Memoryful GoI with recursion

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LOLA(Kyoto), July 5, 2015

$\frac{\text{Memoryful GoI}}{\text{with recursion}}$ (1)

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Memoryful GoI (1)
with recursion (2)

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Memoryful GoI [Hoshino, —, Hasuo ’14]

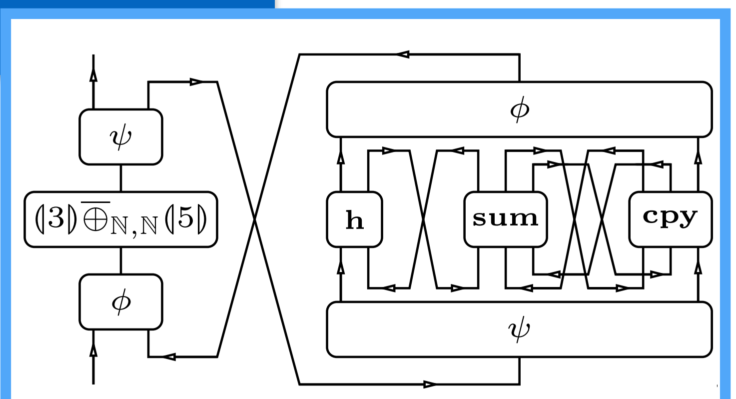
effectful terms

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\, (3 \sqcup 5) \quad : \mathtt{nat}$

sound translation

- based on Geometry of Interaction
- via coalgebraic component calculus

transducers



Geometry of Interaction (GoI)

- semantics of
  - linear logic proofs [Girard '89],
  - functional programming languages

"GoI interpretation"

- token machine representation [Mackie '95]

↓

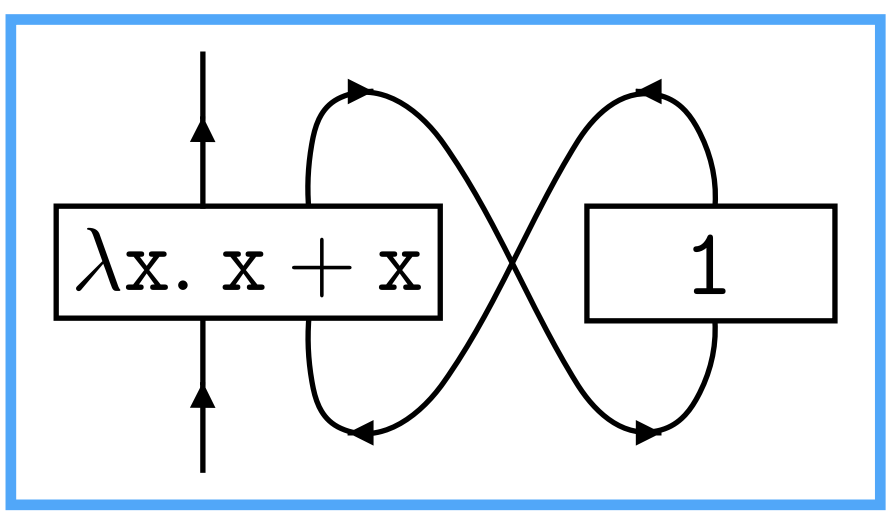
compilation techniques and implementations

[Mackie '95] [Pinto '01] [Ghica '07]

Geometry of Interaction (GoI)

- token machine representation of GoI interpretation

$$(\lambda x.\ x + x)\ 1$$



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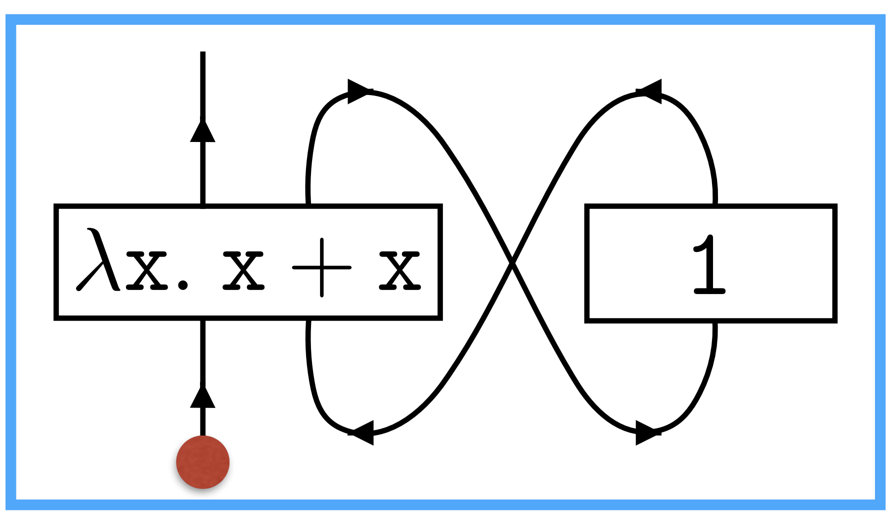
Geometry of Interaction (GoI)

- token machine representation of GoI interpretation

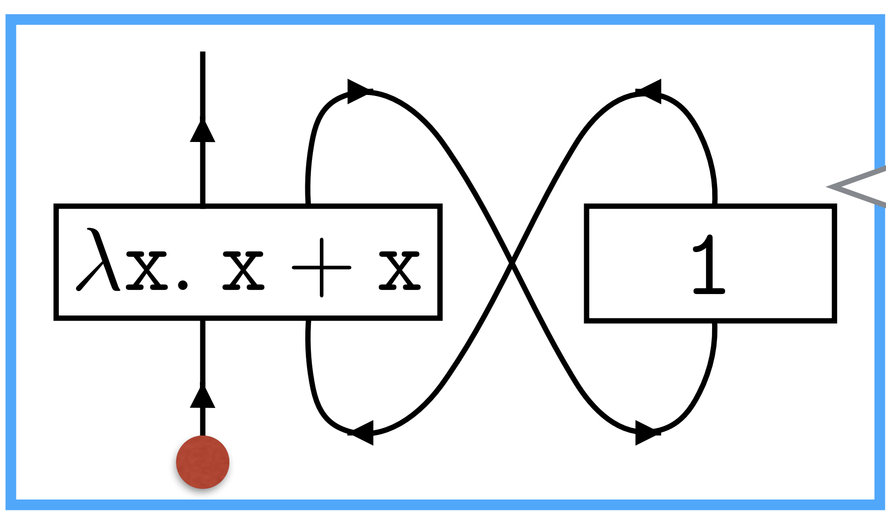
$$(\lambda x.\ x + x)\ 1$$

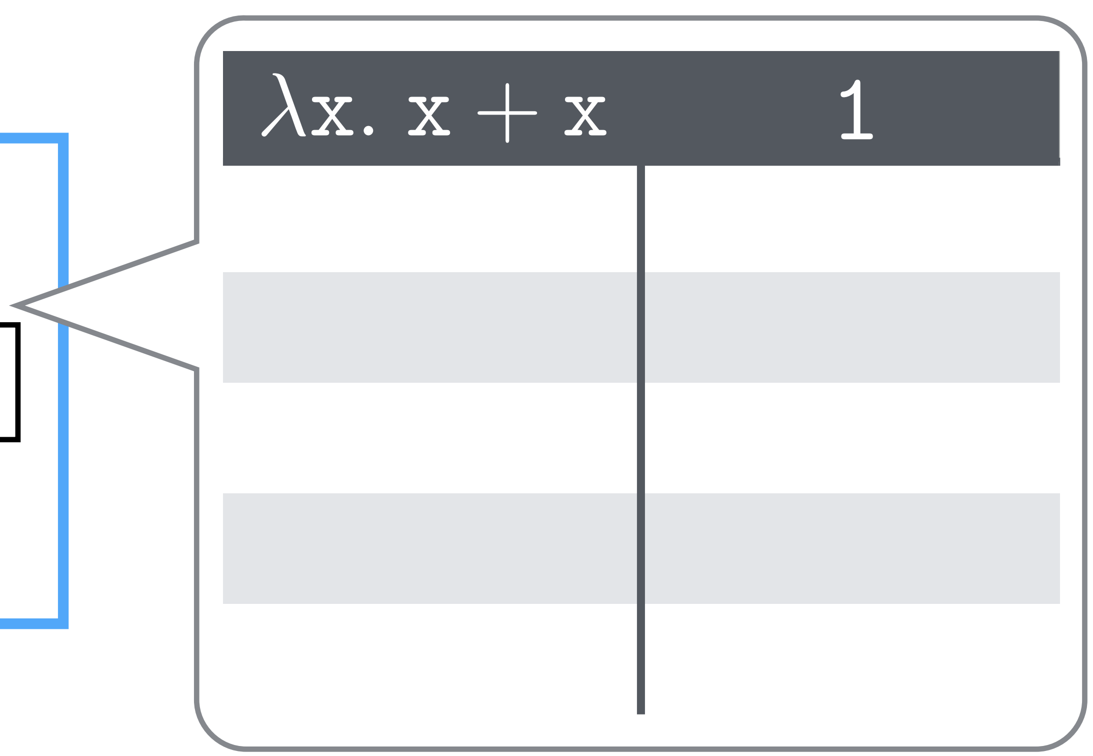


Geometry of Interaction (GoI)

