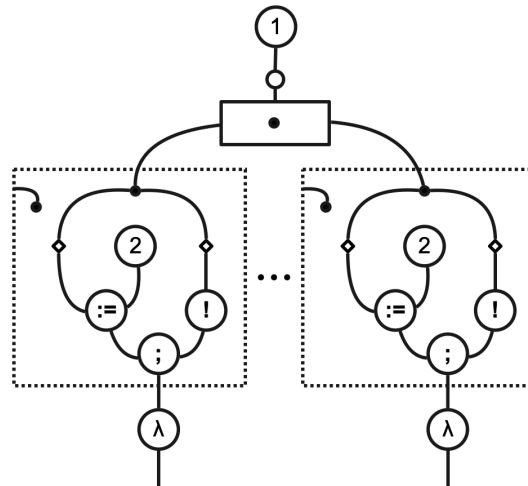
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Idea of *locality*:

analysing behaviour of program fragments,
by tracing sub-graphs during execution

Proof of observational equivalence, using *locality*

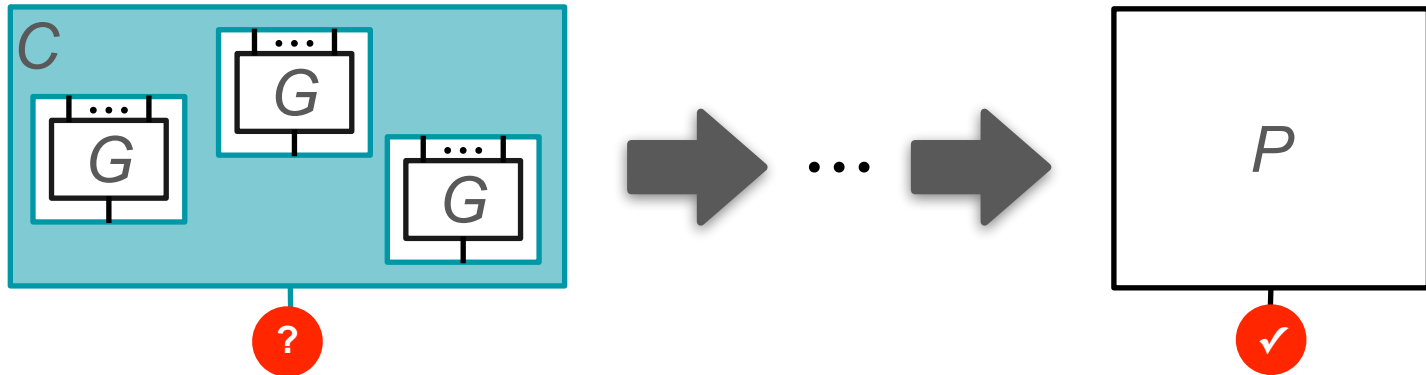
Claim: “Behaviour of a sub-graph G can be matched by behaviour of a sub-graph H .”

Proof of observational equivalence, using *locality*

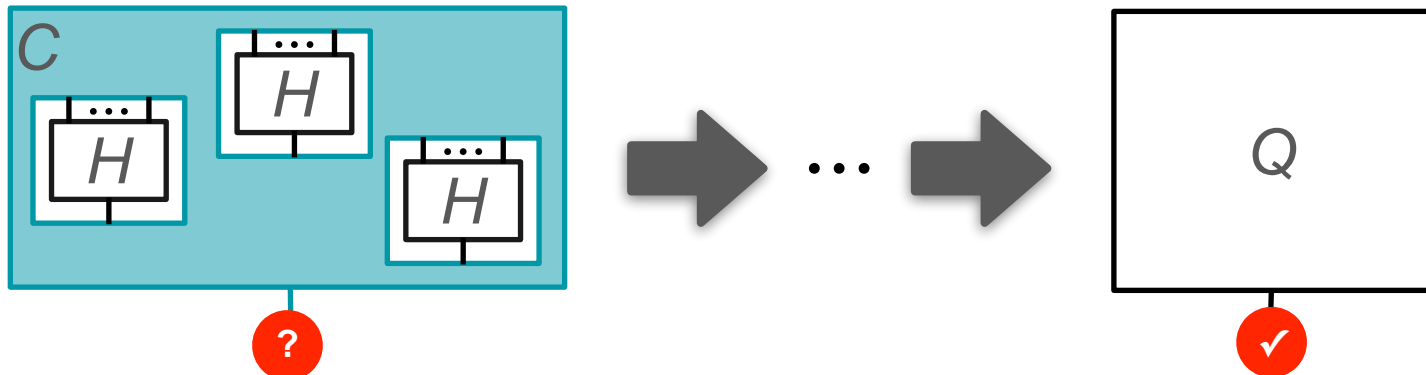
Claim: “Behaviour of a sub-graph G can be matched by behaviour of a sub-graph H .”

For any context C ,

if



then



Proof of observational equivalence, using *locality*

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Proof idea (simplified):

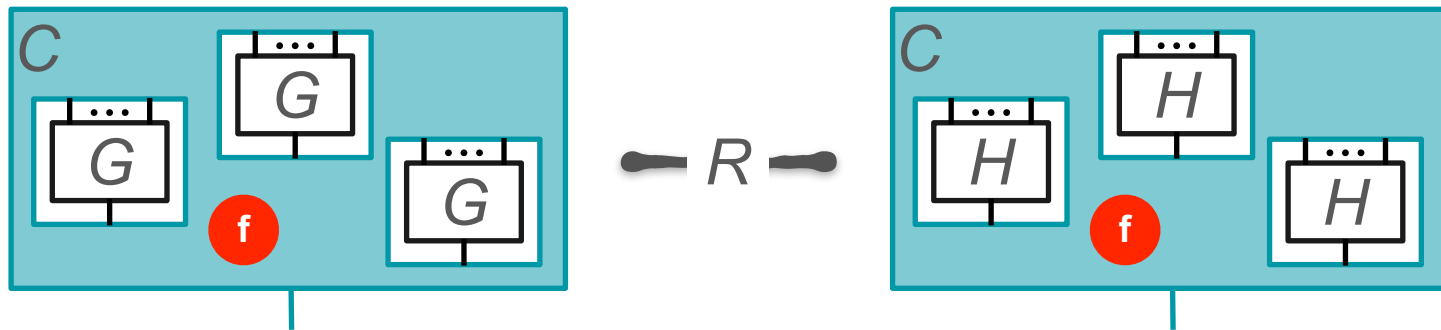
1. take **contextual closure** R of (G, H)
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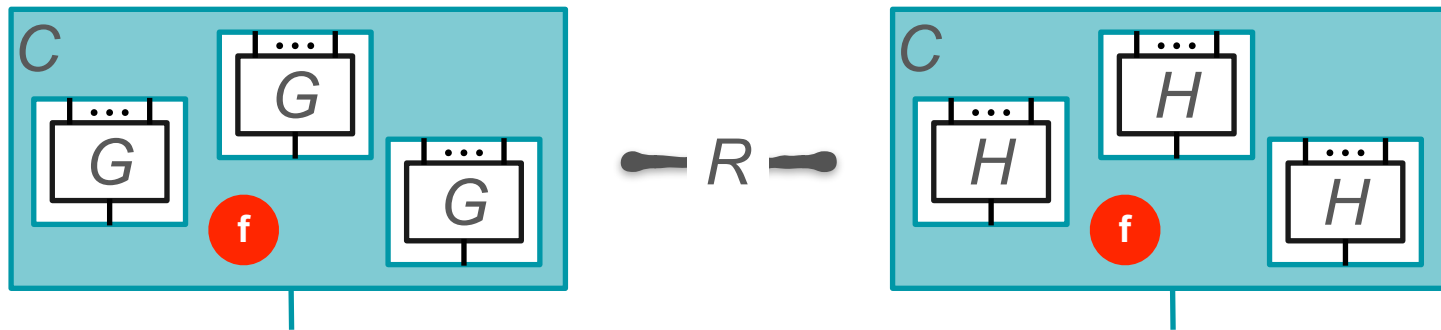
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R is closed under contexts, by definition

