A Functional Perspective on Machine Learning via Programmable Induction and Abduction

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FLOPS (Nagoya), 9 May 2018

Machine Learning

https://xkcd.com/1838/

Machine Learning

modelising

https://xkcd.com/1838/

Machine Learning

using / predicting

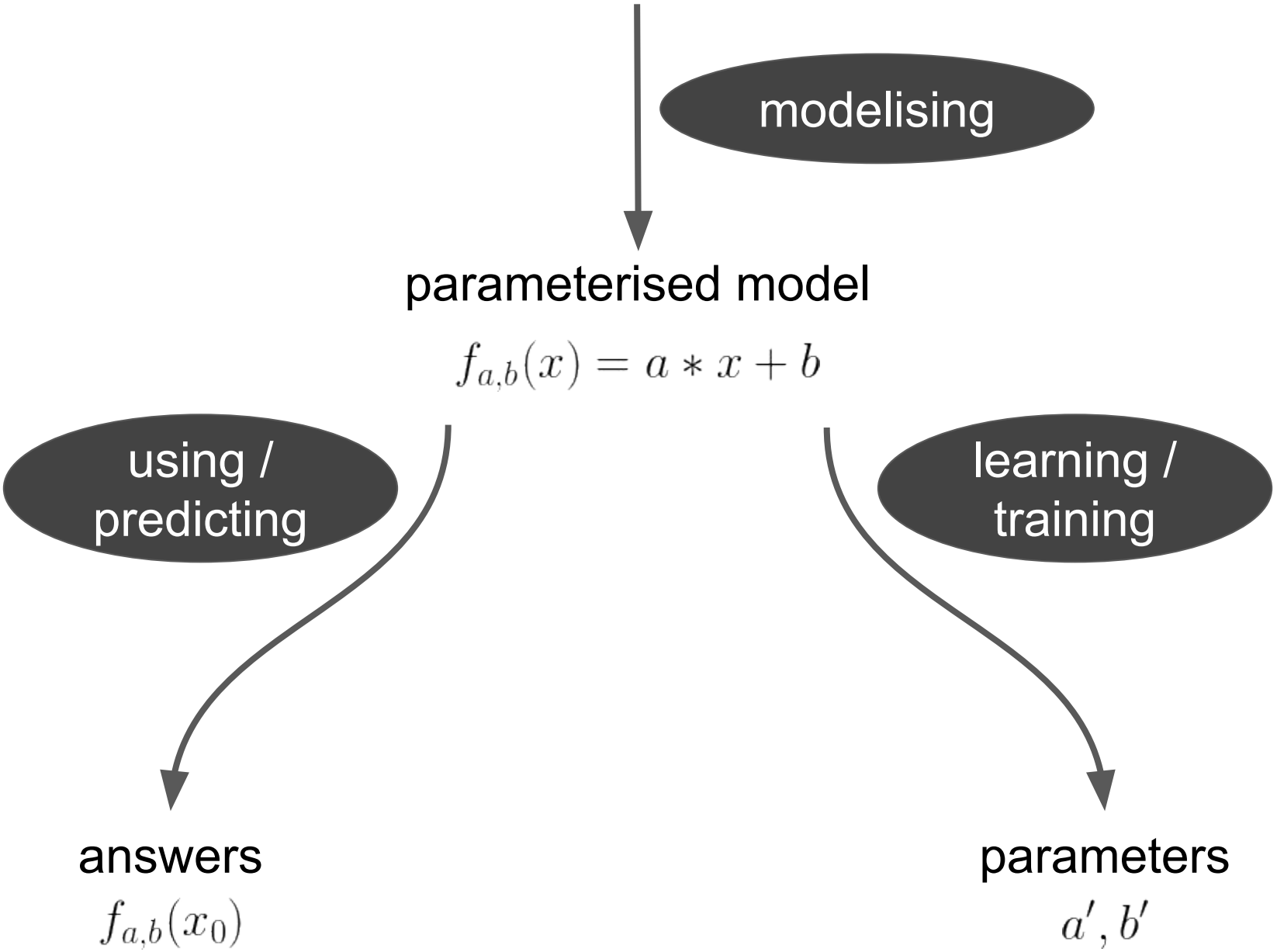
https://xkcd.com/1838/

Machine Learning

learning / training