- token machine representation of GoI interpretation

$$(\lambda x.\ x + x)\ 1$$

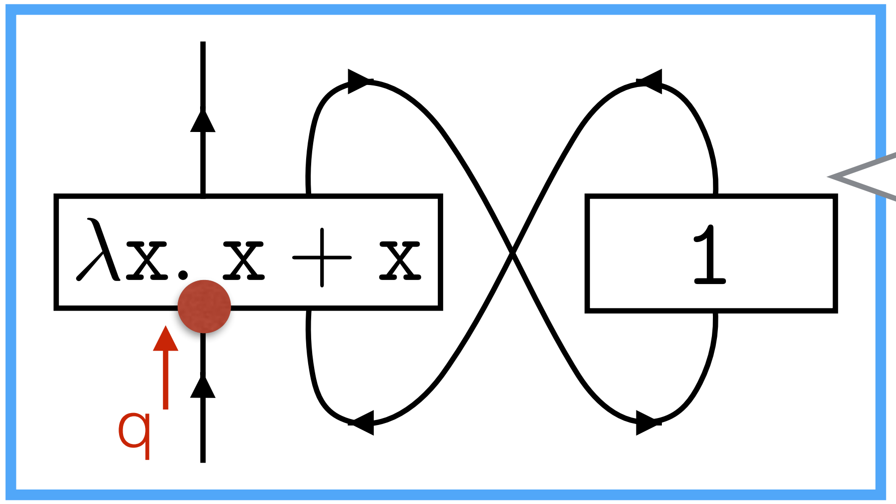


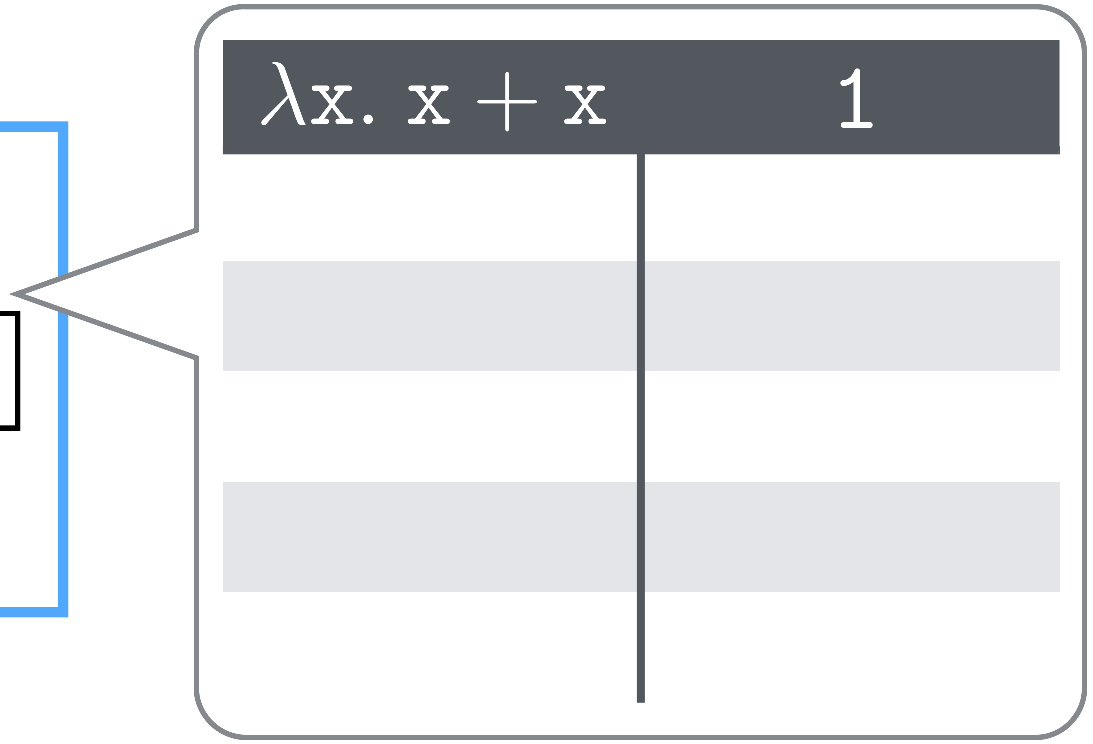


Geometry of Interaction (GoI)

- token machine representation of GoI interpretation

$(\lambda x.\ x + x)\ 1$





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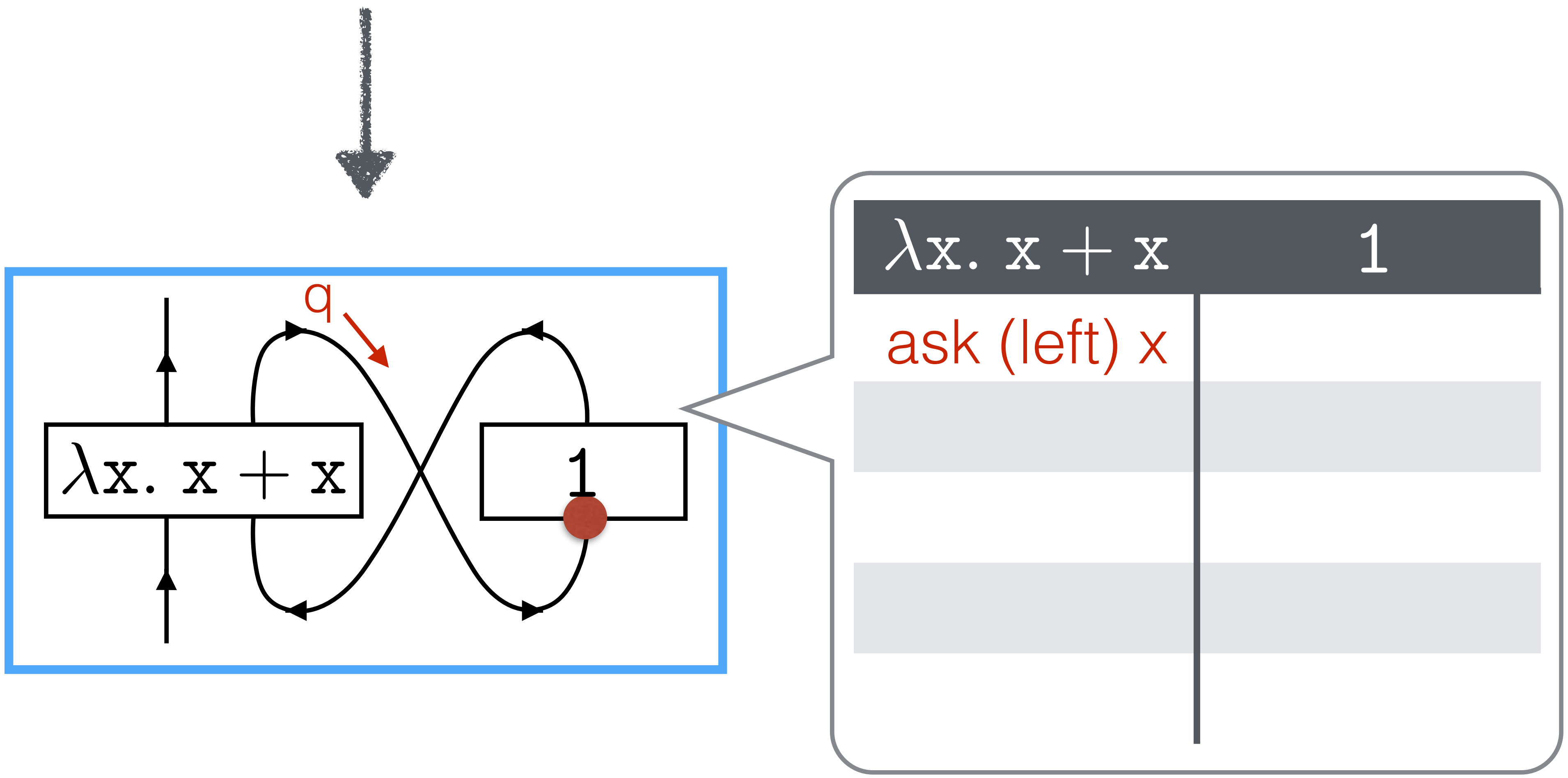
Geometry of Interaction (GoI)

- token machine representation of GoI interpretation

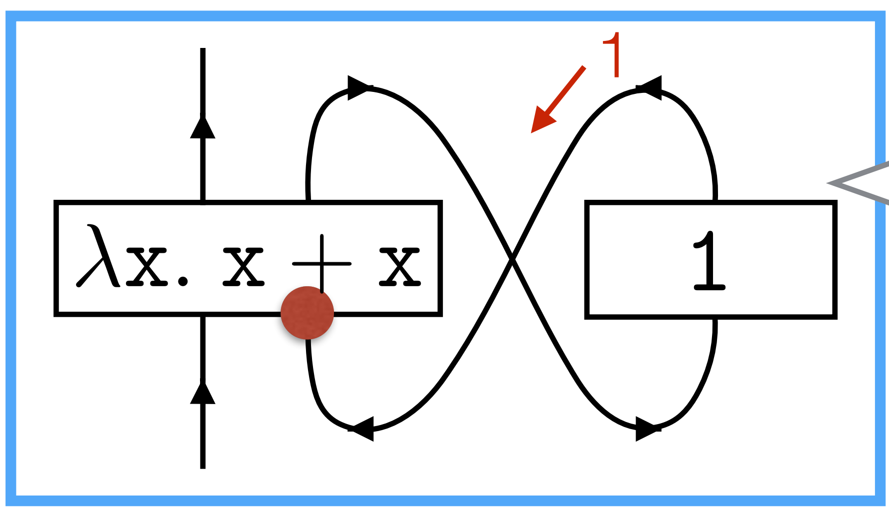
$(\lambda x.\ x + x)\ 1$



Geometry of Interaction (GoI)