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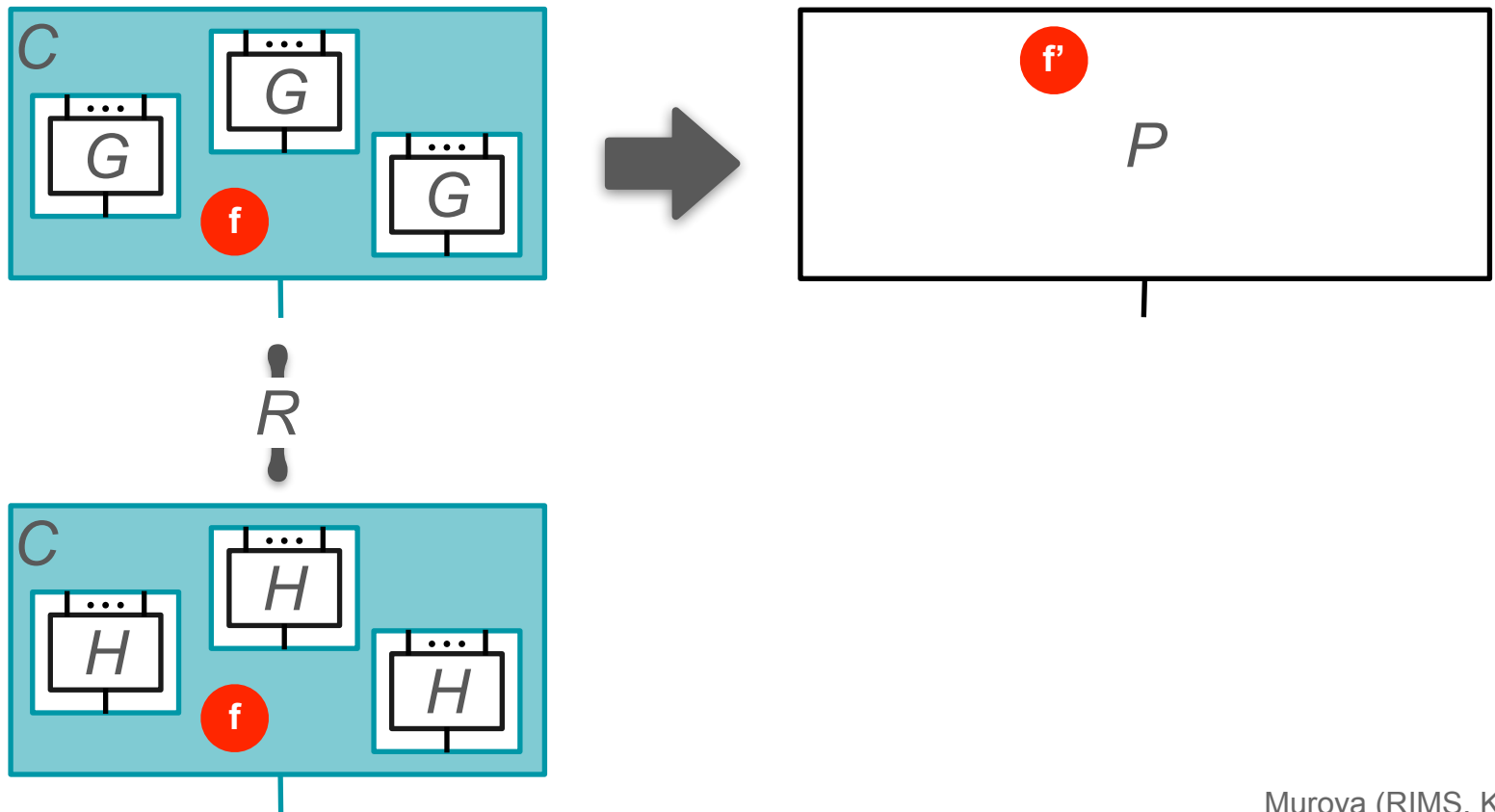
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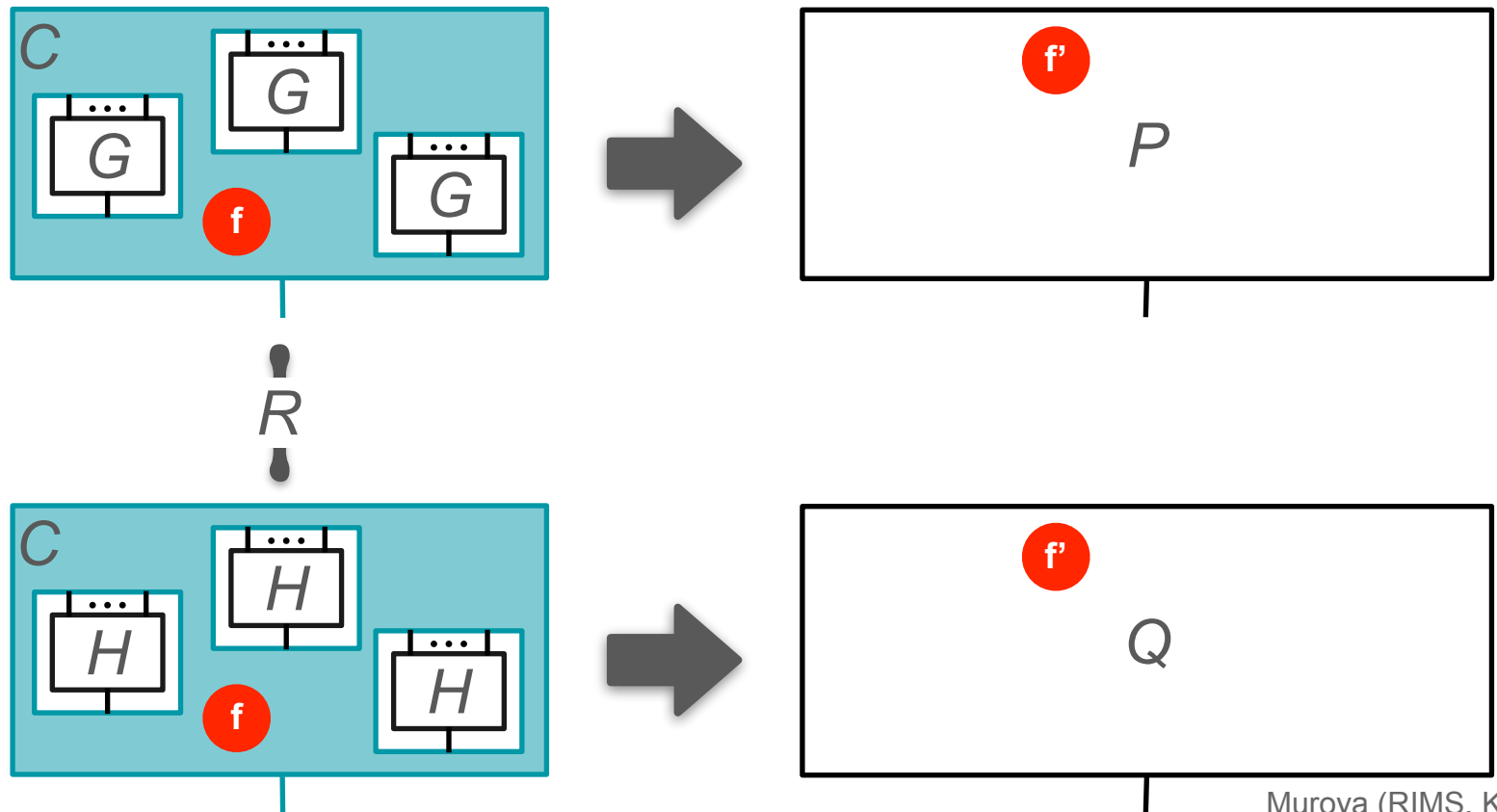
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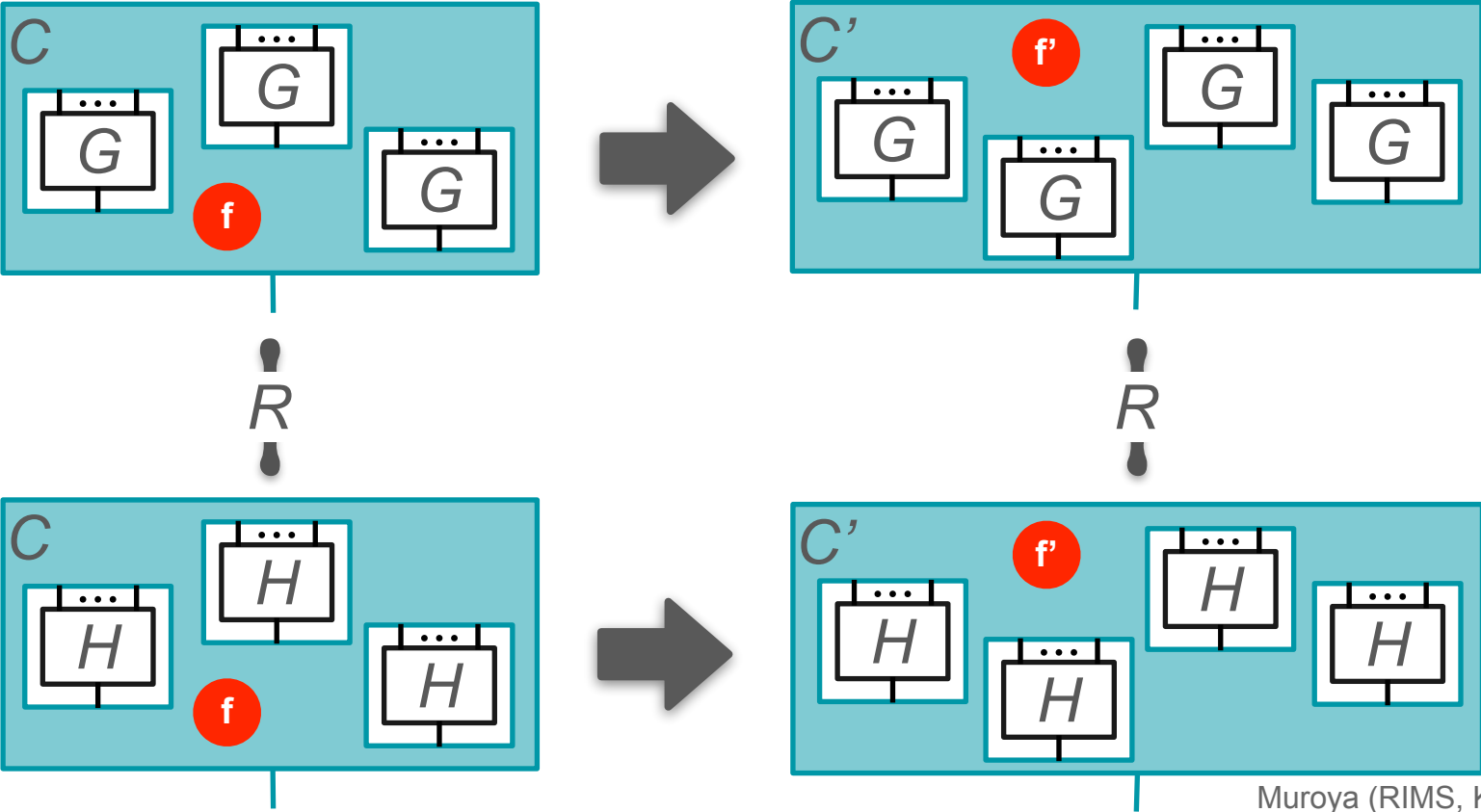
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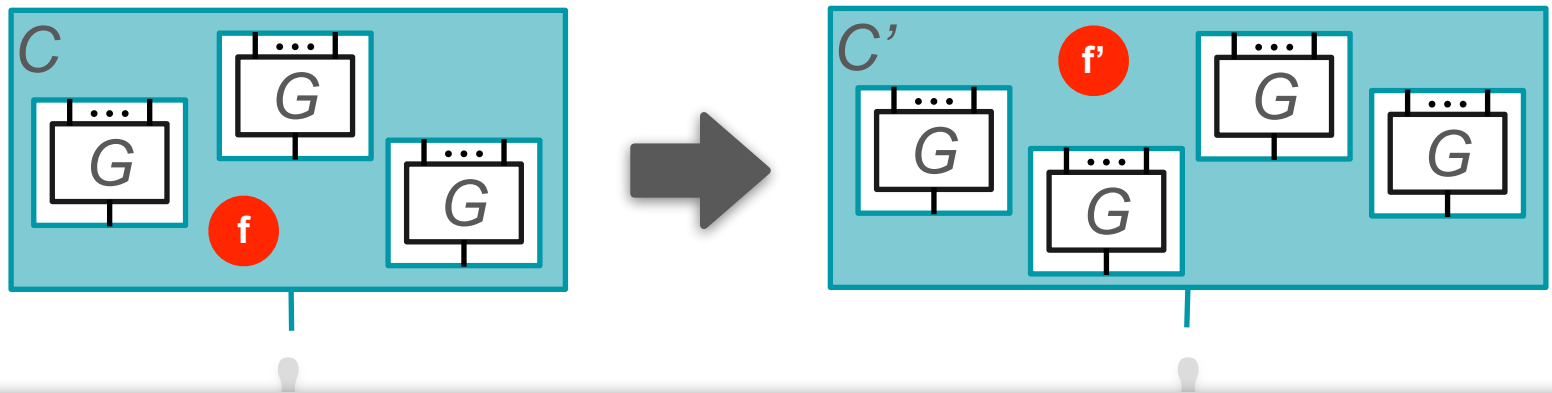
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Idea of *locality*:

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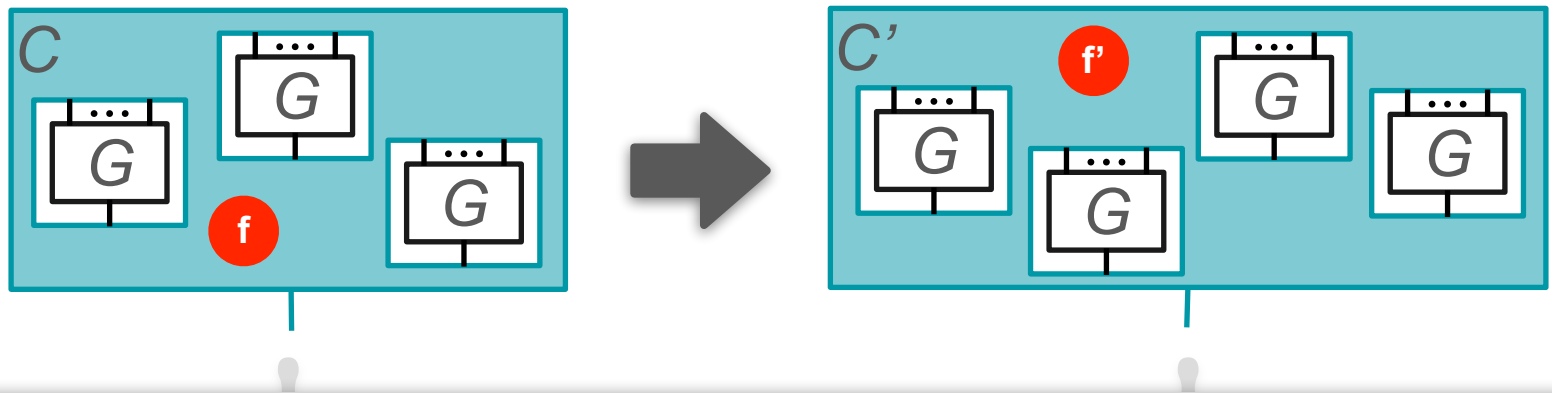
by analysing what happens around the focus during transition

Proof of observational equivalence, using *locality*

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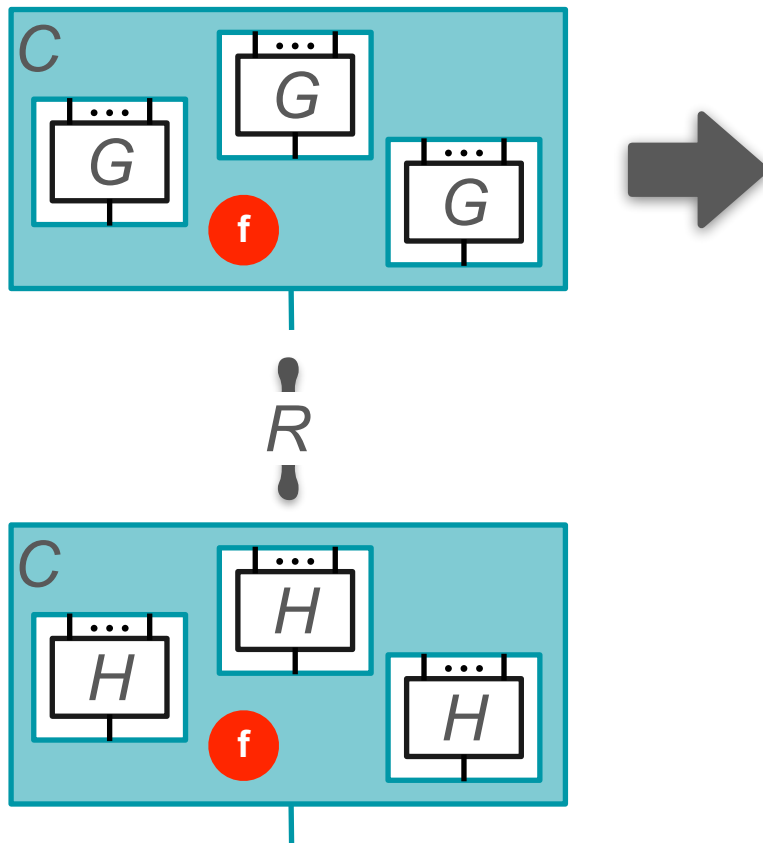
move, or trigger update

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Case (1) move of focus $\textcircled{?}$ or $\textcircled{\checkmark}$ inside context