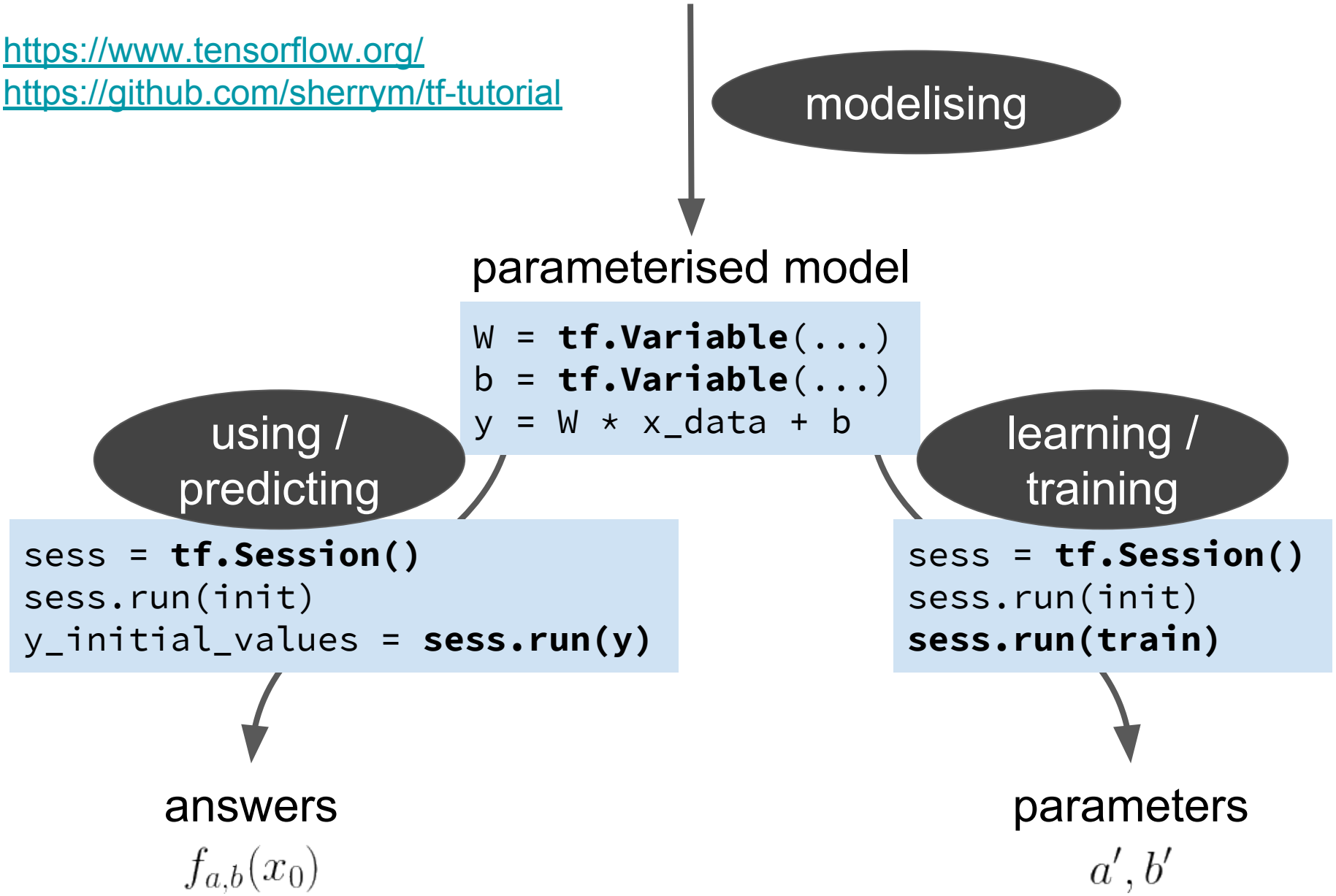
https://xkcd.com/1838/

Machine Learning



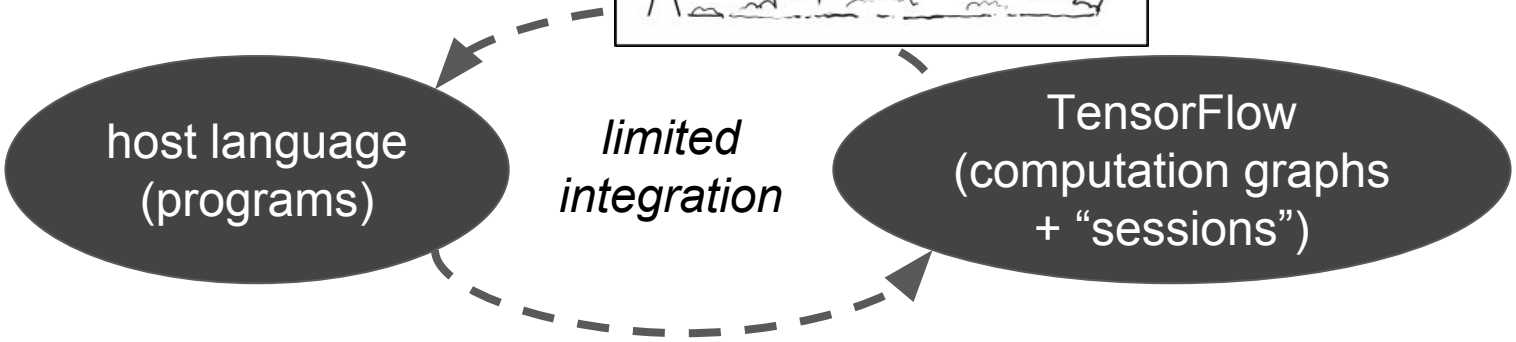
Machine Learning with TensorFlow

https://www.tensorflow.org/
https://github.com/sherrym/tf-tutorial



TensorFlow

- shallow embedded DSL

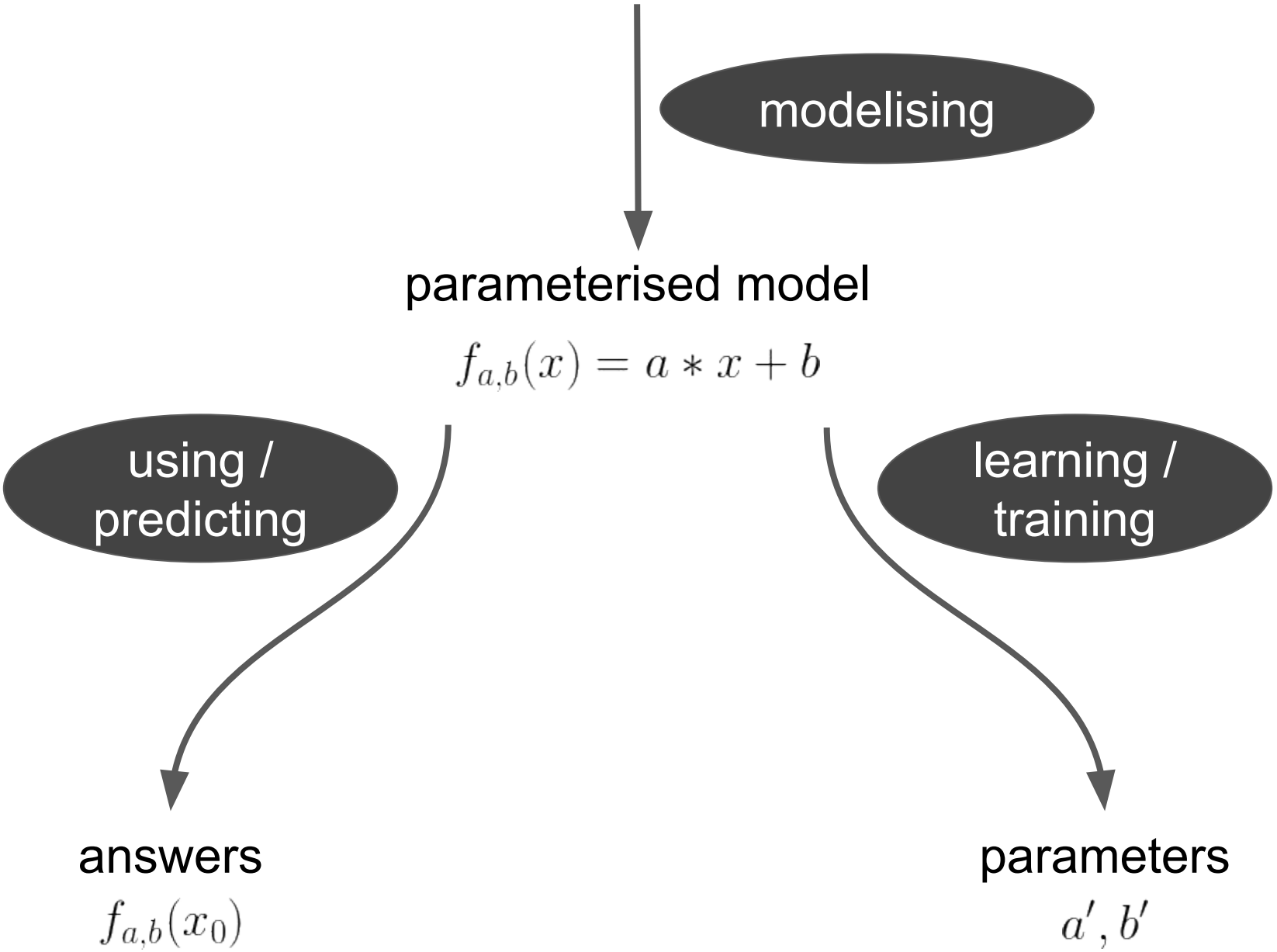


- imperative parameter (“variable”) update

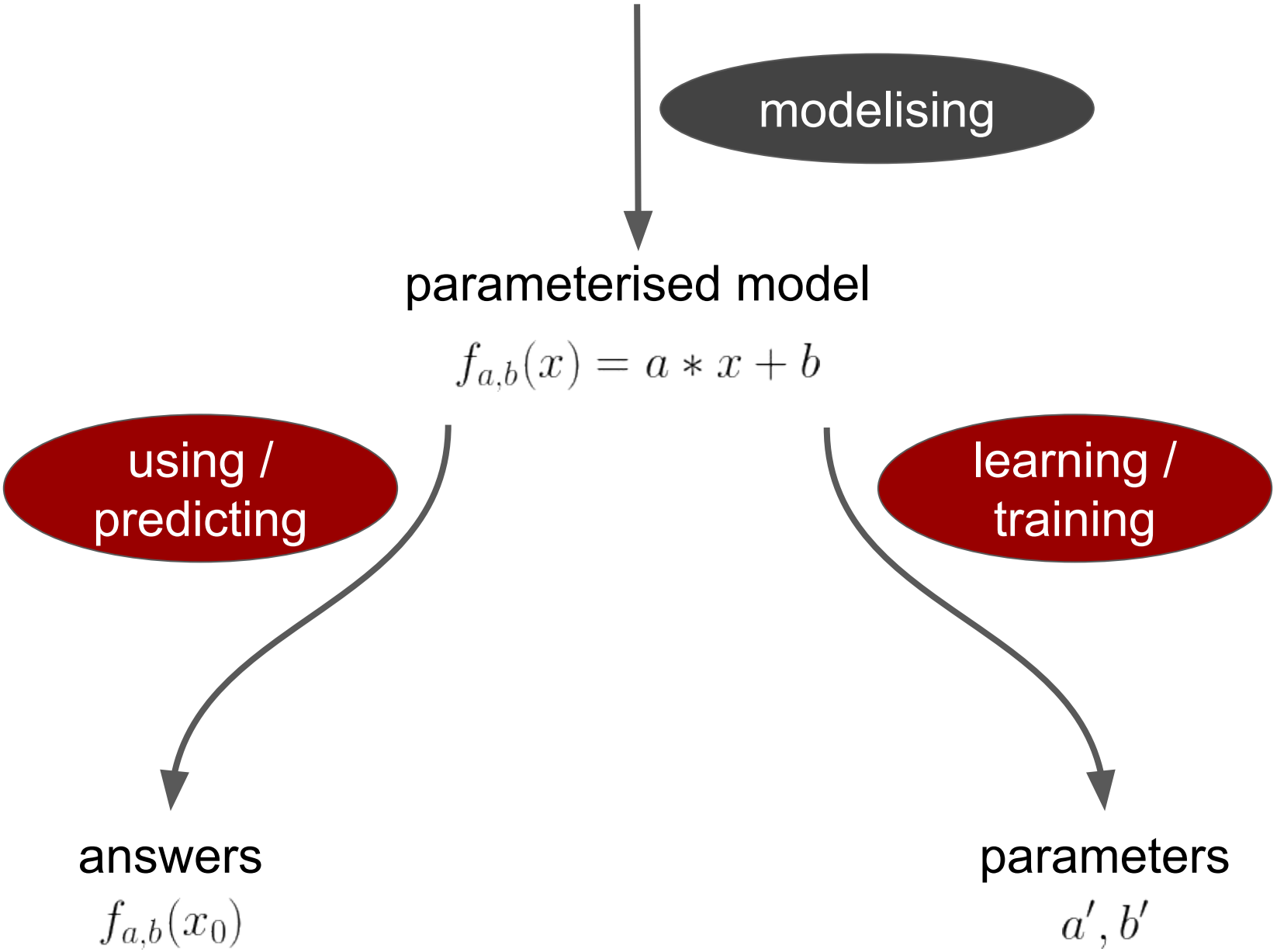
Proper *functional* language?