- token machine representation of GoI interpretation

$(\lambda x.\ x + x)\ 1$

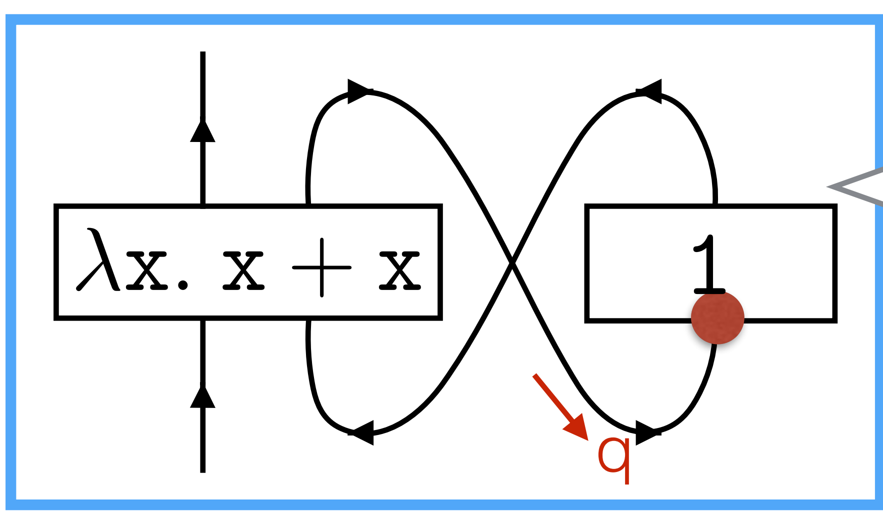


$\lambda x.\ x + x$	1
ask (left) x	
	answer 1

Geometry of Interaction (GoI)

- token machine representation of GoI interpretation

$(\lambda x.\ x + x)\ 1$

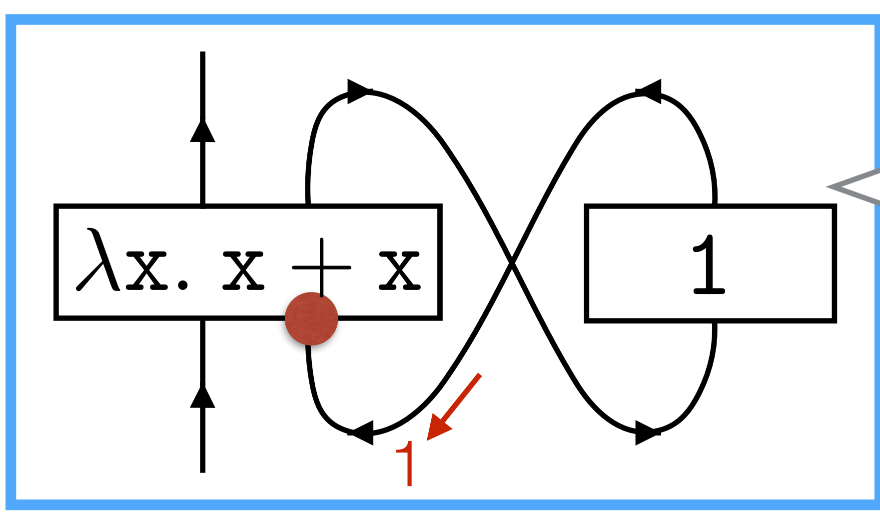


$\lambda x.\ x + x$	1
ask (left) x	
	answer 1
ask (right) x	

Geometry of Interaction (GoI)

- token machine representation of GoI interpretation

$(\lambda \mathrm{x}.\ \mathrm{x} + \mathrm{x})\ 1$

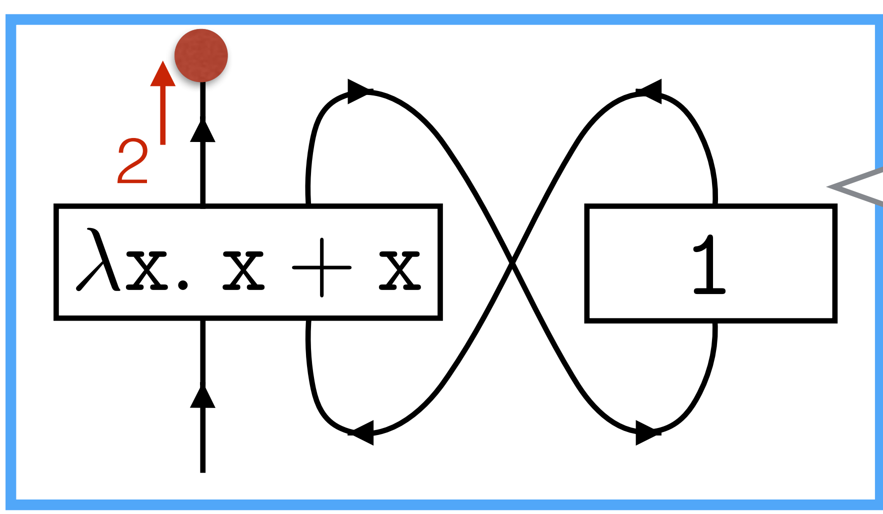


$\lambda \mathrm{x}.\ \mathrm{x} + \mathrm{x}$	1
ask (left) x	
	answer 1
ask (right) x	
	answer 1

Geometry of Interaction (GoI)

- token machine representation of GoI interpretation

$$(\lambda x.\ x + x)\ 1$$



$\lambda x.\ x + x$	1
ask (left) x	
	answer 1
ask (right) x	
	answer 1
answer 2	

Memoryful GoI [Hoshino, —, Hasuo ’14]

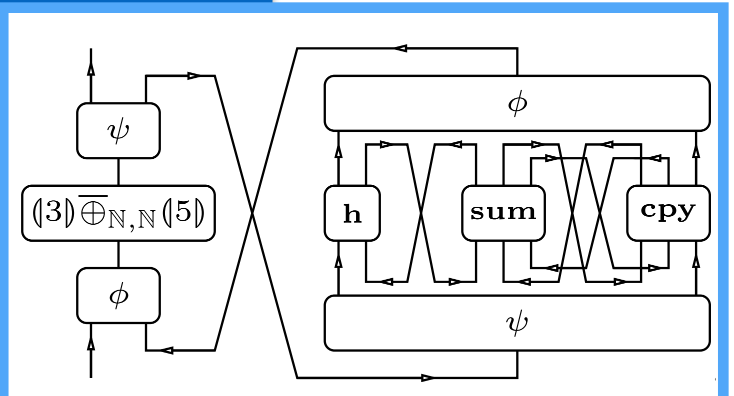
effectful terms

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$

sound translation

- based on Geometry of Interaction
- via coalgebraic component calculus

transducers



Memoryful GoI — Input

effectful terms

transducers

CBV λ-terms with algebraic effects

algebraic operations [Plotkin, Power '01]

- nondeterministic choice $M \sqcup N$
- probabilistic choice $M \sqcup_p N$
- actions on global states

$\mathrm{lookup}_l(M_{v_1}, \ldots, M_{v_{|Val|}})$ $\mathrm{update}_{l,v}(M)$

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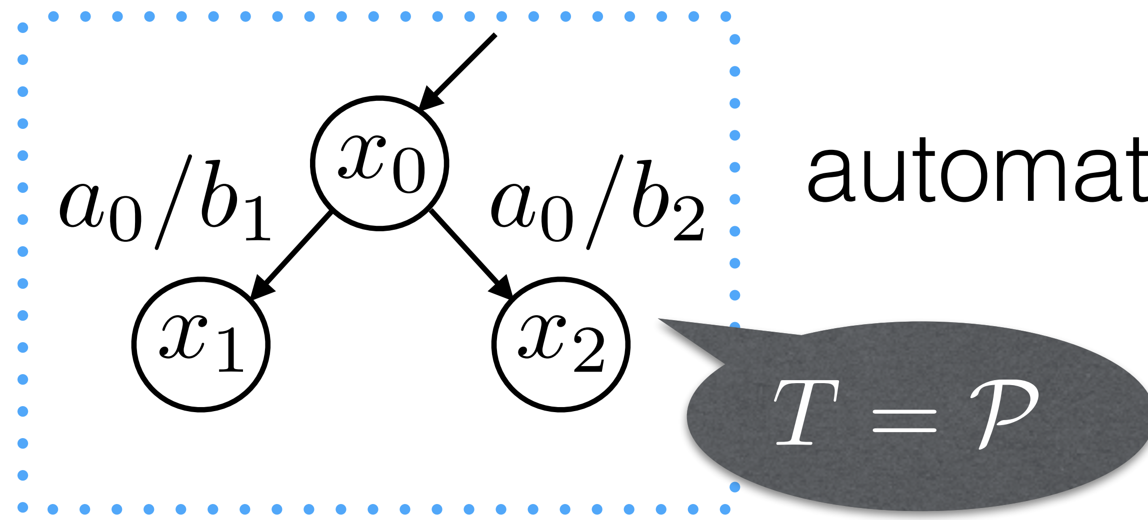
Memoryful GoI — Output