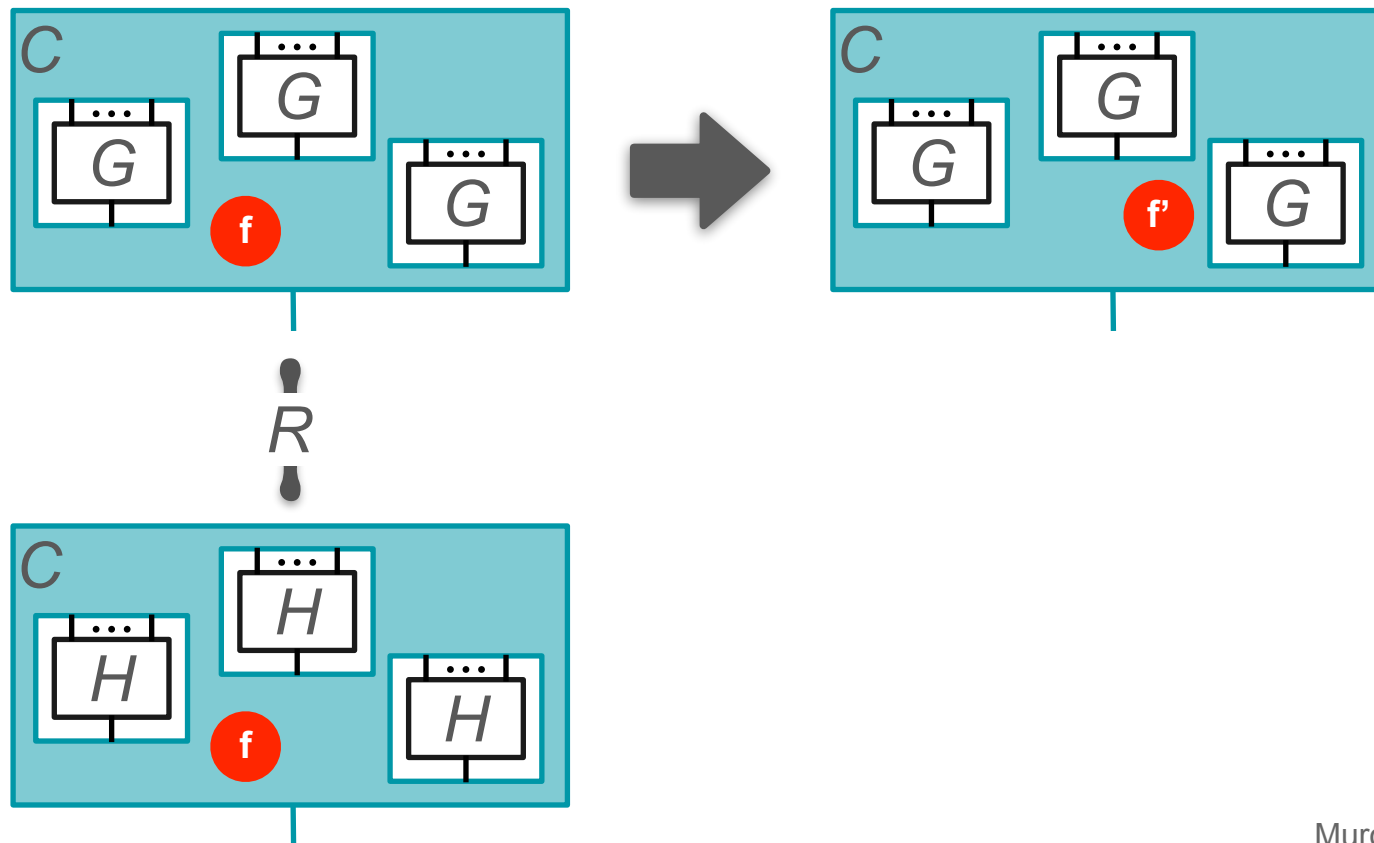


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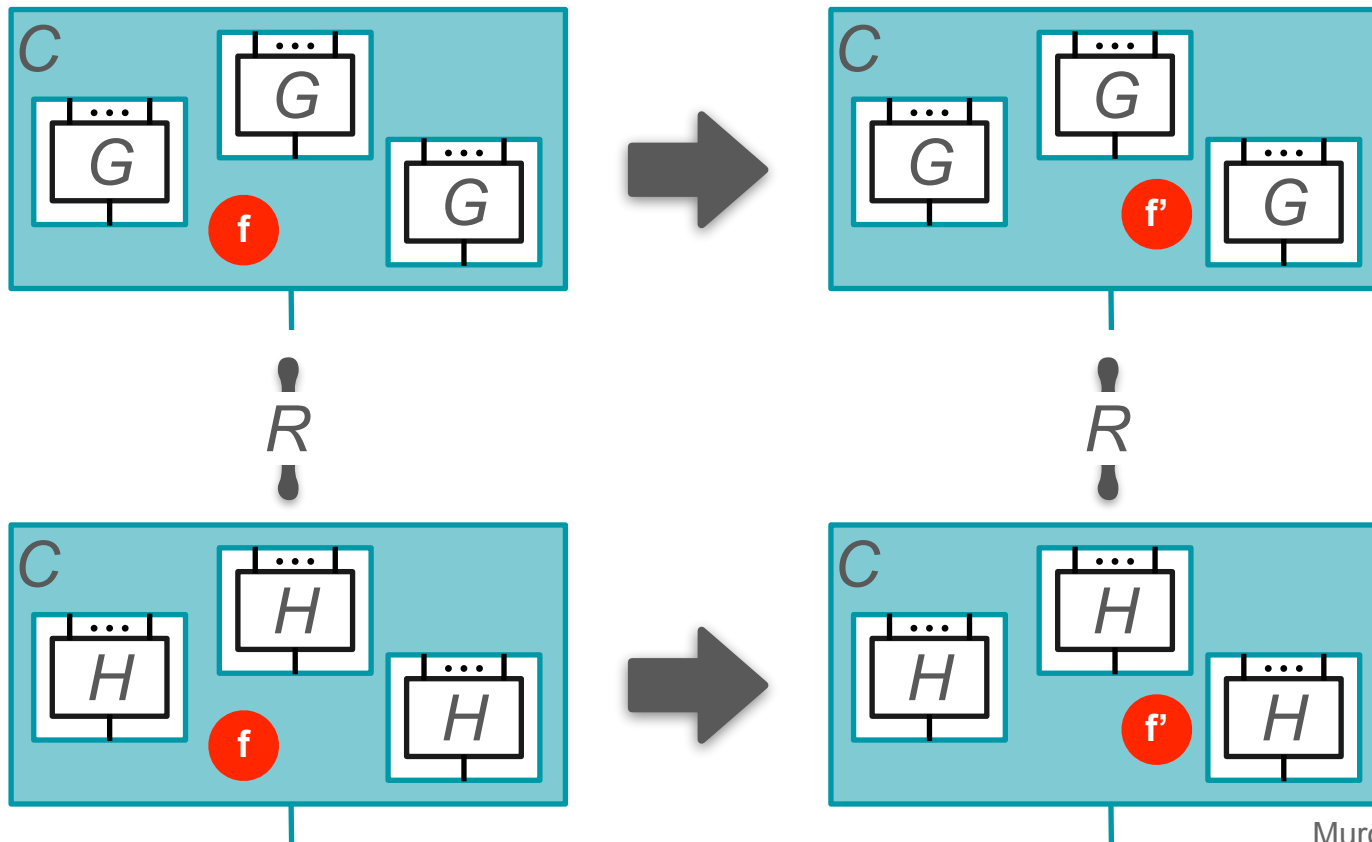


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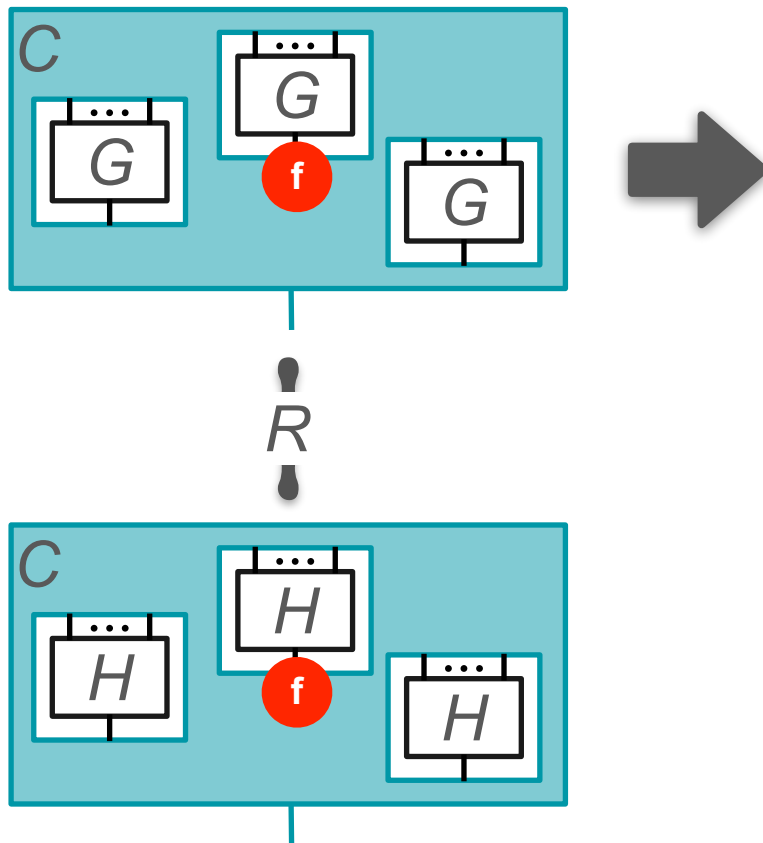


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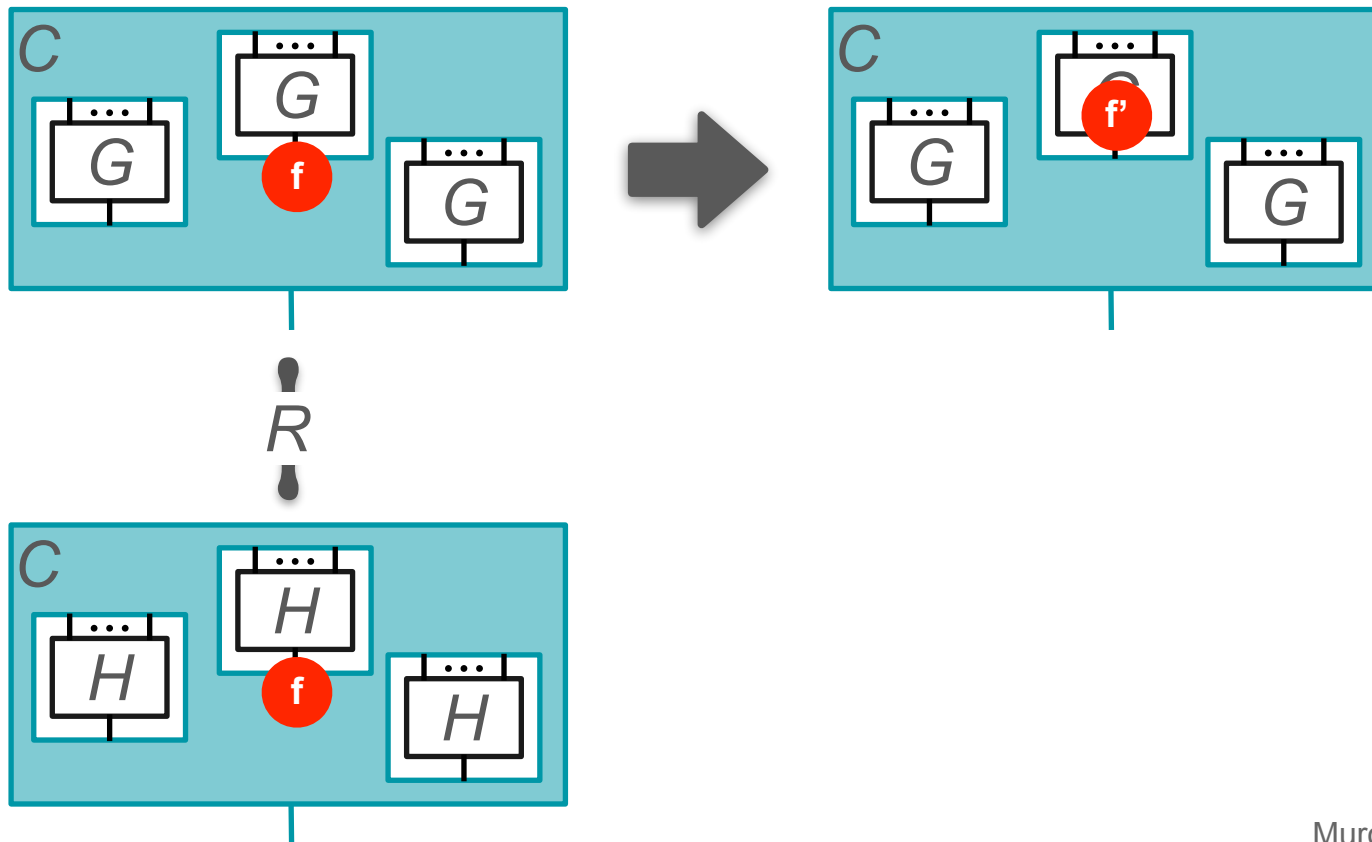


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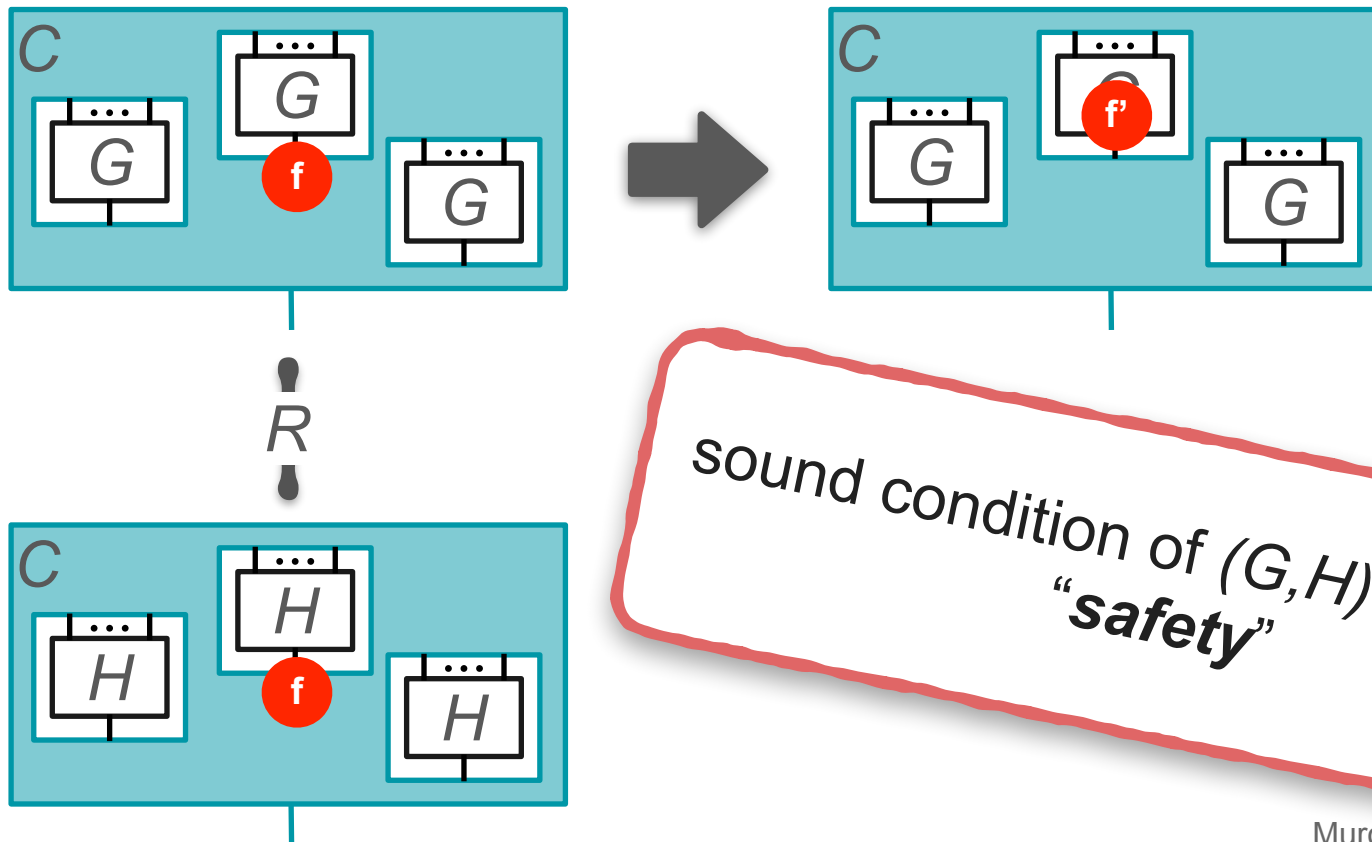