- simple & uniform programming language
  - well-defined operational semantics
- functional parameter update

Machine Learning



Machine Learning



functional programming language for **using & learning**

realisability style
"proofs as programs"

logical framework of **science methodology**

C. S. Peirce "Illustrations of the Logic of Science"

Peirce's Logical Inferences

- *analytic* rule, believable results

- *synthetic* rules, tentative results

Peirce's Logical Inferences

- *analytic* rule, believable results
  - deduction

$$\frac{A \to B \qquad A}{B}$$

- *synthetic* rules, tentative results

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$$\frac{A \qquad B}{A \to B}$$

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$$\frac{A \qquad B}{A \to B}$$

  - abduction

$$\frac{A \to B \qquad B}{A}$$

Peirce's Logical Inferences, *adapted to ML*

- *analytic* rule, believable results
  - deduction

$$\frac{A \to B \qquad A}{B}$$

- *synthetic* rules, tentative results
  - induction

$$\frac{A \times B \;\cdots\; A \times B}{A \to B}$$

create a model from data

  - abduction

$$\frac{A \to B \qquad B}{A}$$

Peirce's Logical Inferences, *adapted to ML*

- *analytic* rule, believable results
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- *synthetic* rules, tentative results
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$$\frac{A \times B \;\cdots\; A \times B}{A \to B}$$

create a model from data

  - abduction

$$\frac{P \to (I \to O) \quad I \to O}{P}$$

find the best parameter of a model to explain data

“Proofs as Programs”

- *analytic* rule, believable results
  - deduction

$$\frac{t\colon A \to B \qquad u\colon A}{t\,u\colon B}$$

- *synthetic* rules, tentative results
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$$\frac{\hat{x}\colon \mathsf{list}\,(A \times B)}{\mathsf{ind}(\hat{x})\colon A \to B}$$

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create a model from data

  - abduction

$$\frac{m\colon P \to (I \to O) \qquad \Omega\colon I \to O}{\mathsf{abd}(m, \Omega)\colon P}$$

find the best parameter of a model to explain data

Towards Programmable Induction & Abduction

$$\frac{\hat{x}\colon \mathsf{list}\ (A \times B)}{\mathsf{ind}(\hat{x})\colon A \to B}$$

interpolation
exterpolation

$$\frac{m\colon P \to (I \to O) \quad \Omega\colon I \to O}{\mathsf{abd}(m, \Omega)\colon P}$$

generic optimisation
(e.g. gradient descent)

Towards Programmable Induction & Abduction

$$\frac{\hat{x}\colon \mathsf{list}\ (A \times B)}{\mathsf{ind}(\hat{x})\colon A \to B}$$

metric space

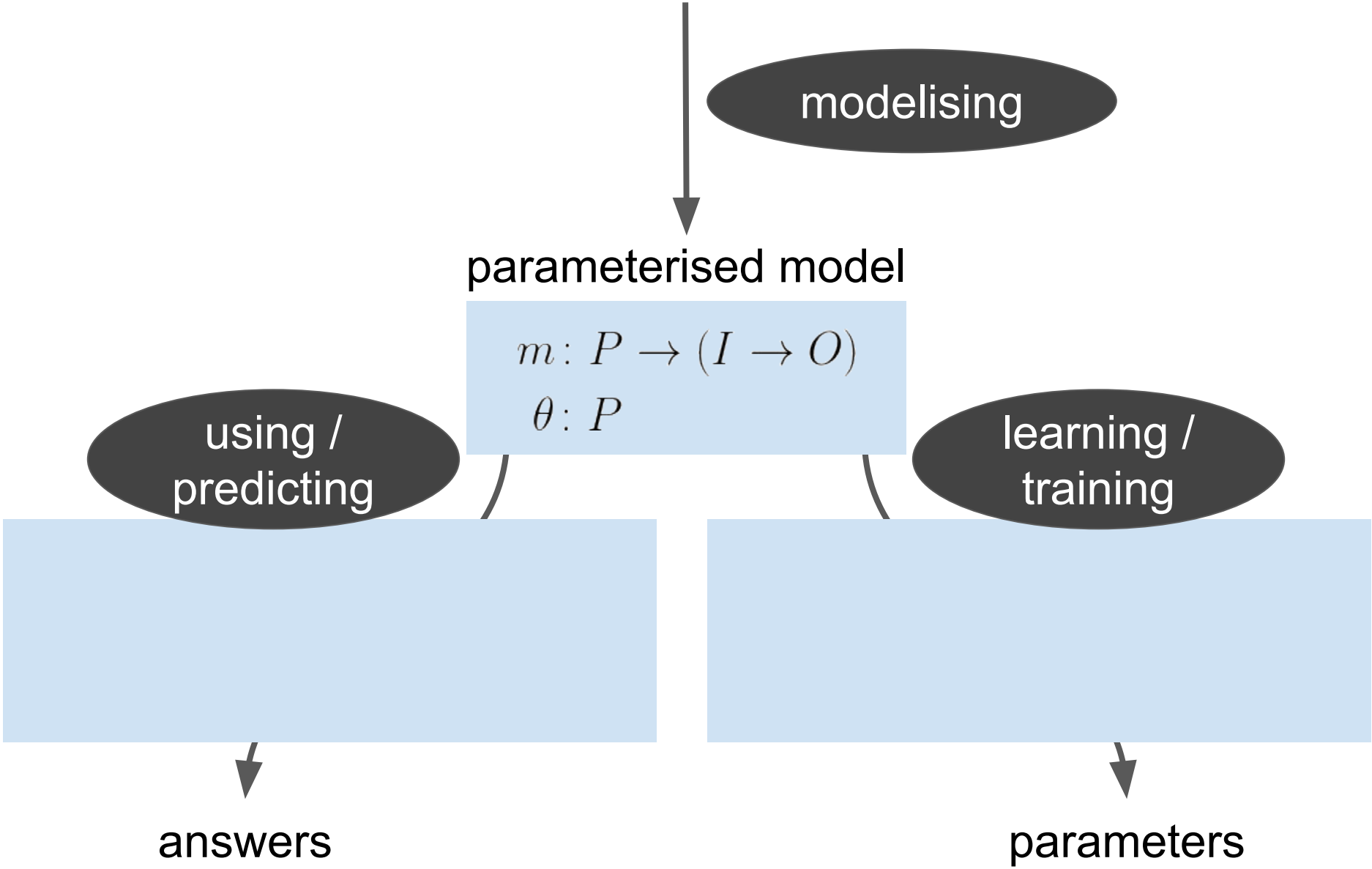
interpolation
exterpolation

$$\frac{m\colon P \to (I \to O) \quad \Omega\colon I \to O}{\mathsf{abd}(m, \Omega)\colon P}$$

vector space

generic optimisation
(e.g. gradient descent)

Machine Learning with Induction & Abduction



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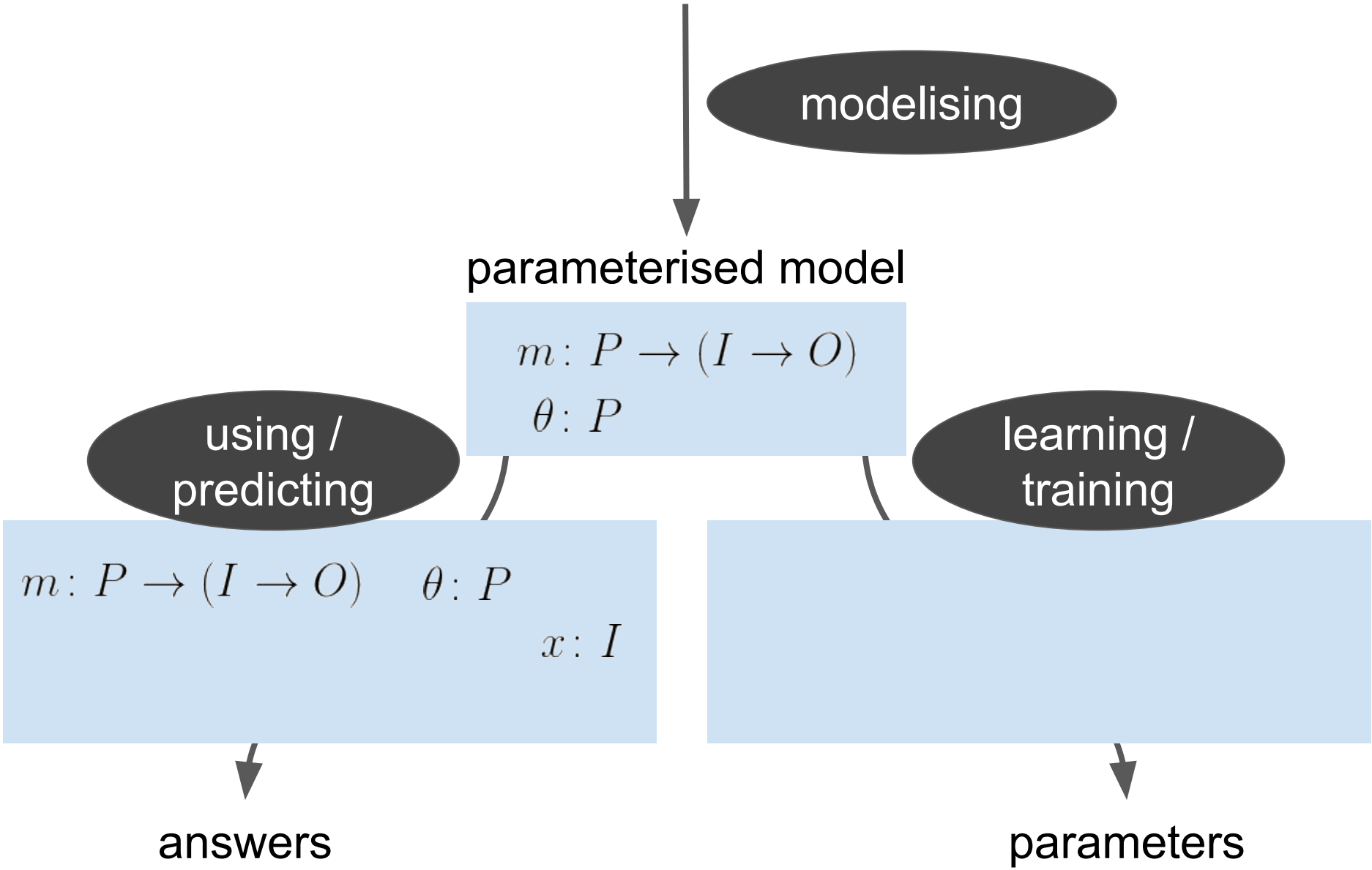