effectful terms

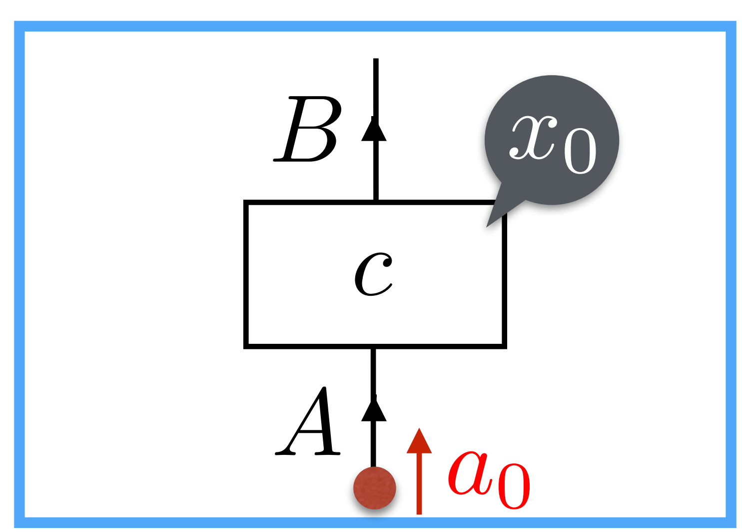
transducers

stream transducers (Mealy machines)

$$(X,\ c\colon X \times A \to T(X \times B),\ x_0 \in X)\colon A \to B$$



automaton style



string diagram style

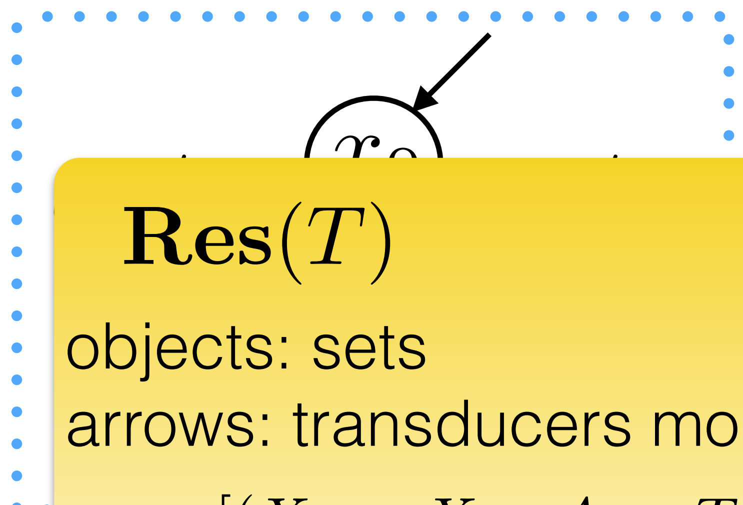
Memoryful GoI — Output

effectful terms

transducers

stream transducers (Mealy machines)

$$(X,\ c\colon X \times A \to T(X \times B),\ x_0 \in X)\colon A \rightharpoonup B$$



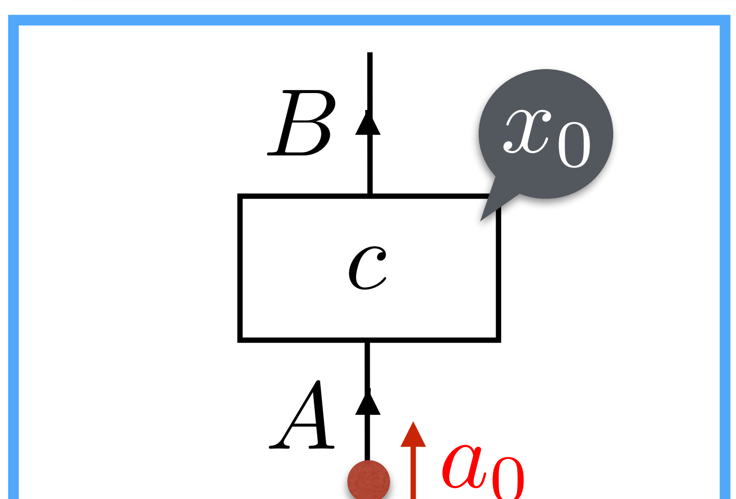
automaton style

$\mathbf{Res}(T)$

objects: sets

arrows: transducers modulo behavioral equivalence

$$[(X,\ c\colon X \times A \to T(X \times B),\ x_0 \in X)]_{\simeq}\colon A \rightharpoonup B$$



string diagram style

Memoryful GoI — Output

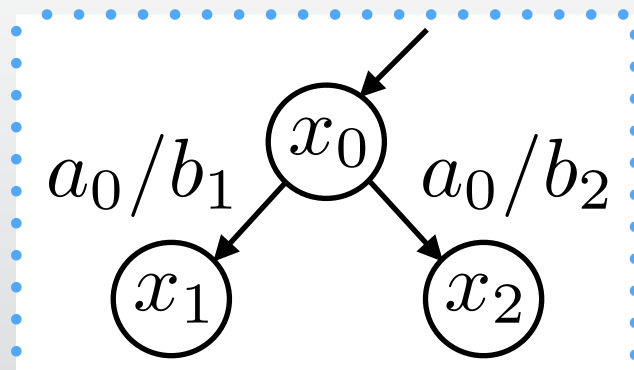
effectful terms

transducers

stream transducers (Mealy machines)

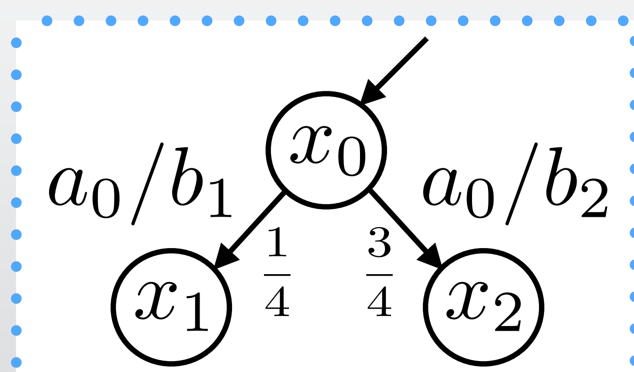
$$(X,\ c\colon X \times A \to T(X \times B),\ x_0 \in X)\colon A \rightharpoonup B$$

$$T = \mathcal{P} \quad (x_0, a_0) \longmapsto \{(x_1, b_1), (x_2, b_2)\}$$



nondeterministic computation

$$T = \mathcal{D} \quad (x_0, a_0) \longmapsto \begin{bmatrix} (x_1, b_1) \mapsto 1/4, \\ (x_2, b_2) \mapsto 3/4 \end{bmatrix}$$



probabilistic computation

$$T = \mathcal{S} = (1 + (-) \times S)^S$$

computation with global states

Memoryful GoI [Hoshino, —, Hasuo ’14]

effectful terms

$(\lambda x : \mathtt{nat}.\, x + x)\,(3 \sqcup 5) \quad : \mathtt{nat}$

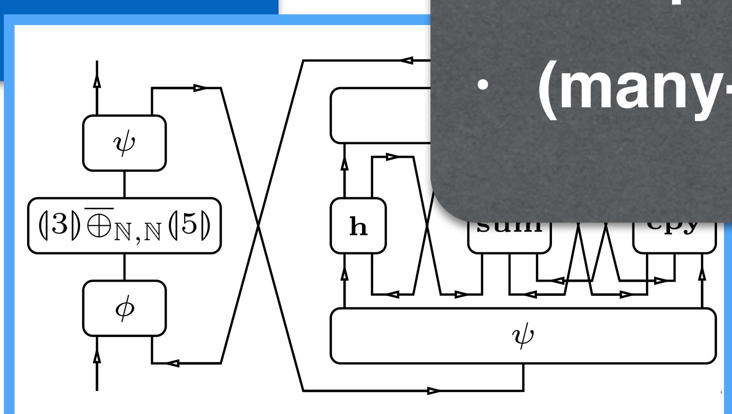
sound translation

- based on Geometry of Interaction
- via coalgebraic component calculus

[Barbosa ’03] [Hasuo, Jacobs ’11]