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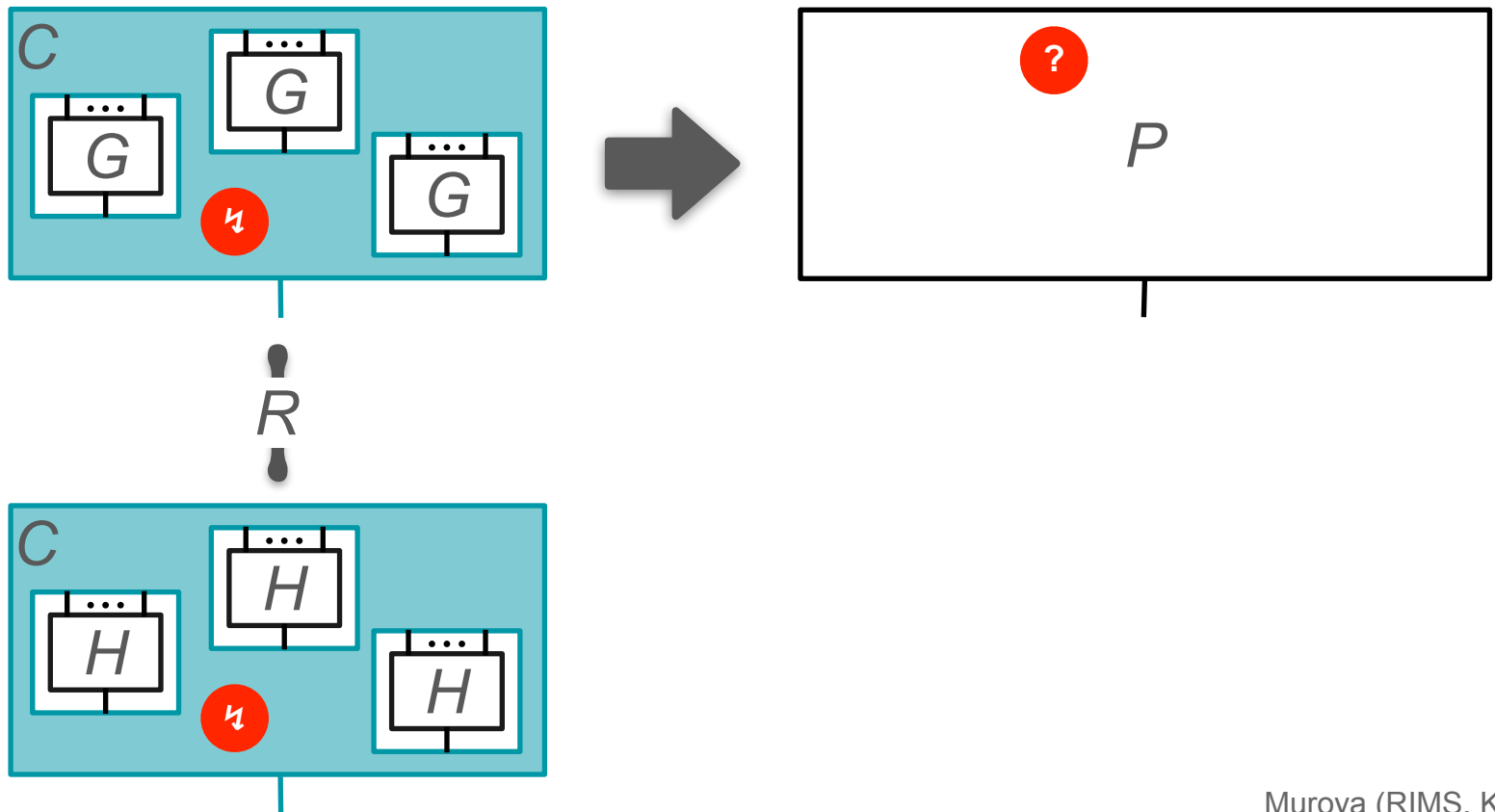


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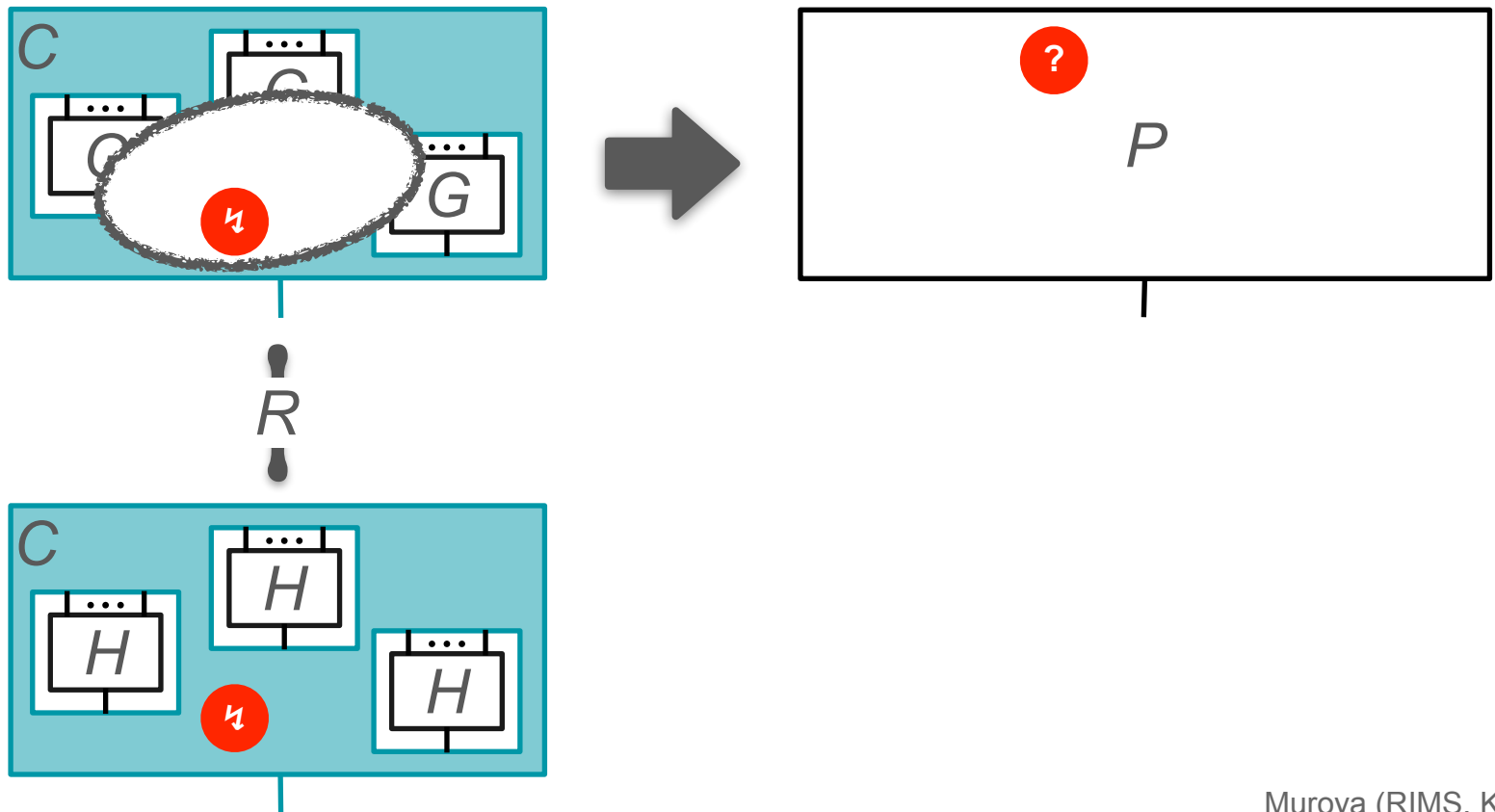