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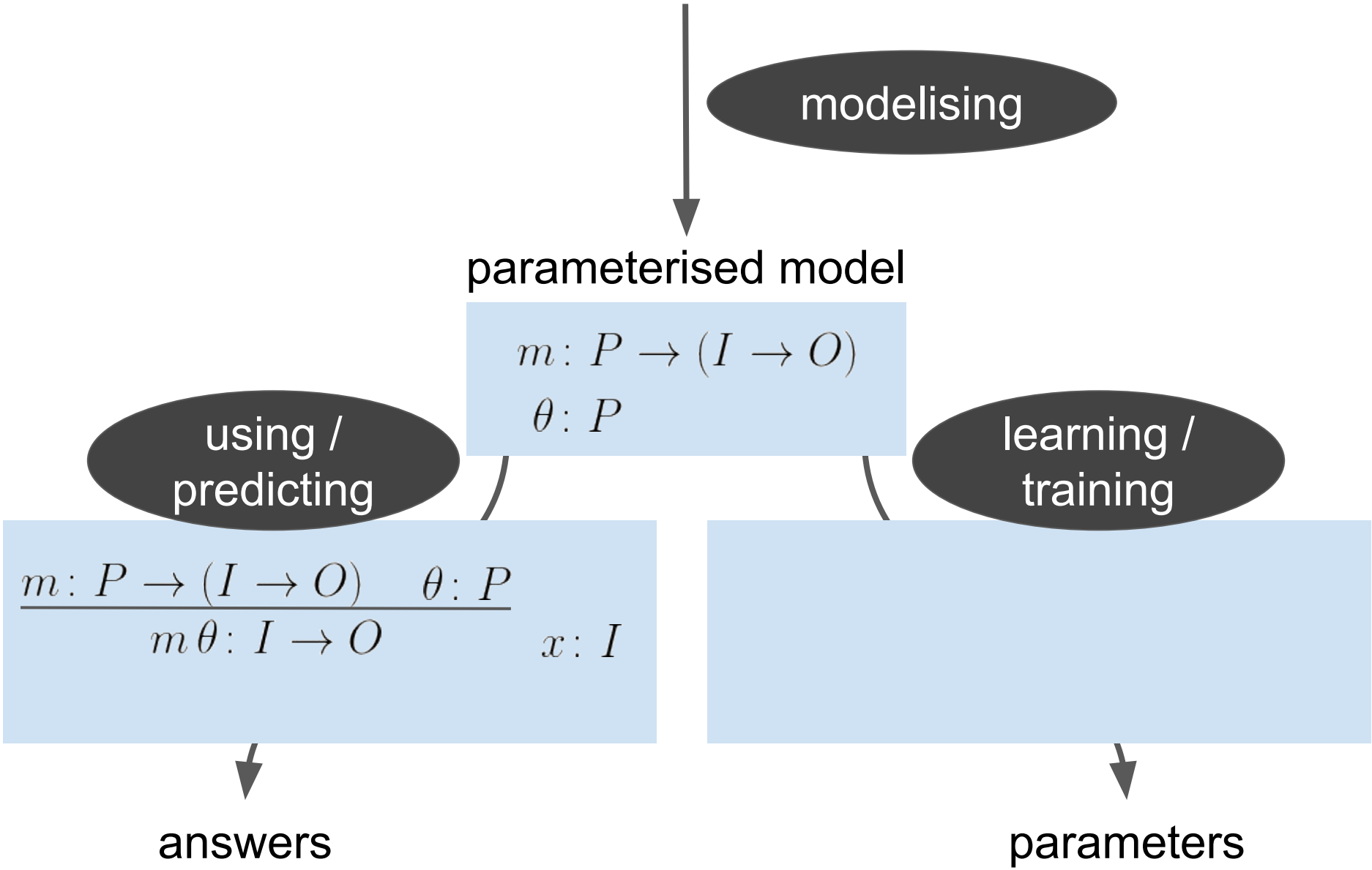

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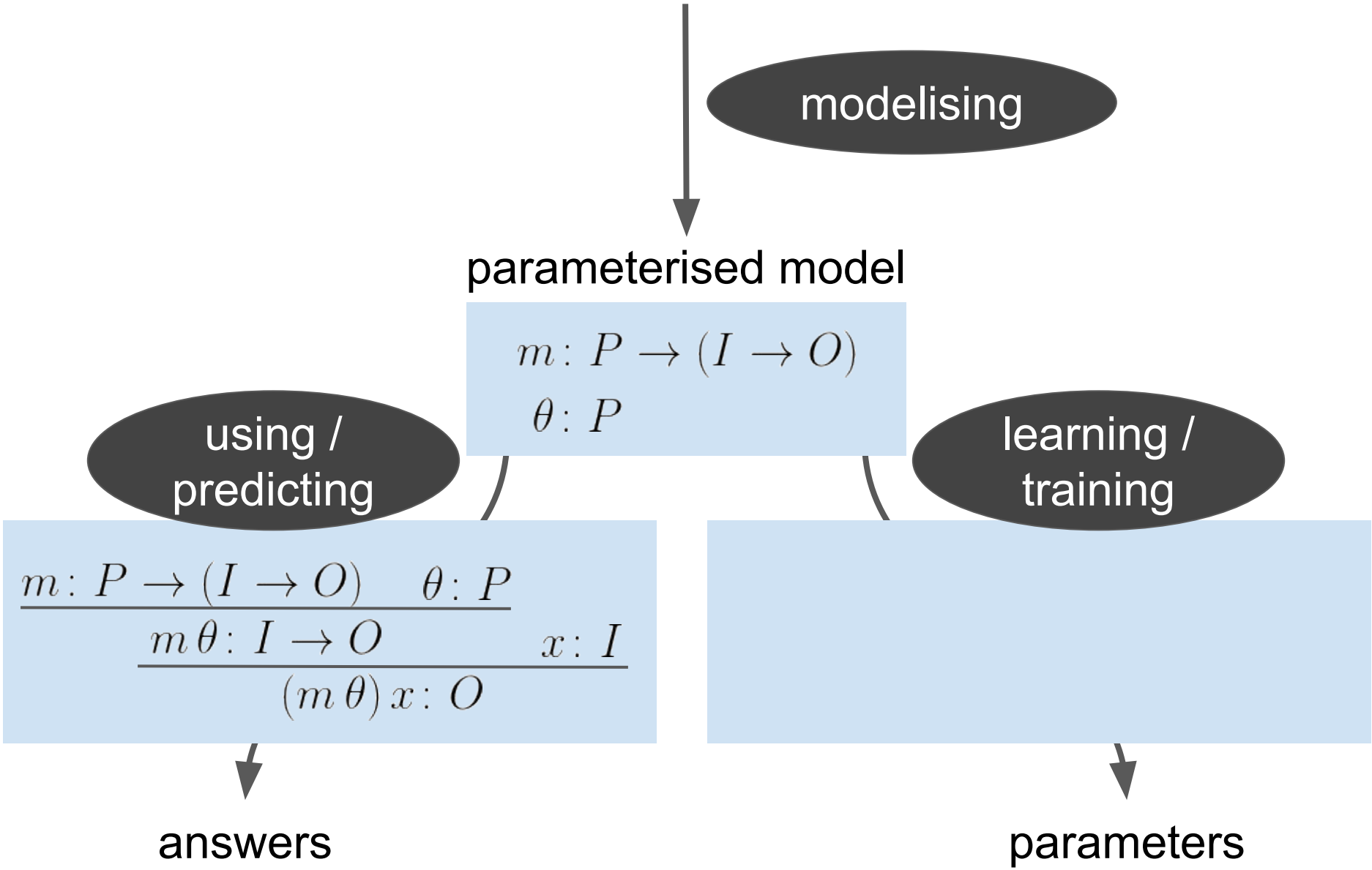