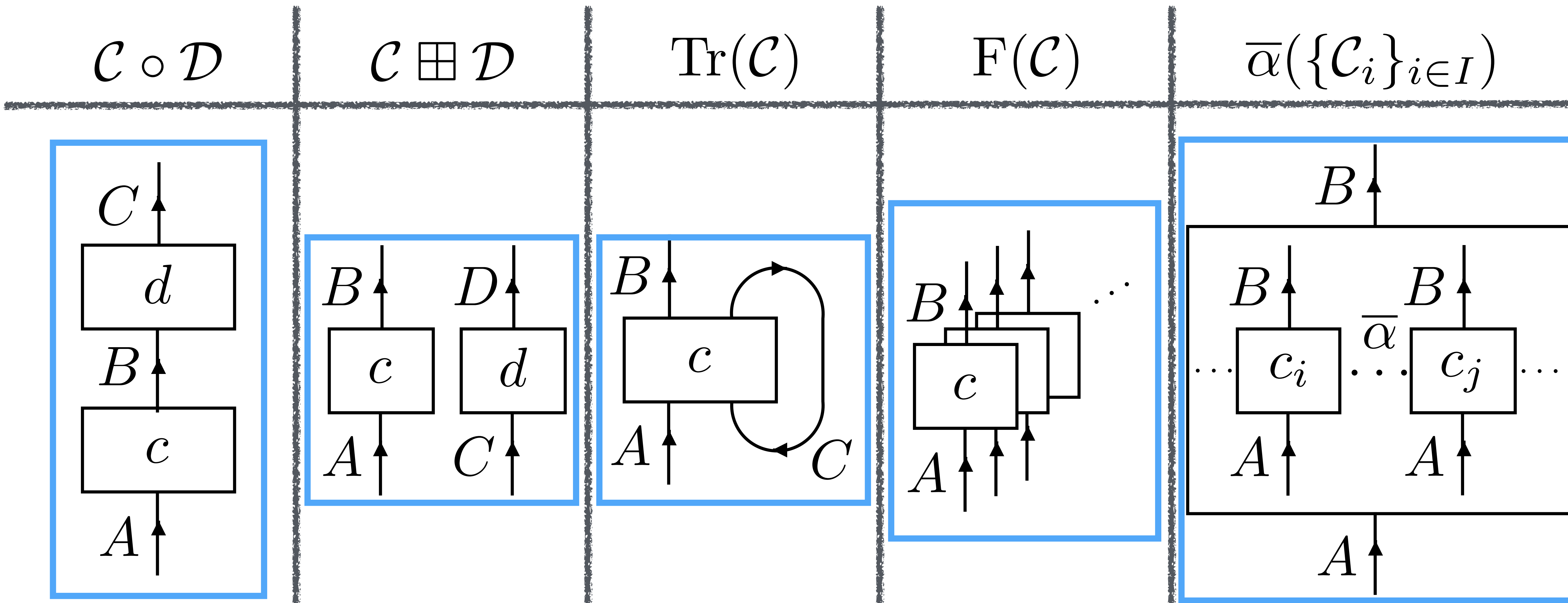
- composition operators for software components
- (many-sorted) process calculus

transducers



Memoryful GoI — Translation

<u>Def. (component calculus)</u>



(α: I-ary algebraic operation)

Memoryful GoI — Translation

Def. (component calculus)

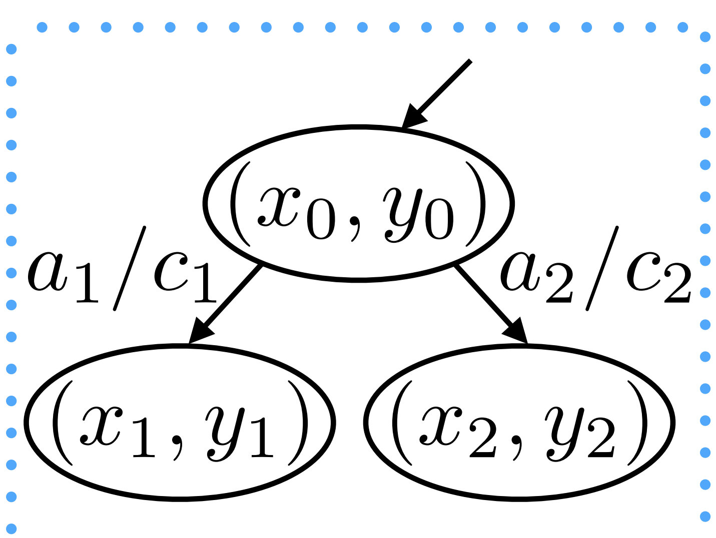
sequential composition

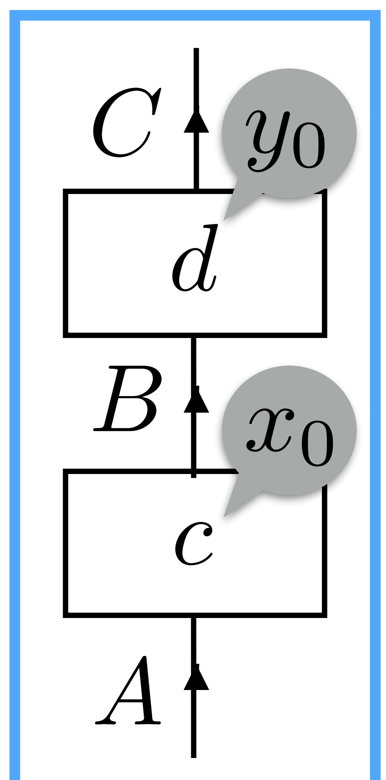
$\mathcal{C} \circ \mathcal{D}$

parallel composition

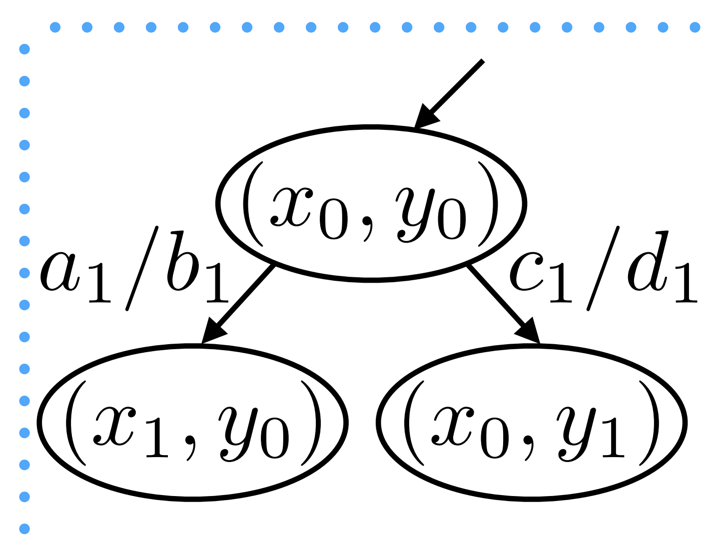
$\mathcal{C} \boxplus \mathcal{D}$

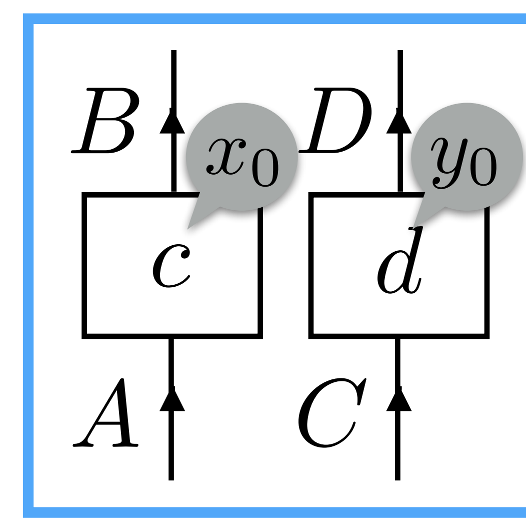
$$\begin{pmatrix} Y, \\ Y \times B \xrightarrow{d} T(Y \times C), \\ y_0 \in Y \end{pmatrix} \circ \begin{pmatrix} X, \\ X \times A \xrightarrow{c} T(X \times B), \\ x_0 \in X \end{pmatrix} = \begin{pmatrix} X \times Y, \\ \cdots \\ (x_0, y_0) \in X \times Y \end{pmatrix}$$





$$\begin{pmatrix} X, \\ X \times A \xrightarrow{c} T(X \times B), \\ x_0 \in X \end{pmatrix} \boxplus \begin{pmatrix} Y, \\ Y \times C \xrightarrow{d} T(Y \times D), \\ y_0 \in Y \end{pmatrix} = \begin{pmatrix} X \times Y, \\ \cdots \\ (x_0, y_0) \in X \times Y \end{pmatrix}$$