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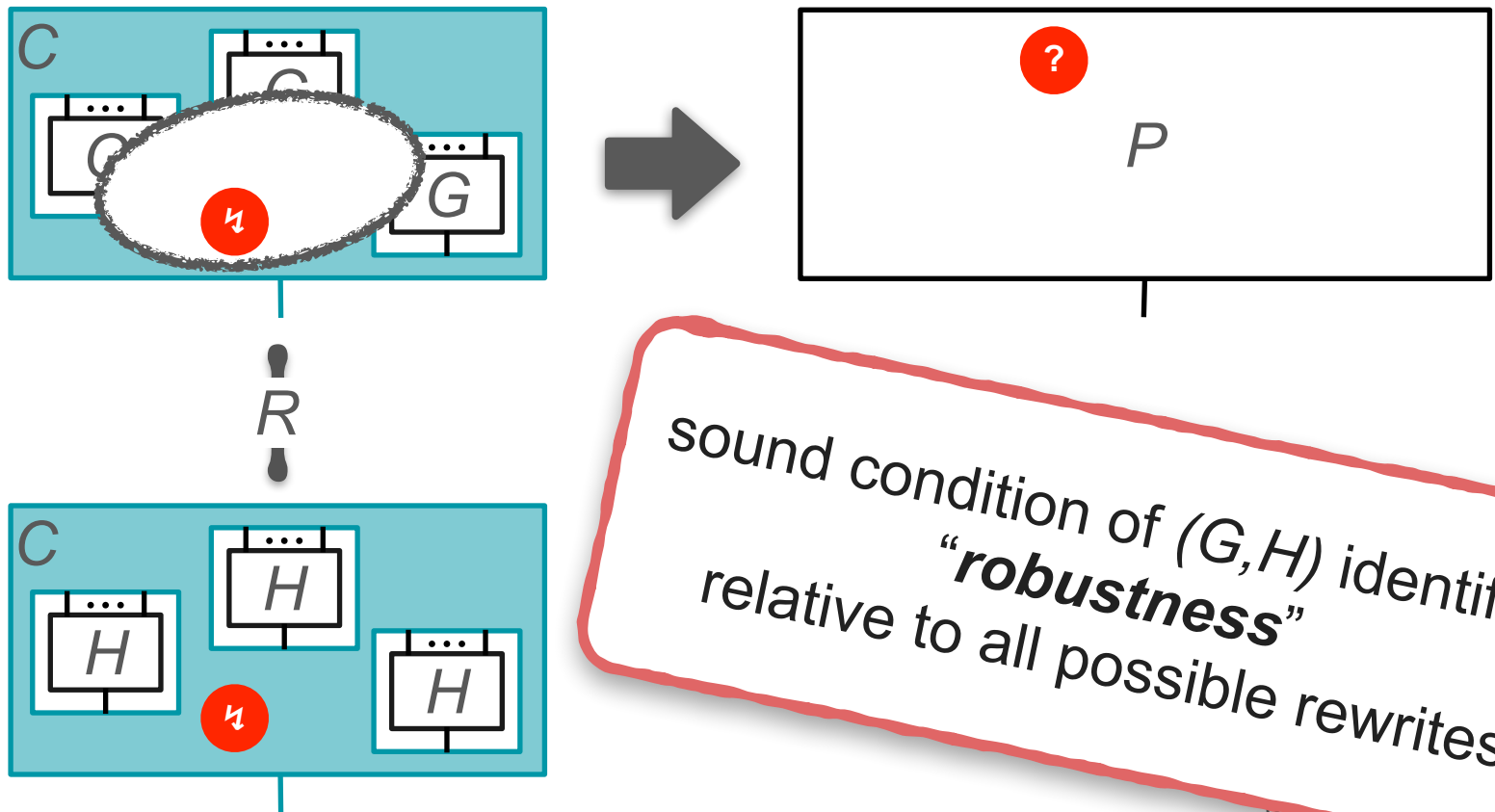


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Characterisation Theorem

Robust and safe template induce observational equivalences.
(for deterministic & “reasonable” languages)

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Observation:

ordinary simulations do not always suffice...

Complication of simulation notion

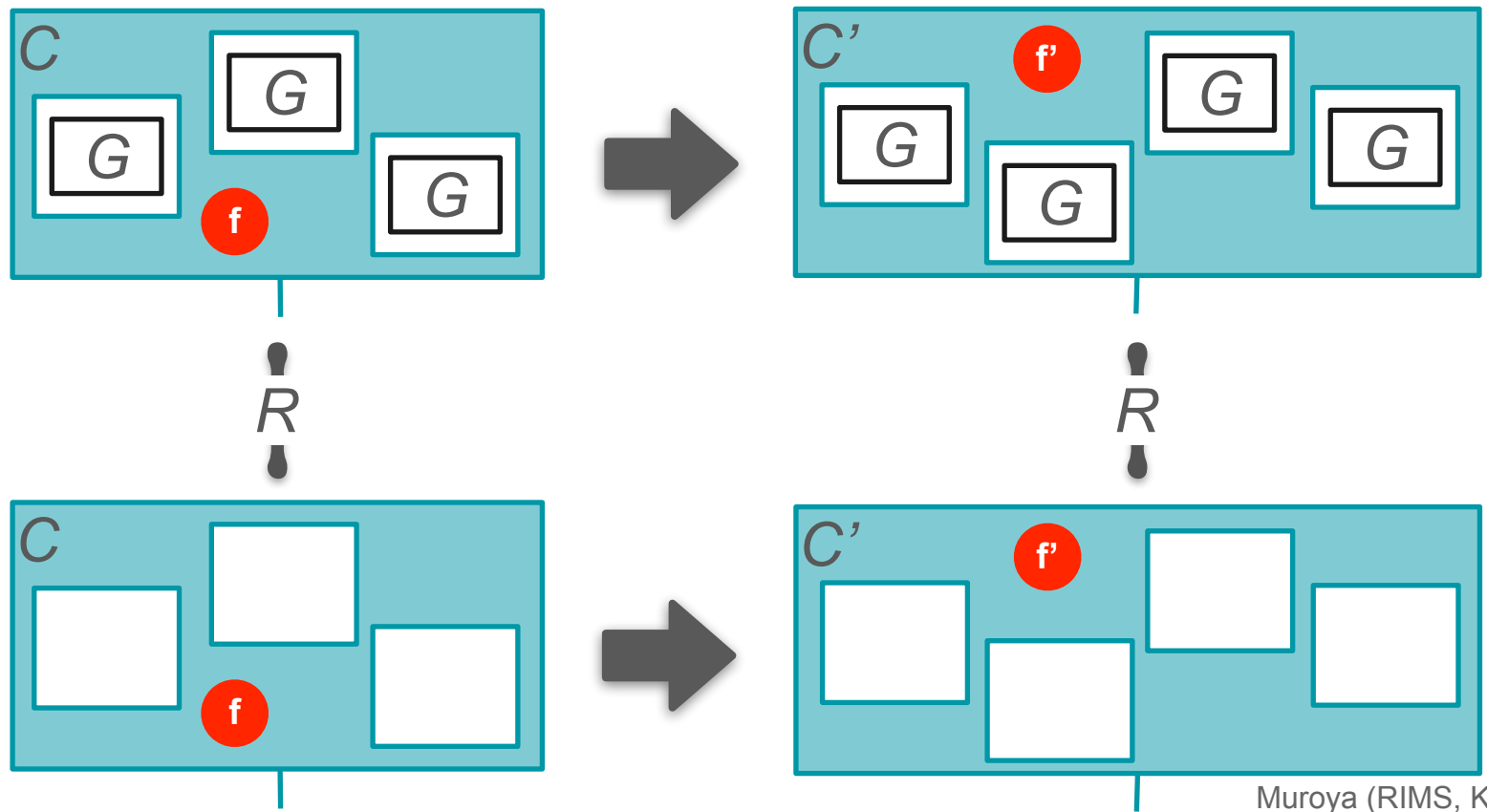
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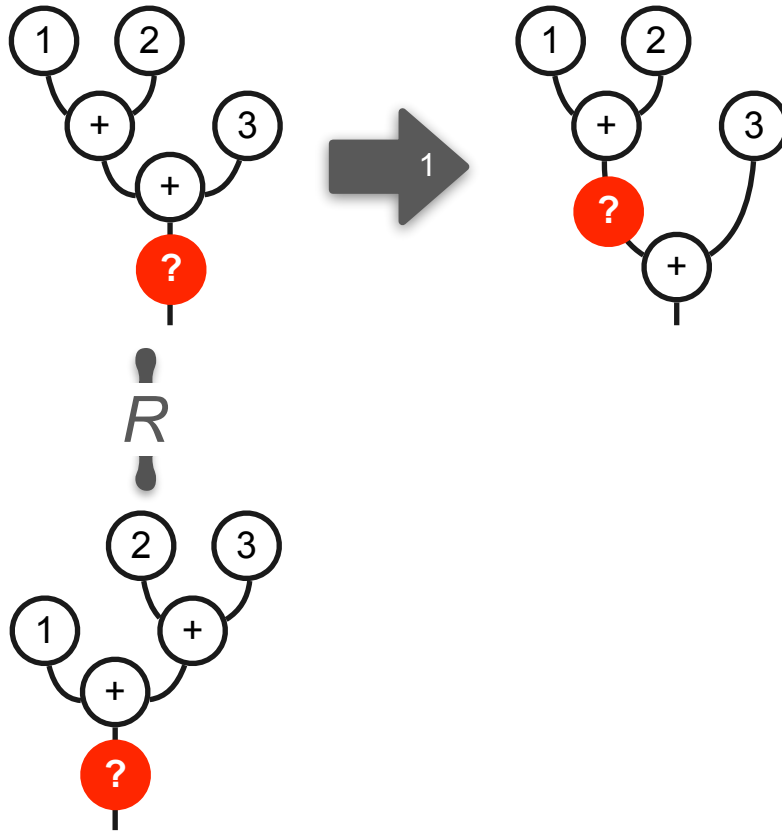
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Complication of simulation notion