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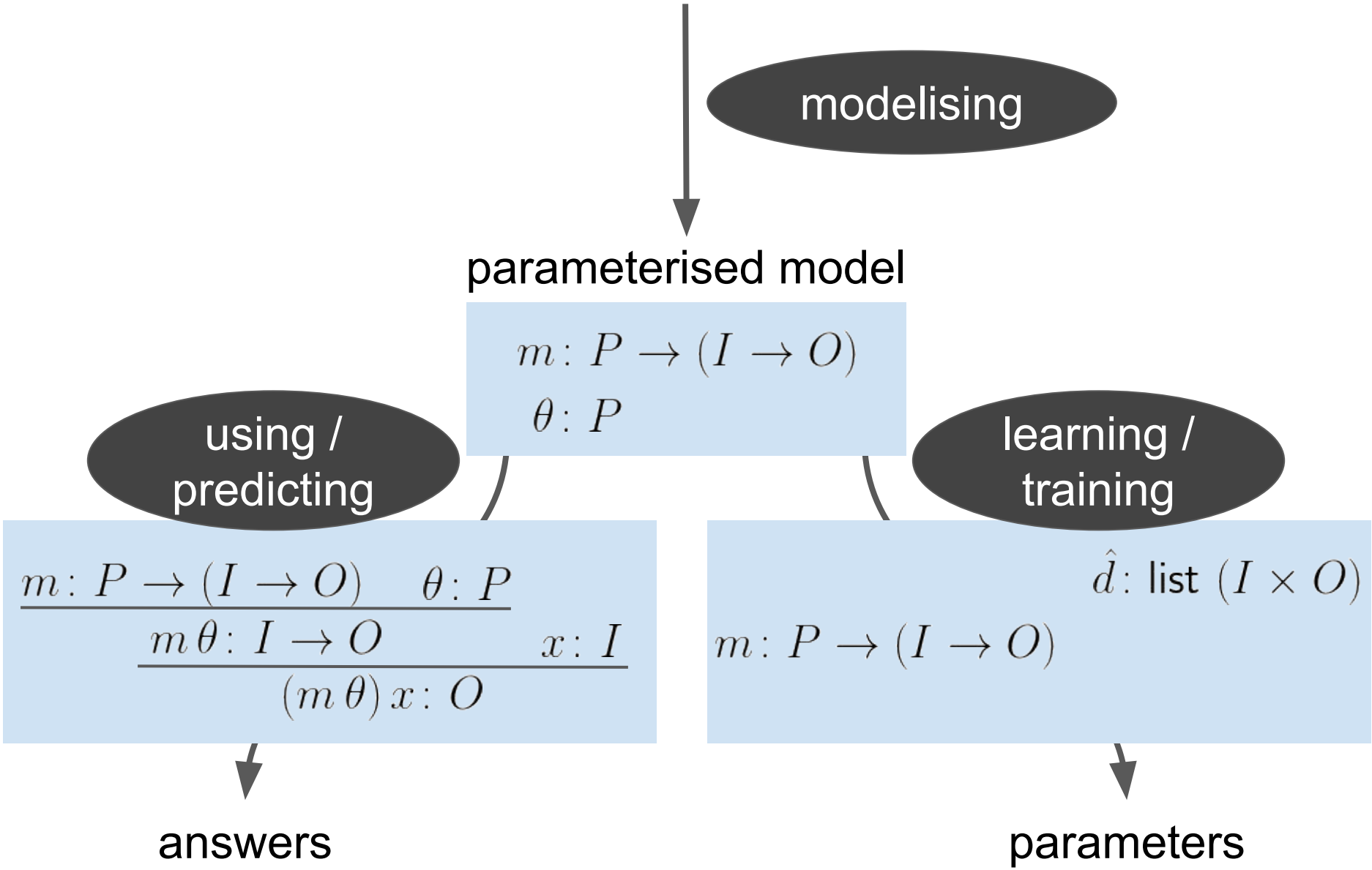

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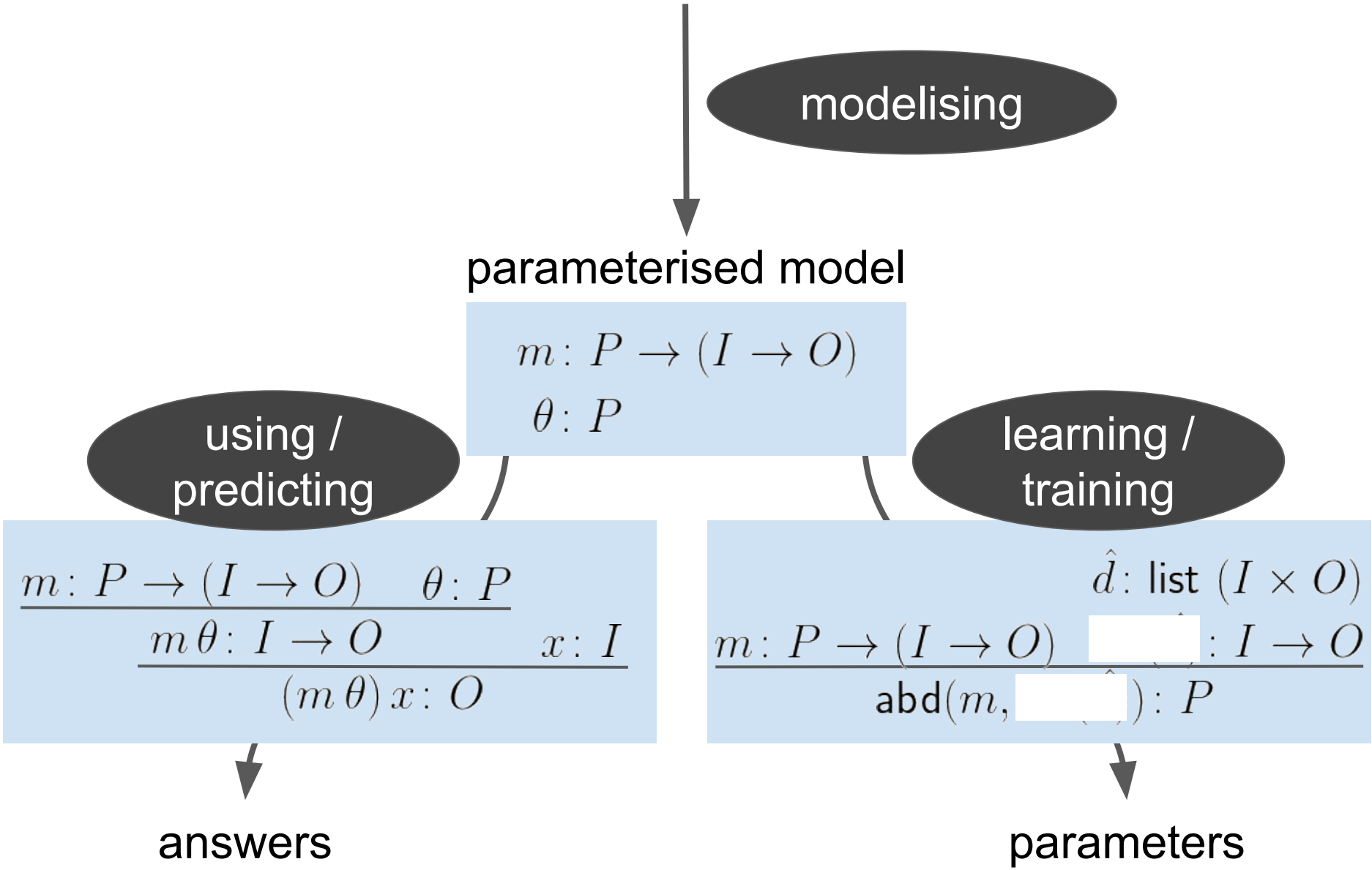