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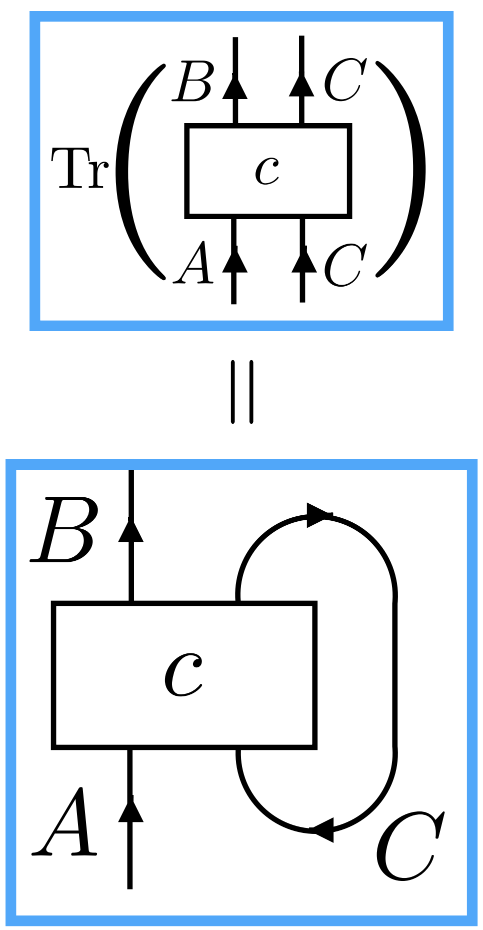
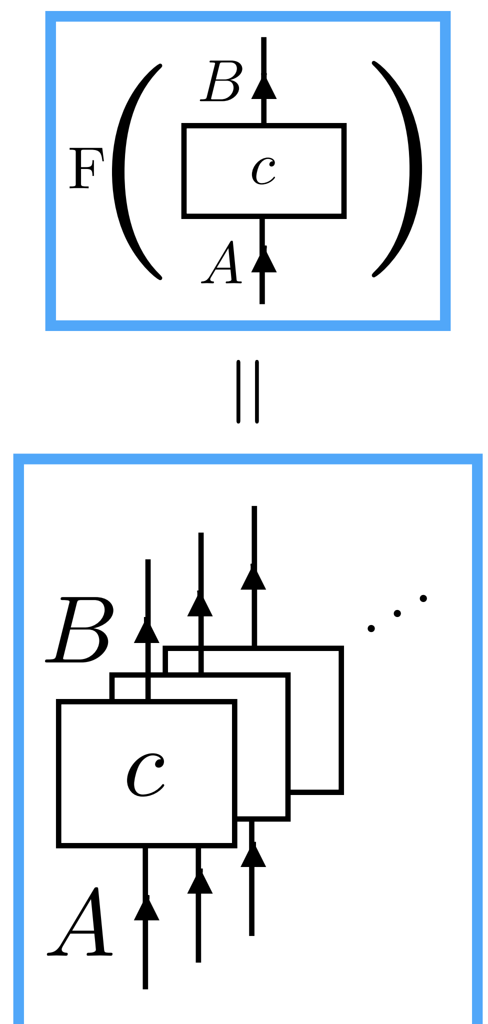
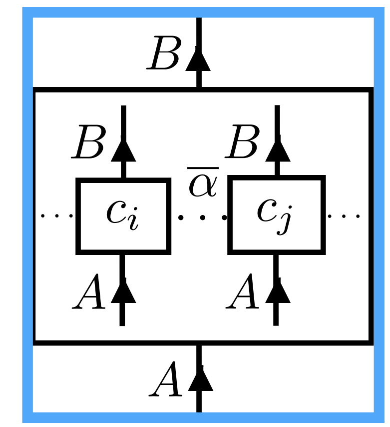
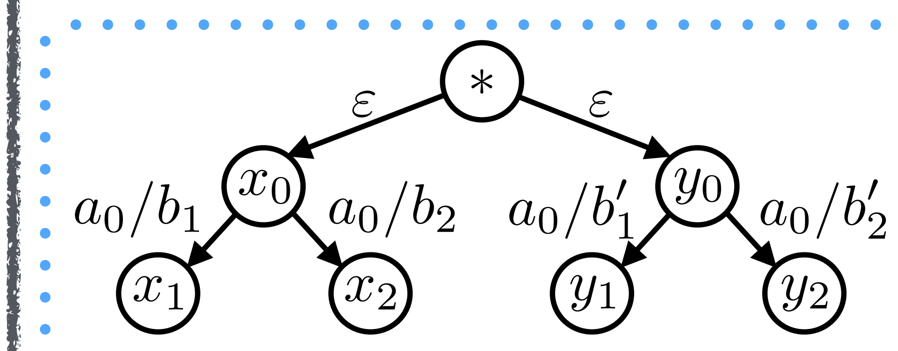
Memoryful GoI — Translation

<u>Def. (component calculus)</u>

trace operator

countable copy operator

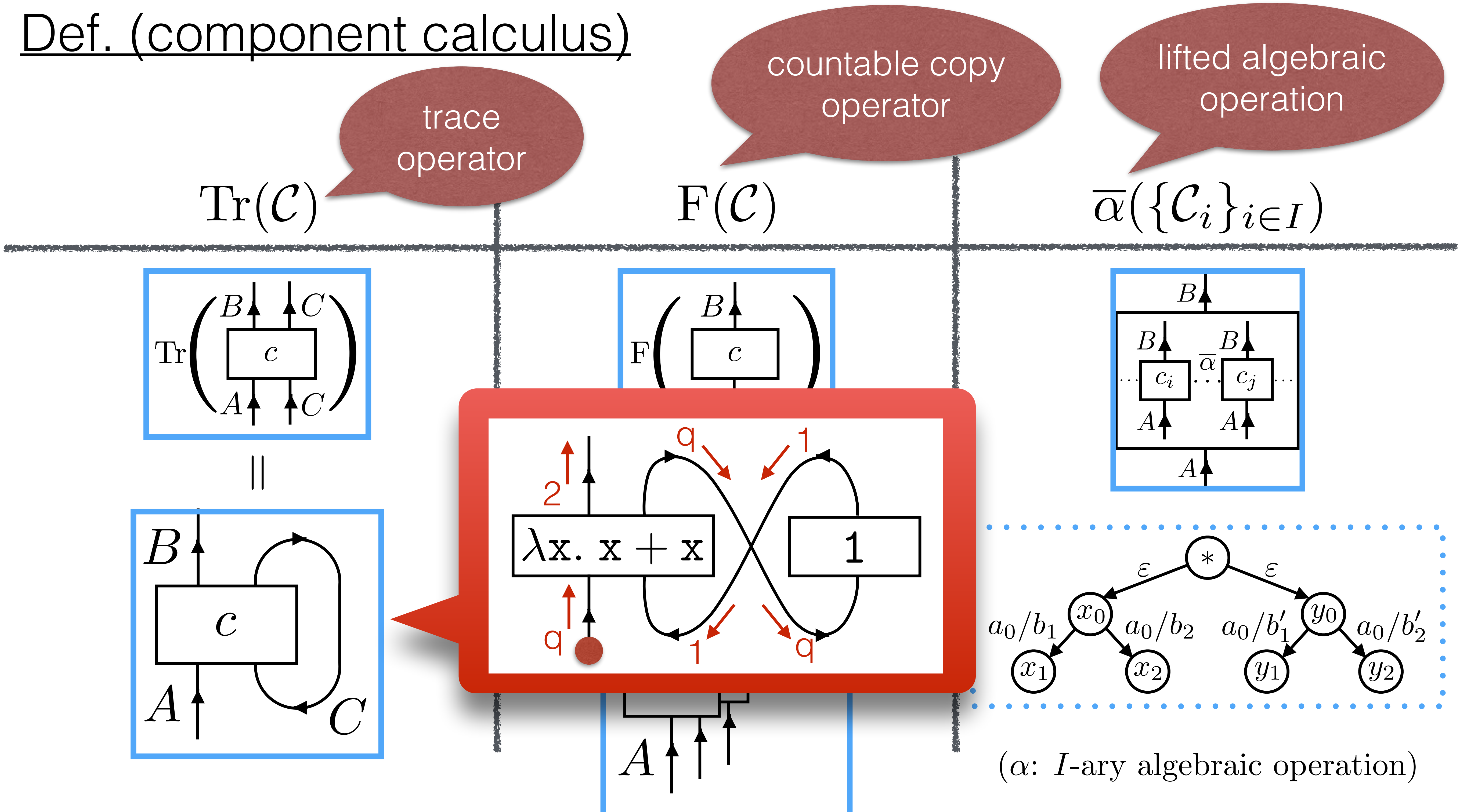
lifted algebraic operation

$\mathrm{Tr}(\mathcal{C})$	$\mathrm{F}(\mathcal{C})$	$\overline{\alpha}(\{\mathcal{C}_i\}_{i\in I})$
		 

(α: I-ary algebraic operation)

Memoryful GoI — Translation

Def. (component calculus)



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Memoryful GoI — Translation

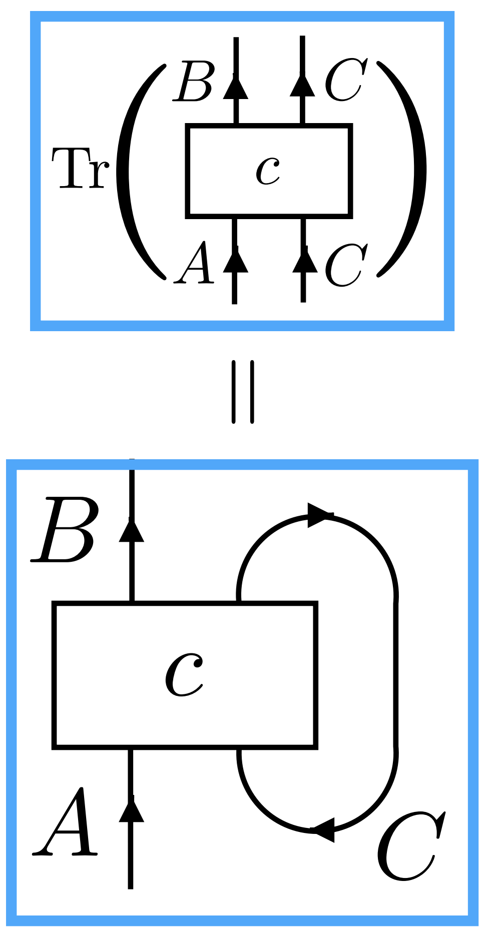
Def. (component calculus)