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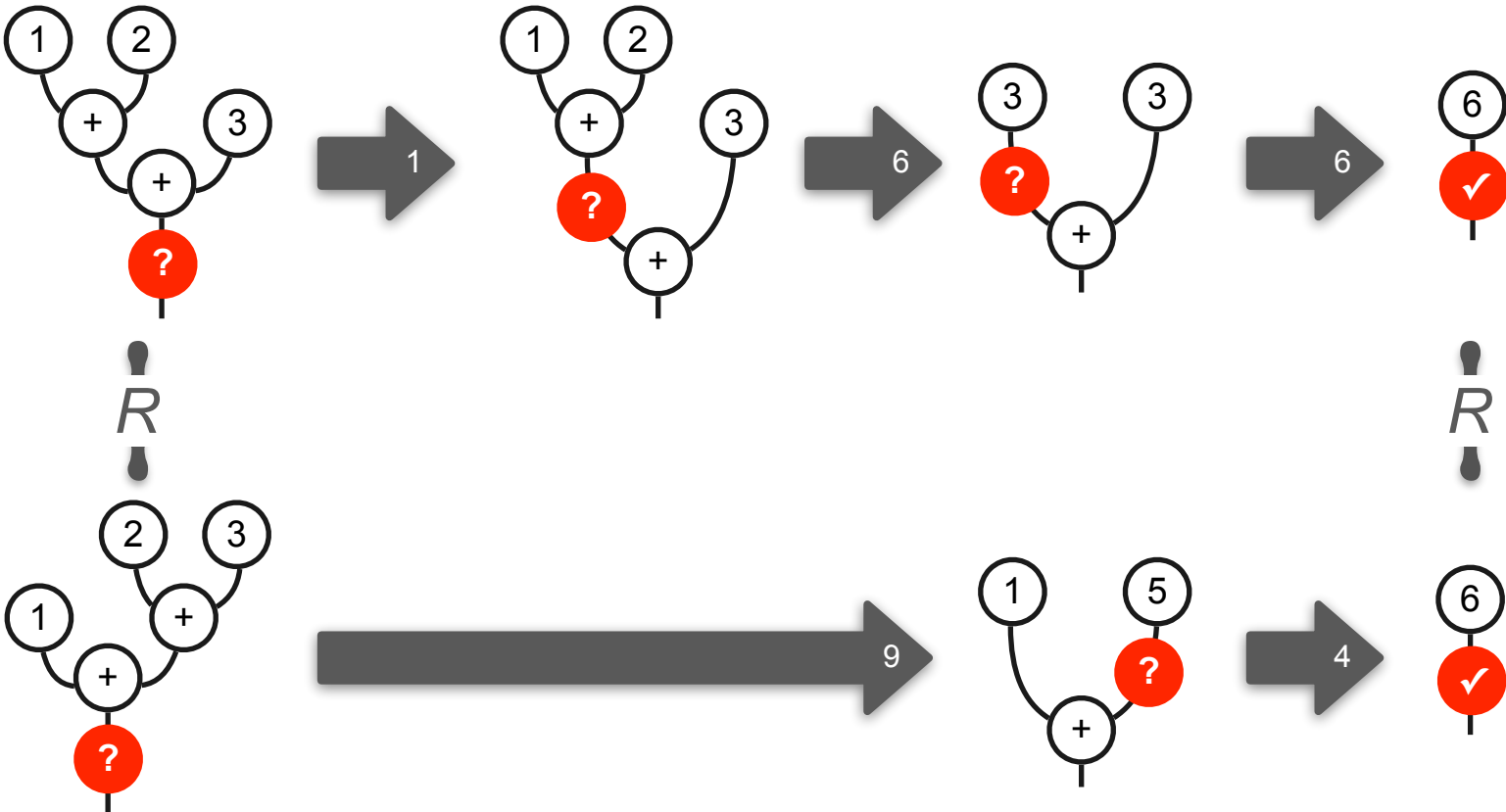
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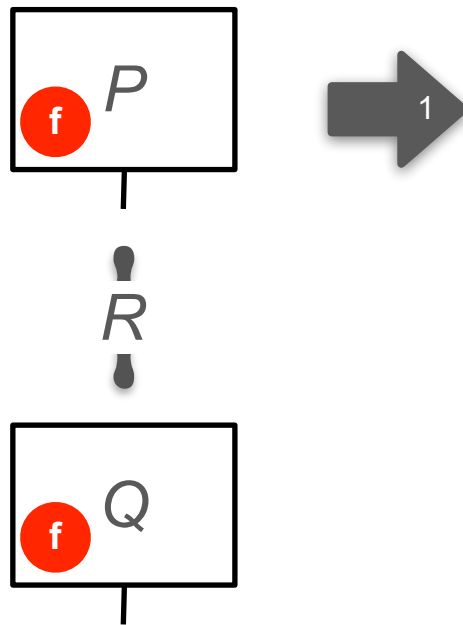
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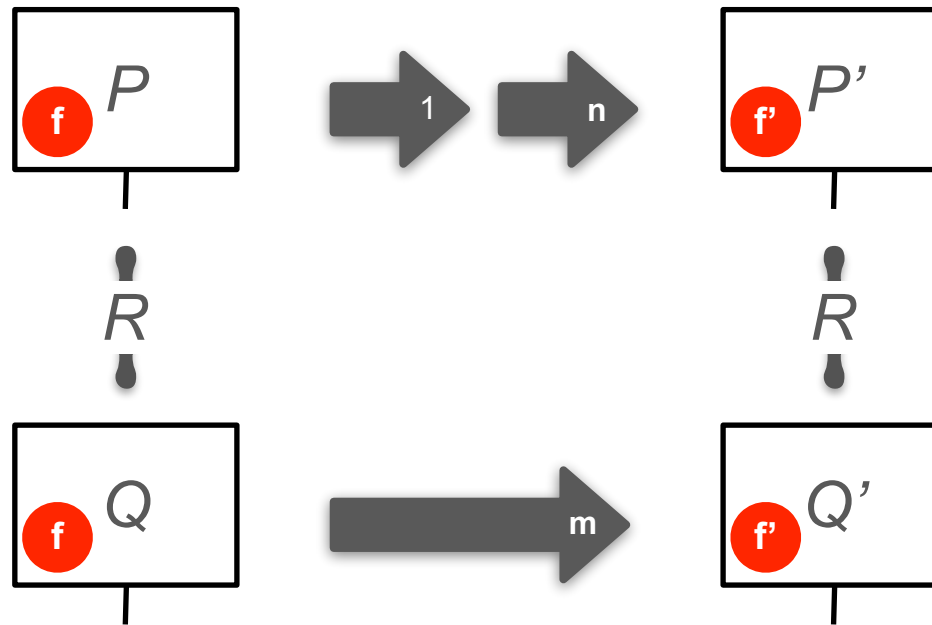
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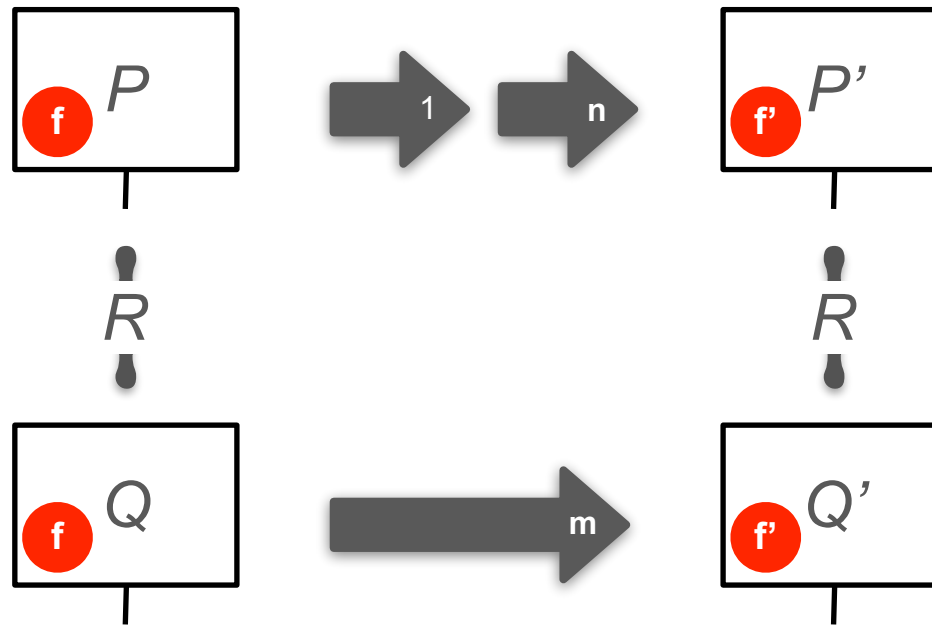
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- Soundness fails, in the presence of nondeterminism.

Complication of simulation notion

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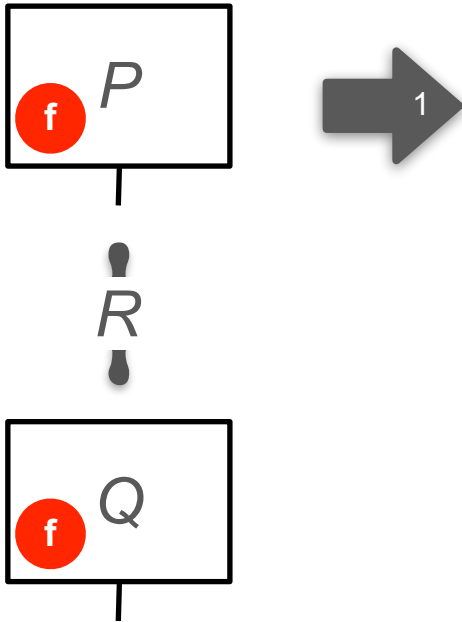


- ✗ working on graphs modulo structural equivalence
- ✓ using up-to technique with structural equivalence