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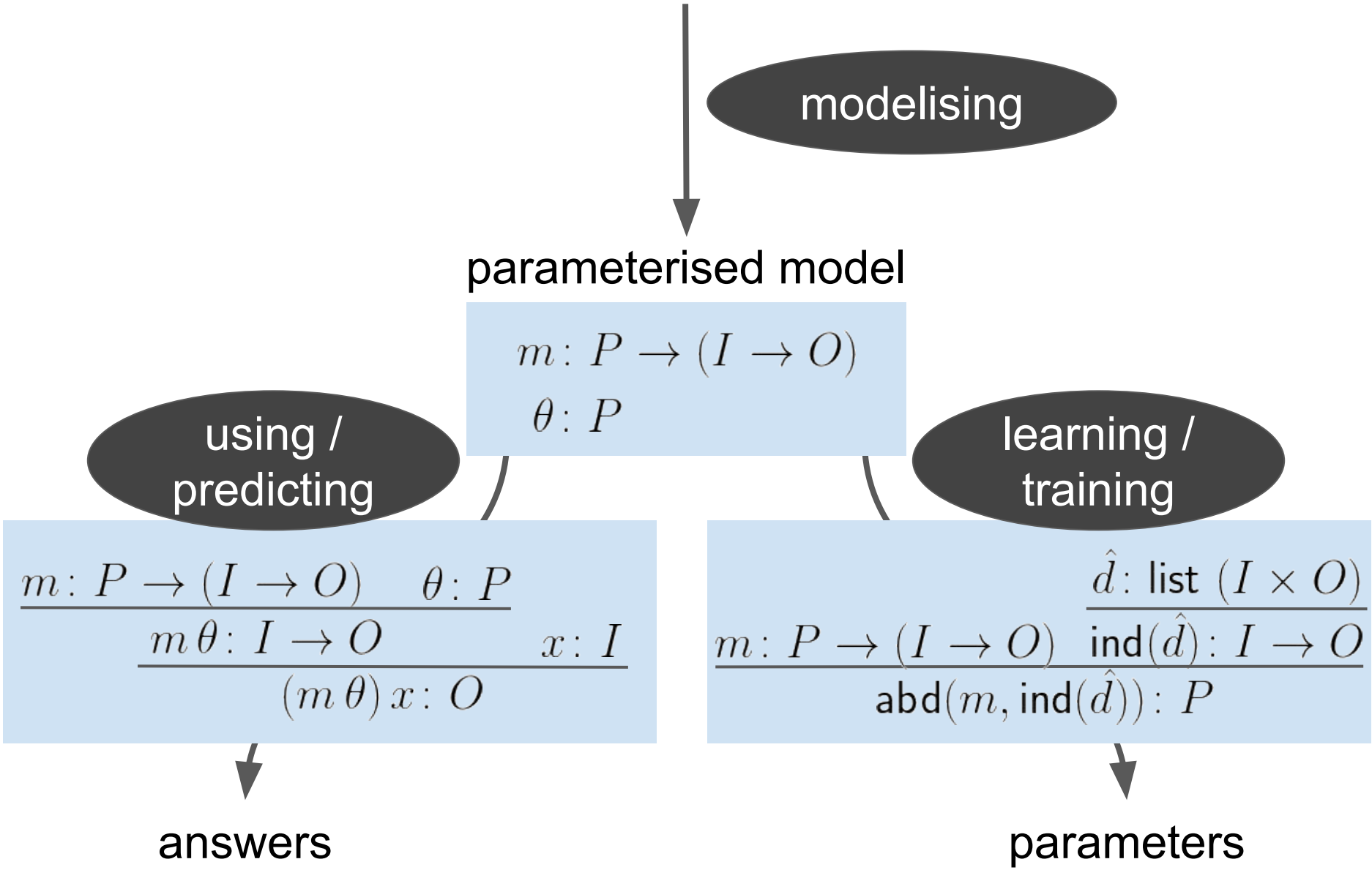

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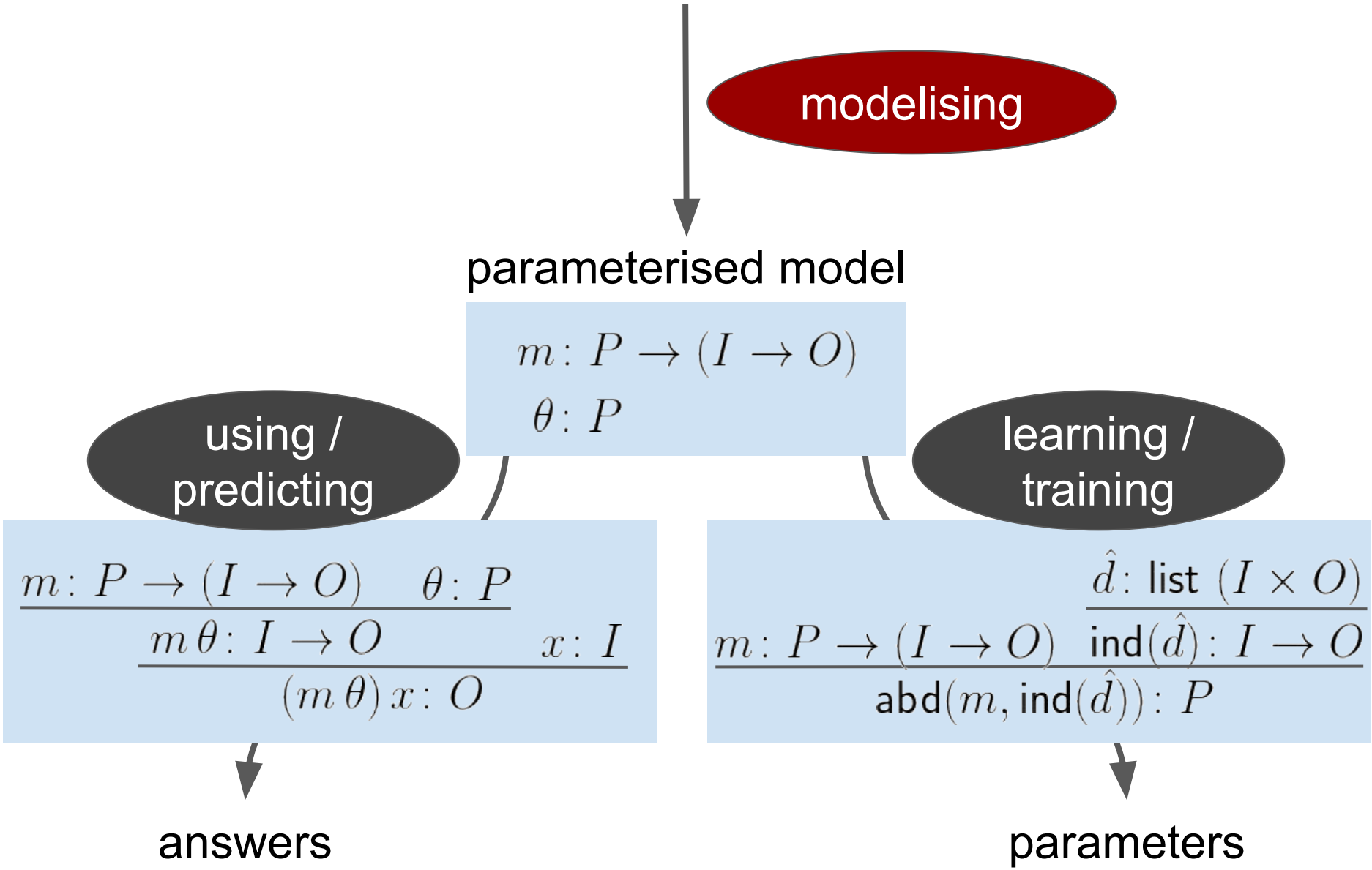
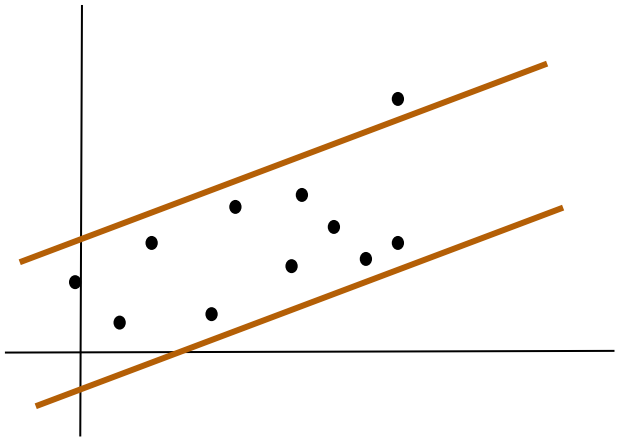