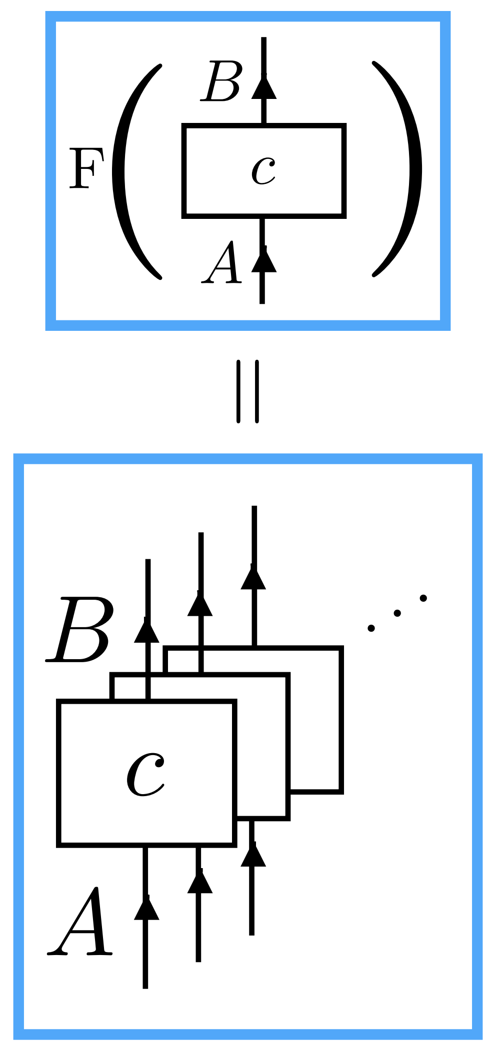
trace operator

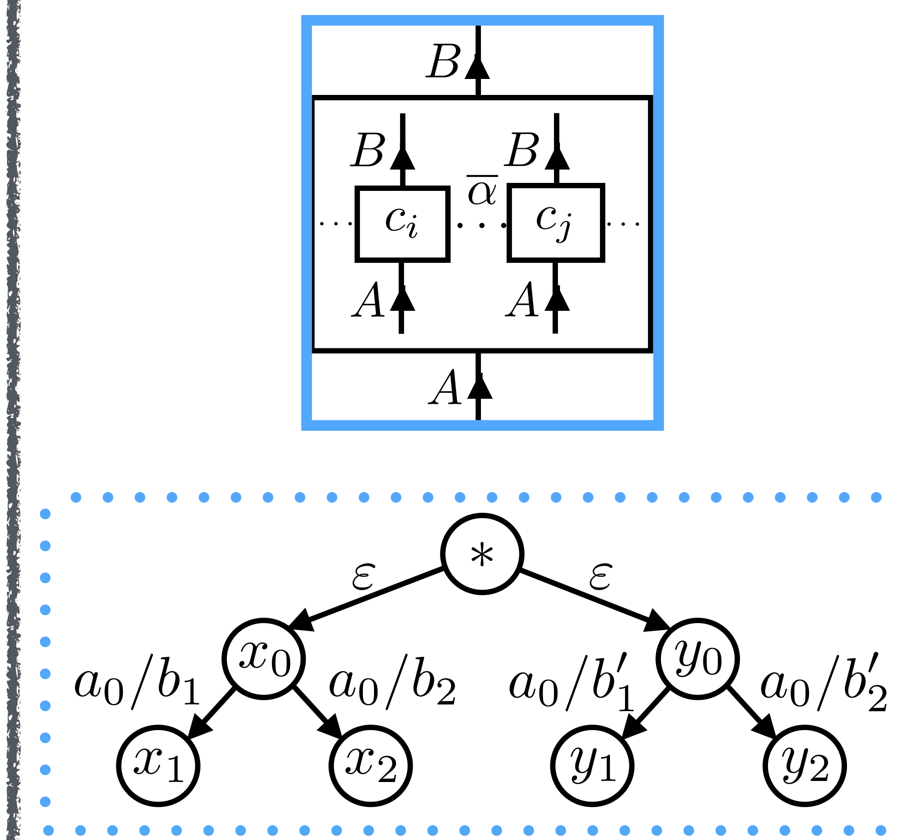
countable copy operator

lifted algebraic operation

$\mathrm{Tr}(\mathcal{C})$	$\mathrm{F}(\mathcal{C})$	$\overline{\alpha}(\{\mathcal{C}_i\}_{i \in I})$







(α: I-ary algebraic operation)

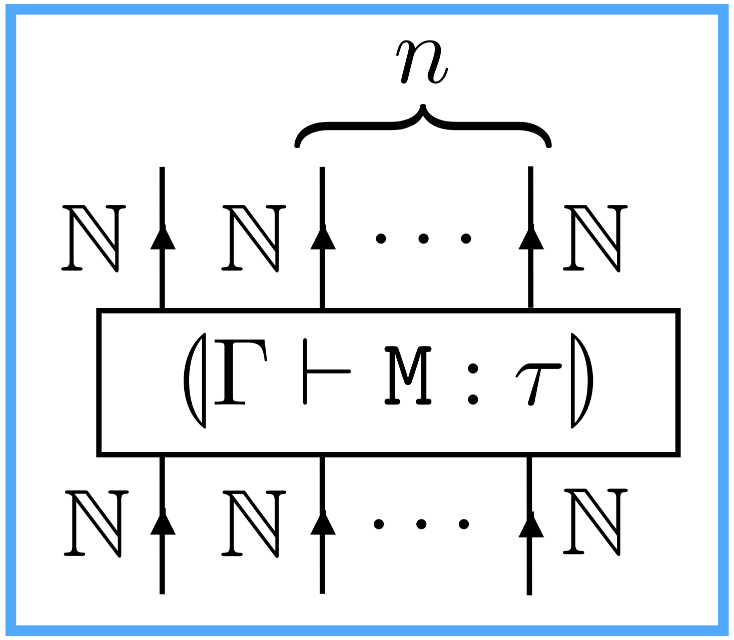
Memoryful GoI — Translation

Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

For a type judgement $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$ $(\Gamma = \mathtt{x}_1 : \tau_1, \ldots, \mathtt{x}_n : \tau_n)$

we inductively define

$(\!|\Gamma \vdash \mathtt{M} : \tau|\!) =$



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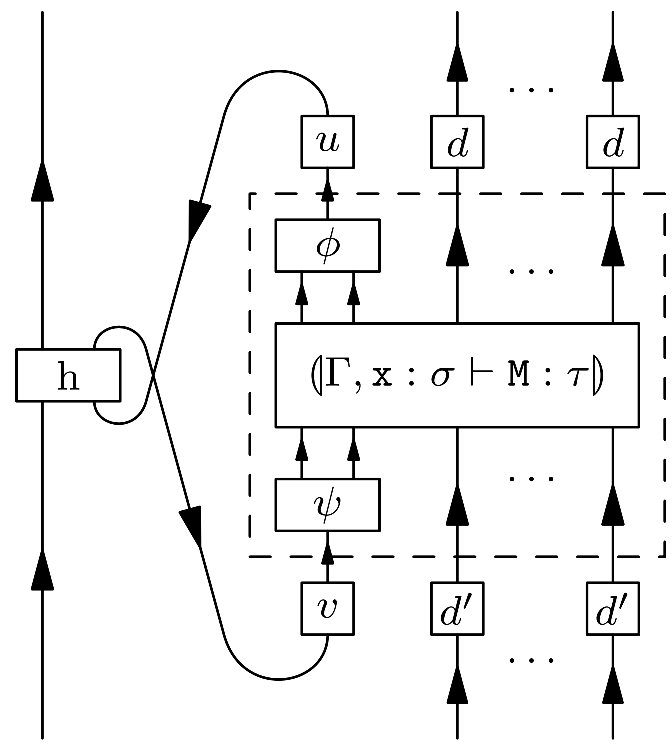
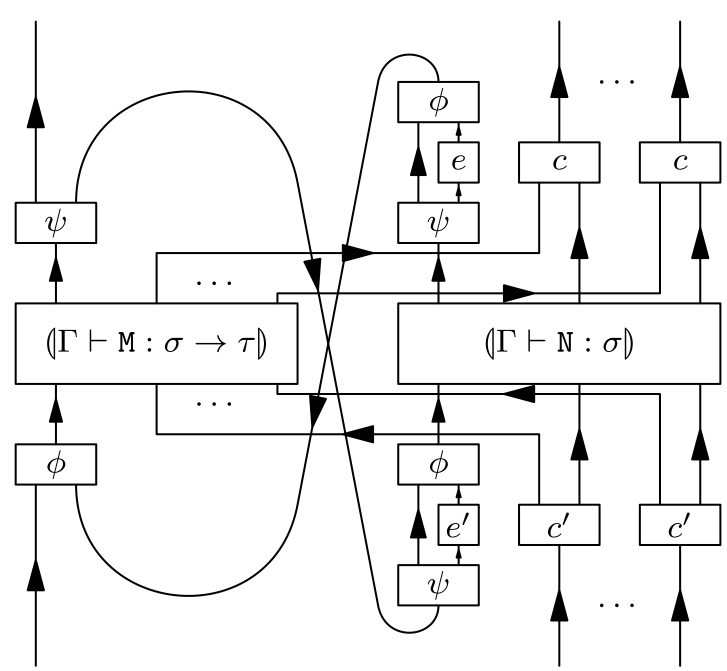
Memoryful GoI — Translation