Complication of simulation notion

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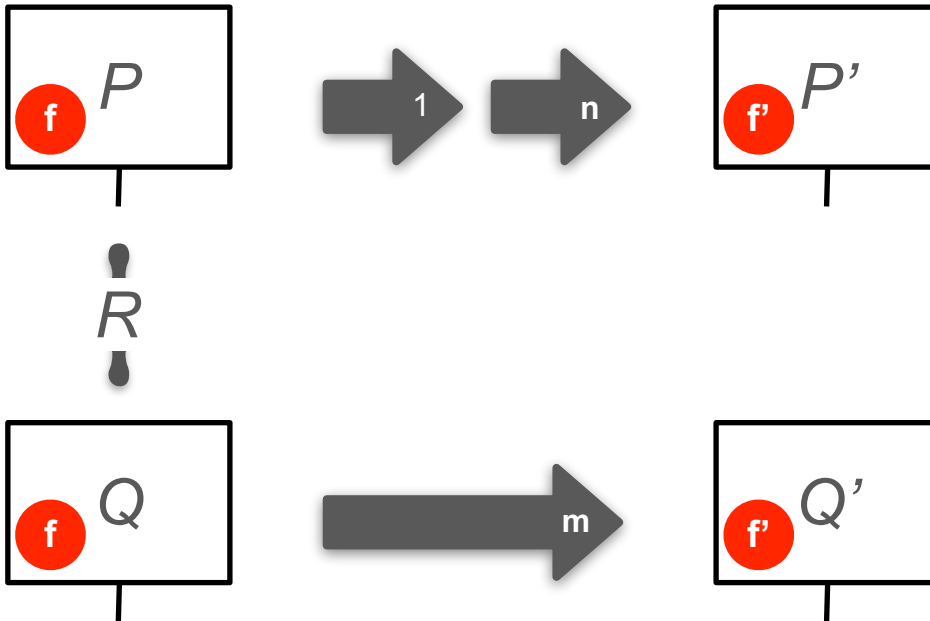
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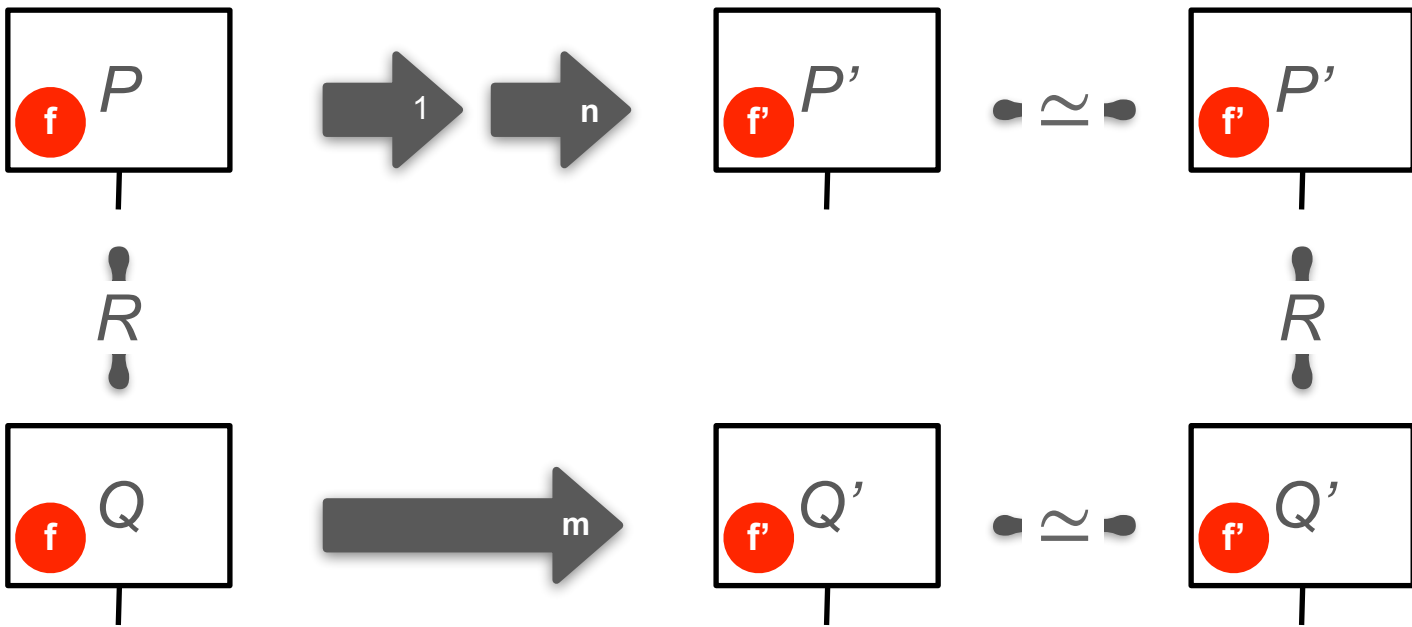
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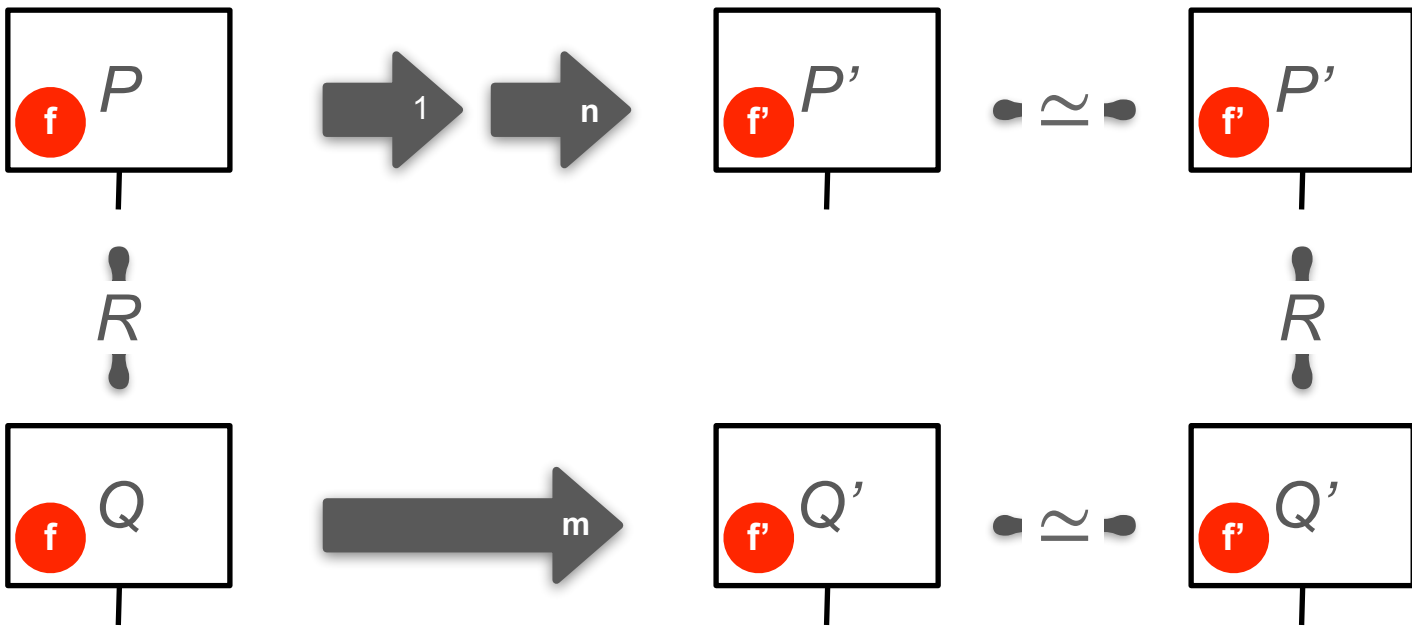
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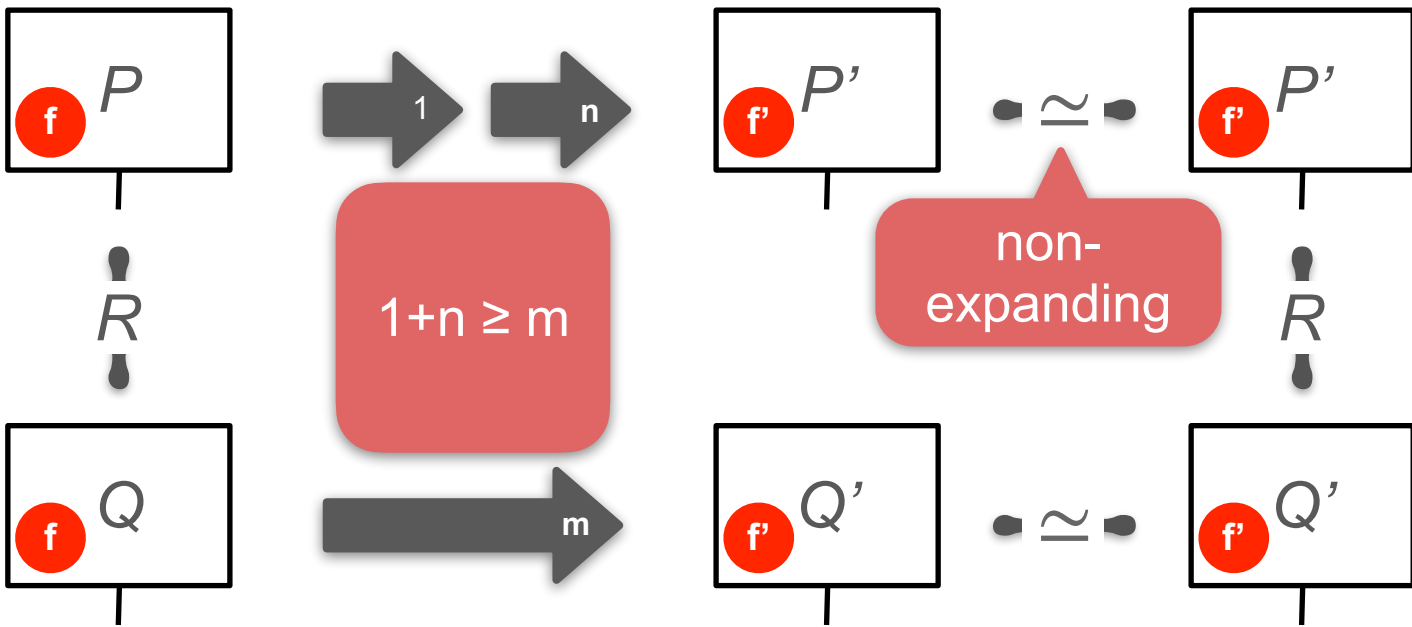


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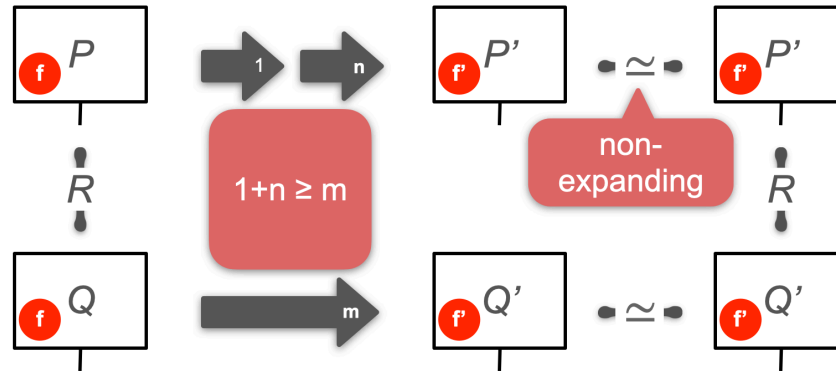
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Conclusion

- a (general) framework for analysing and proving robustness of observational equivalence
 - hypernet semantics: a *graphical* abstract machine
 - *local & step-wise* reasoning to prove observational equivalence, with the concept of *robustness*
- current key limitation: determinism

Future directions

- complication of simulation notion



- What causes complication of simulation notion?
- How can this complication be justified?
- Are there relevant simulation notions?
- How can we deal with nondeterminism?

Future directions

- Sand's improvement theory
(incorporating cost reduction in observational equivalence)
- The number of steps can already be dealt with,
by the quantitative restrictions on the weak up-to
simulation.