Parameter Management in Modelising

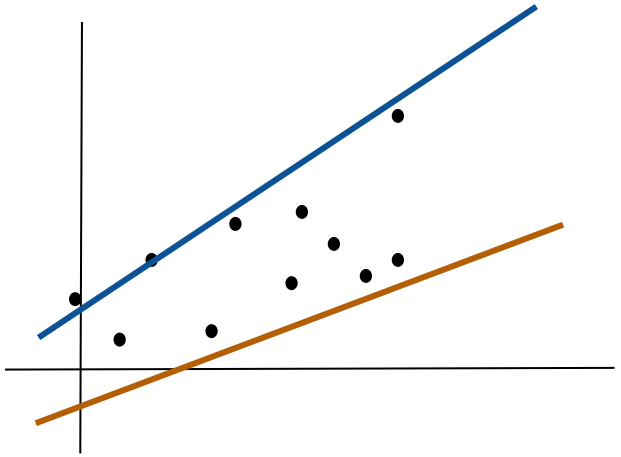
```
let pl v x = v[0] * x + v[1]


let pci v = (pl v[0,1], pl v[0,2])
let pwr v = (pl v[0,1], pl v[2,3])
```

linear regression with confidence interval



weighted regression



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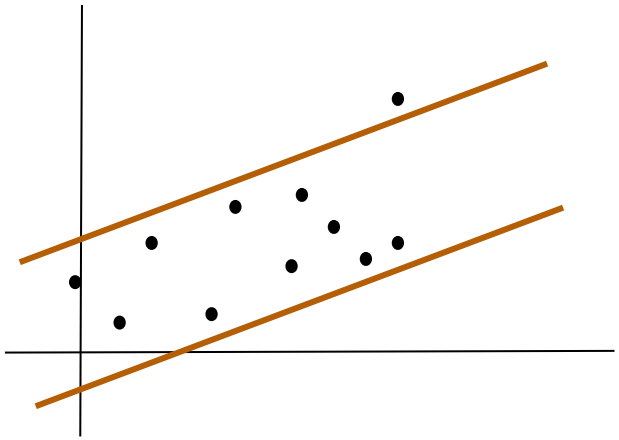
error-prone

linear regression with confidence interval



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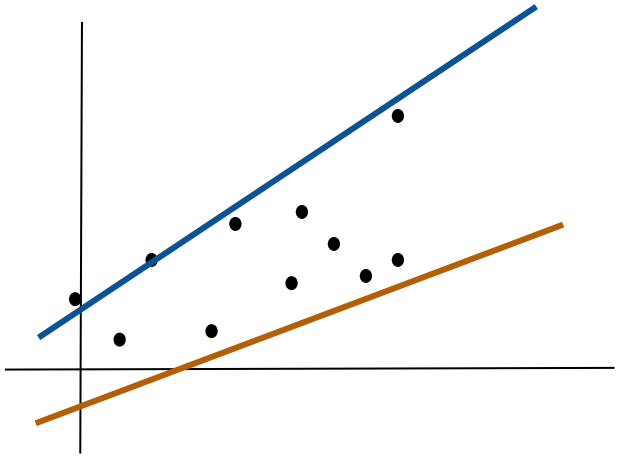

Modelising with *Embedded* Parameters

```
let pl' a b x = a * x + b

let ci = let a = {0} in
         (pl' a {1}, pl' a {2})
let wr = (pl' {0} {1}, pl' {2} {3})
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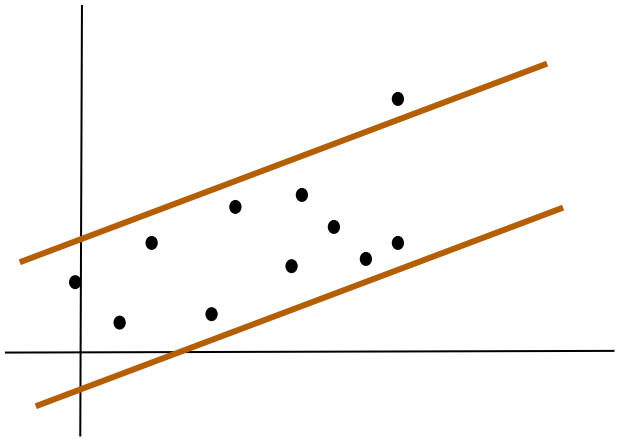
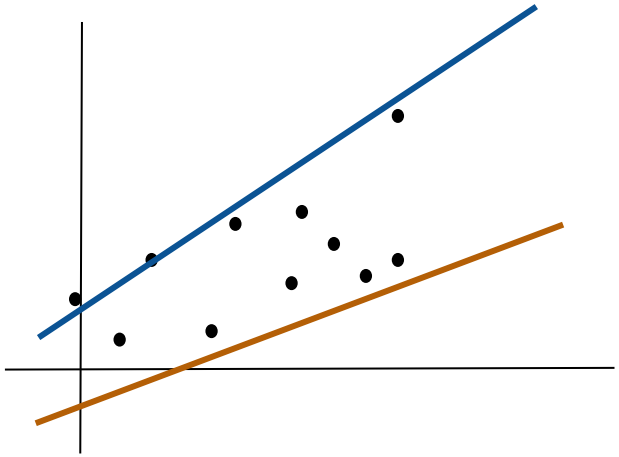
linear regression with confidence interval



weighted regression



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let wr = (pl' {0} {1}, pl' {2} {3})

;; pl'    : F -> F -> F -> F
;; ci,wr : (F -> F) * (F -> F)
```

$$\frac{}{\{k\} : \mathbb{F}}$$

```
let pl v x = v[0] * x + v[1]

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;; pl       : P -> F -> F
;; pci,pwr : P -> ((F -> F) * (F -> F))
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Modelising with *Embedded* Parameters

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;; pl'    : F -> F -> F -> F
;; ci,wr : (F -> F) * (F -> F)
```

$$\frac{}{\{k\} : \mathbb{F}}$$

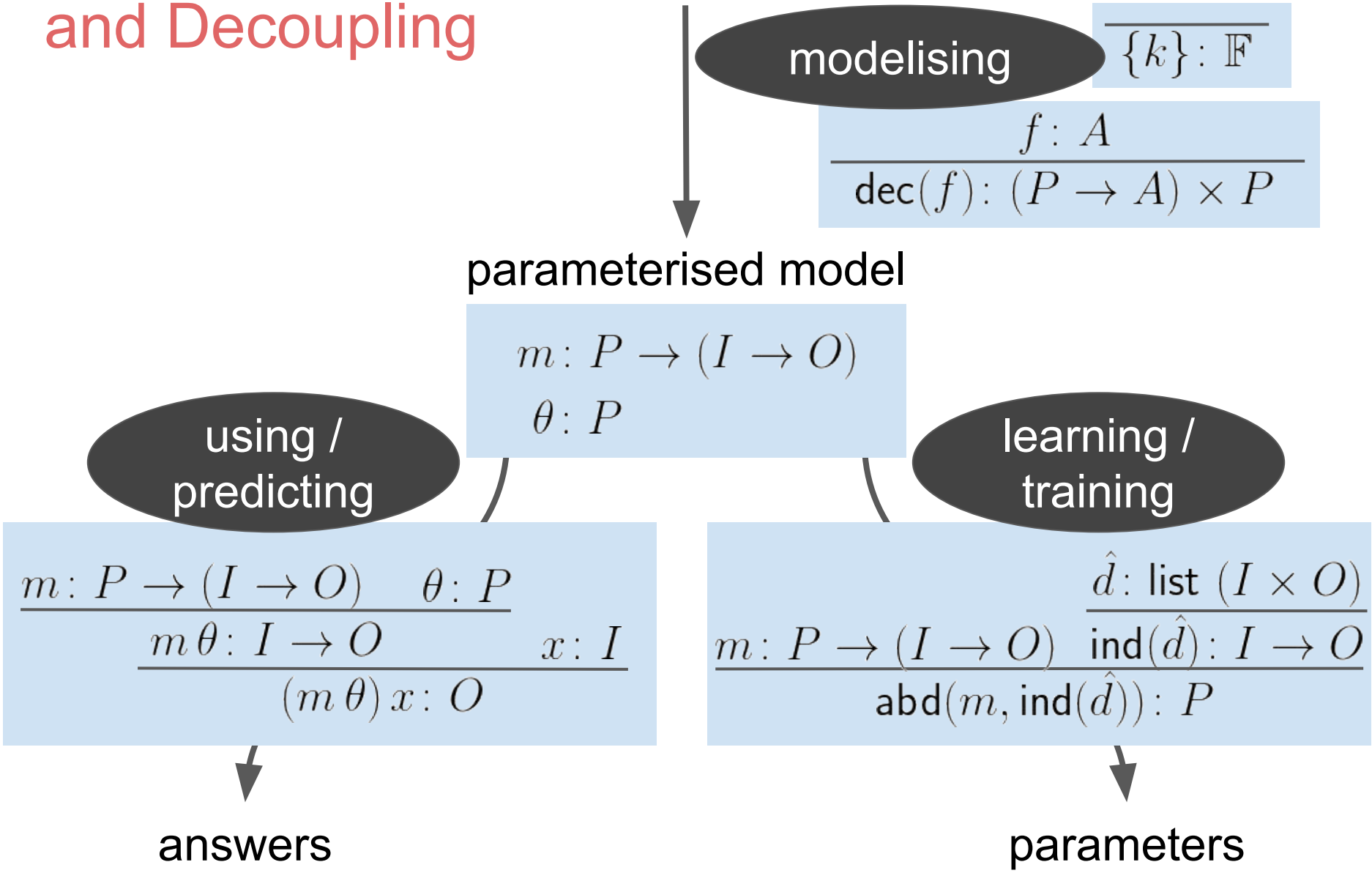
...and *Decoupling*

$$\frac{f : A}{\mathsf{dec}(f) : (P \to A) \times P}$$

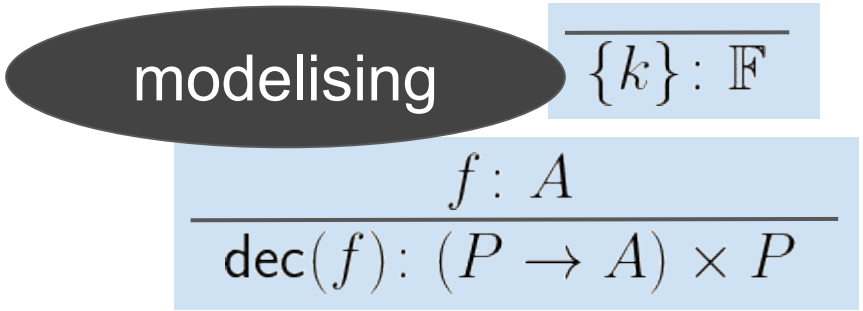
```
let pl v x = v[0] * x + v[1]

let pci v = (pl v[0,1], pl v[0,2])
let pwr v = (pl v[0,1], pl v[2,3])

;; pl       : P -> F -> F
;; pci,pwr : P -> ((F -> F) * (F -> F))
```

```
dec(ci) ~ (pci, [0;1;2])
dec(wr) ~ (pwr, [0;1;2;3])
```

Machine Learning with Induction, Abduction and Decoupling



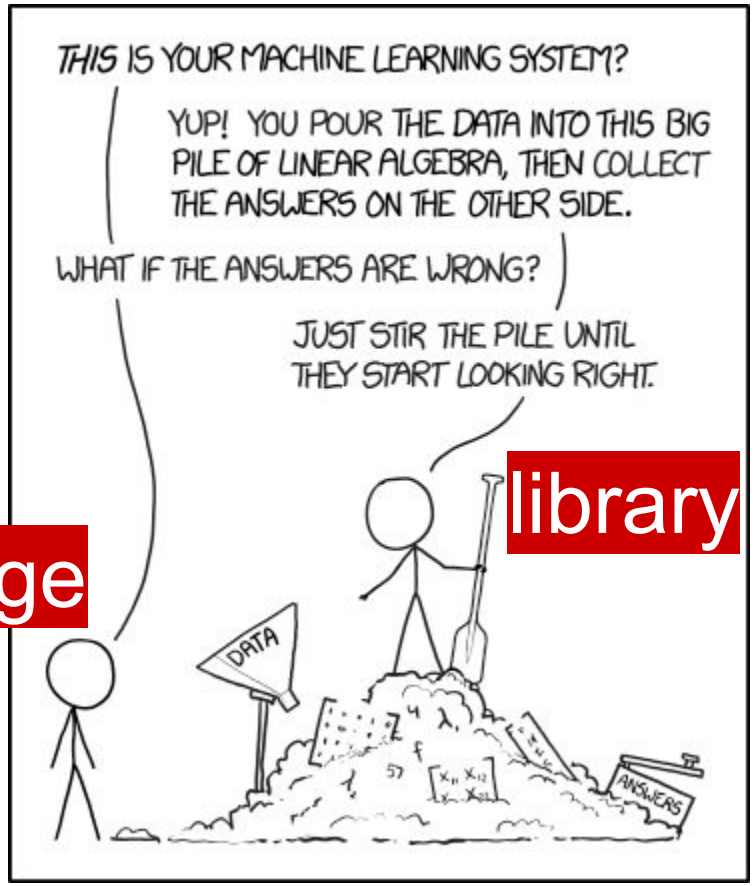
DecML, or Idealised TensorFlow



- as an extended simply-typed lambda-calculus
  - type soundness & beta-law, using graph-rewriting operational semantics [-, Cheung & Ghica, LICS '18 to appear]
  - graph-rewriting visualiser https://cwtsteven.github.io/GoI-TF-Visualiser/CBV-with-CBN-embedding/index.html
- as a PPX extension of OCaml
  - https://github.com/DecML/decml-ppx
    https://github.com/reubenrowe/ocaml-decml (to be merged)
  - (non-trivial) translation to OCaml in pre-processing

Machine Learning with Induction, Abduction and Decoupling

host language

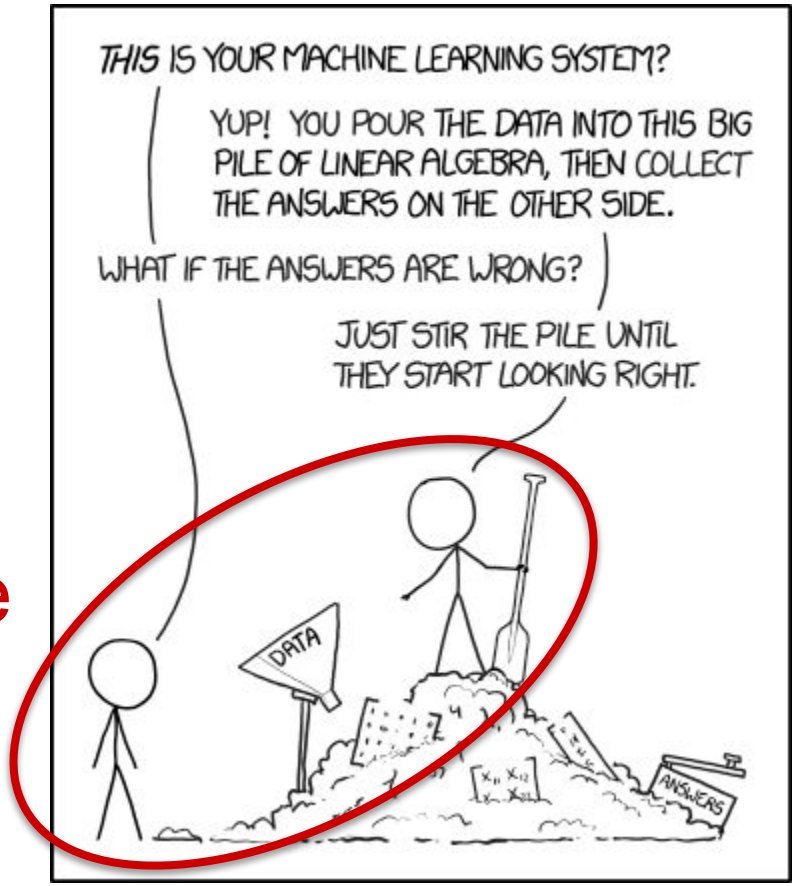


library

https://xkcd.com/1838/

Machine Learning with Induction, Abduction and Decoupling

Let them be *united* :-)



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Machine Learning with Induction, Abduction and Decoupling

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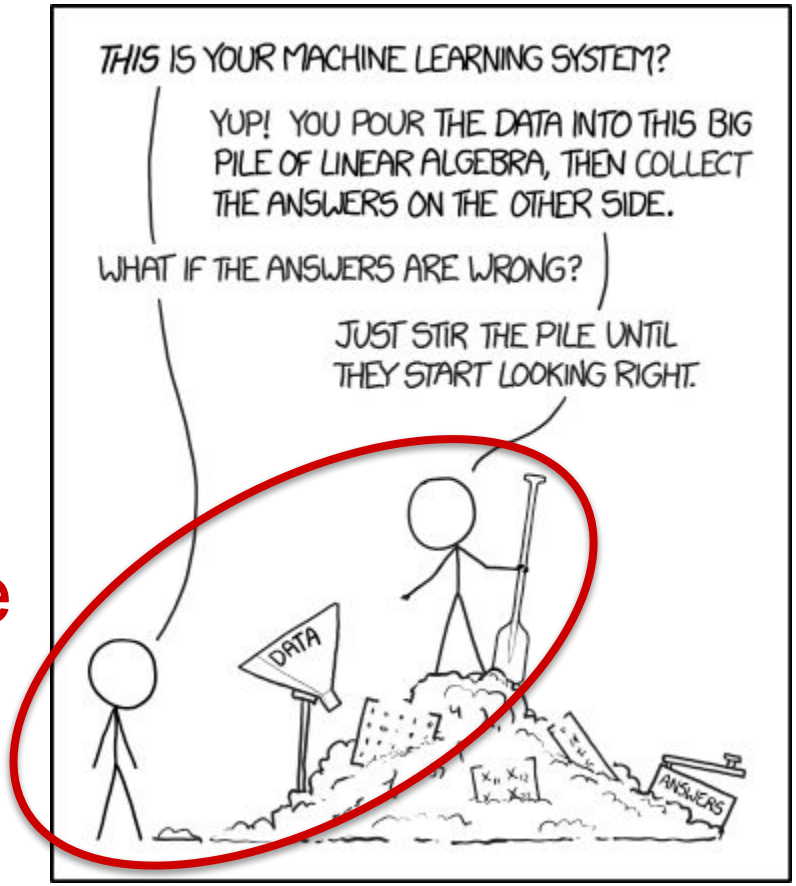

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probability TBA