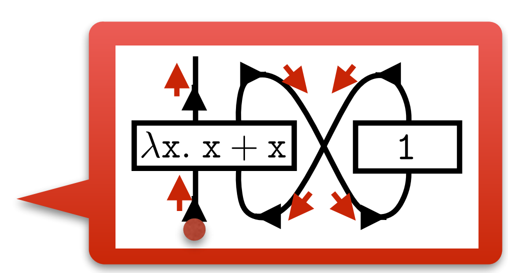
Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

$(\!|\Gamma \vdash \lambda\mathtt{x} : \sigma.\,\mathtt{M} : \sigma \to \tau|\!) =$



$(\!|\Gamma \vdash \mathtt{M}\ \mathtt{N} : \tau|\!) =$

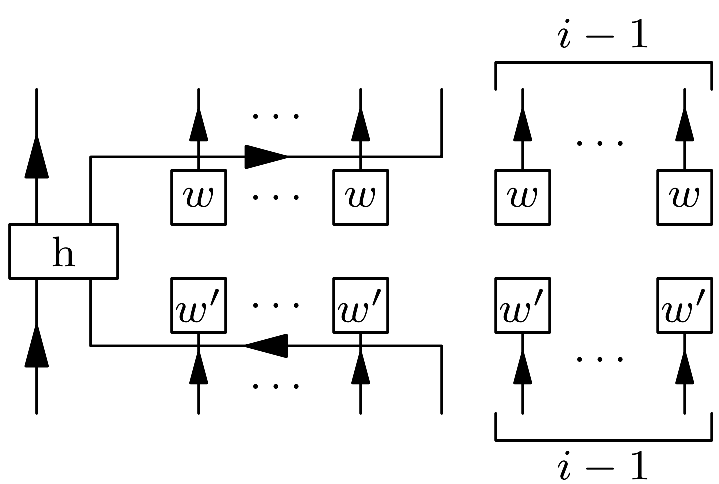




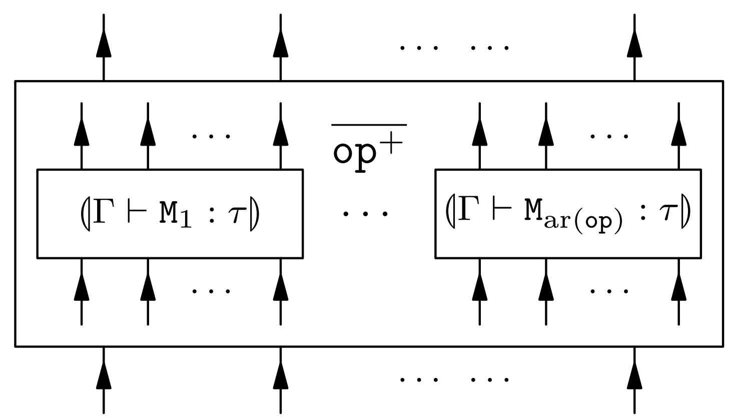
Memoryful GoI — Translation

Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

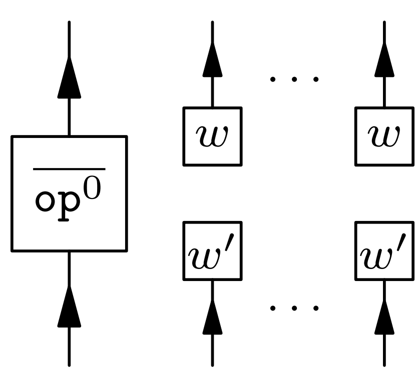
$(\!|\Gamma \vdash \mathtt{x}_i : \tau_i|\!) =$



$(\!|\Gamma \vdash \mathtt{op}^+(\mathtt{M}_1, \ldots, \mathtt{M}_{\mathrm{ar}(\mathtt{op})}) : \tau|\!) =$



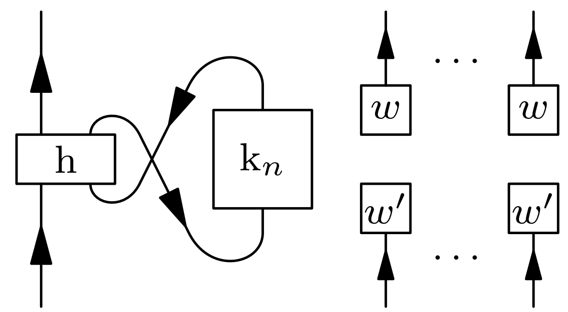
$(\!|\Gamma \vdash \mathtt{op}^0() : \tau|\!) =$



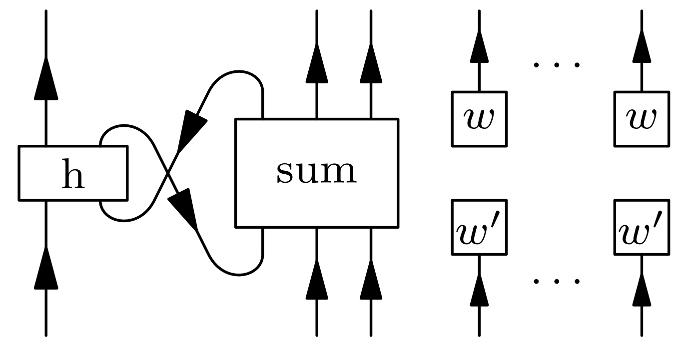
Memoryful GoI — Translation

Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

$$(\!|\Gamma \vdash \underline{\mathtt{n}} : \mathtt{nat}|\!) =$$

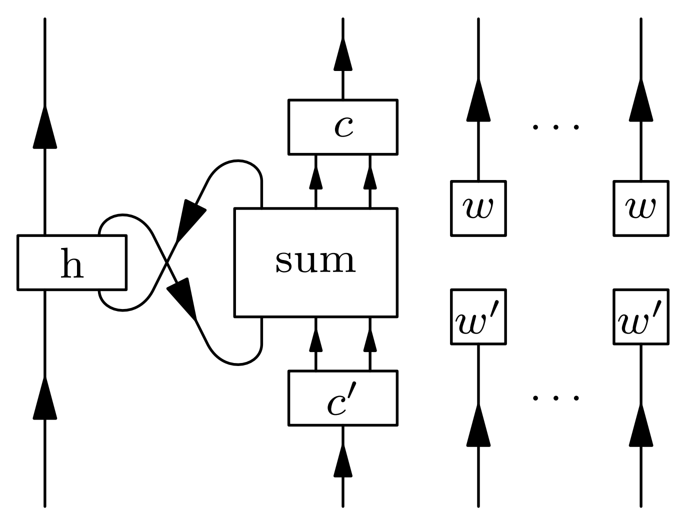


$$(\!|\Gamma, \mathtt{x} : \mathtt{nat}, \mathtt{y} : \mathtt{nat} \vdash \mathtt{x} + \mathtt{y} : \mathtt{nat}|\!) =$$



(if $\mathtt{x} \not\equiv \mathtt{y}$)

$$(\!|\Gamma, \mathtt{x} : \mathtt{nat} \vdash \mathtt{x} + \mathtt{x} : \mathtt{nat}|\!) =$$



Memoryful GoI — Translation

Theorem III.3 (soundness of $(\!|-|\!)$). *For closed terms M and N of the base type* $\mathtt{nat}$, $\vdash M = N : \mathtt{nat}$ *implies* $(\!|\vdash M : \mathtt{nat}|\!) \simeq (\!|\vdash N : \mathtt{nat}|\!)$.

- (almost full fragment of) Moggi's equations for computational lambda-calculus
- equations for algebraic operations

$$M \sqcup M = M$$

$$E[\mathrm{opr}(M_1, \ldots, M_n)] = \mathrm{opr}(E[M_1], \ldots, E[M_n])$$

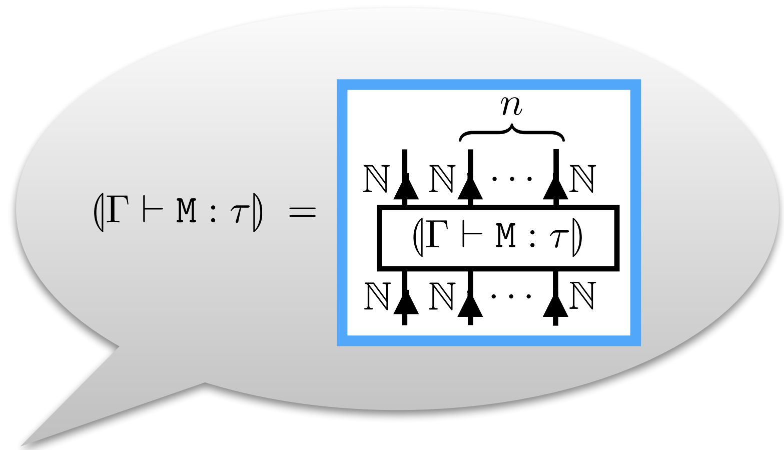
$$(\lambda x.\, M)\,(N_1 \sqcup N_2) = (\lambda x.\, M)\, N_1 \;\sqcup\; (\lambda x.\, M)\, N_2$$

behavioral equivalence

Memoryful GoI [Hoshino, —, Hasuo '14]

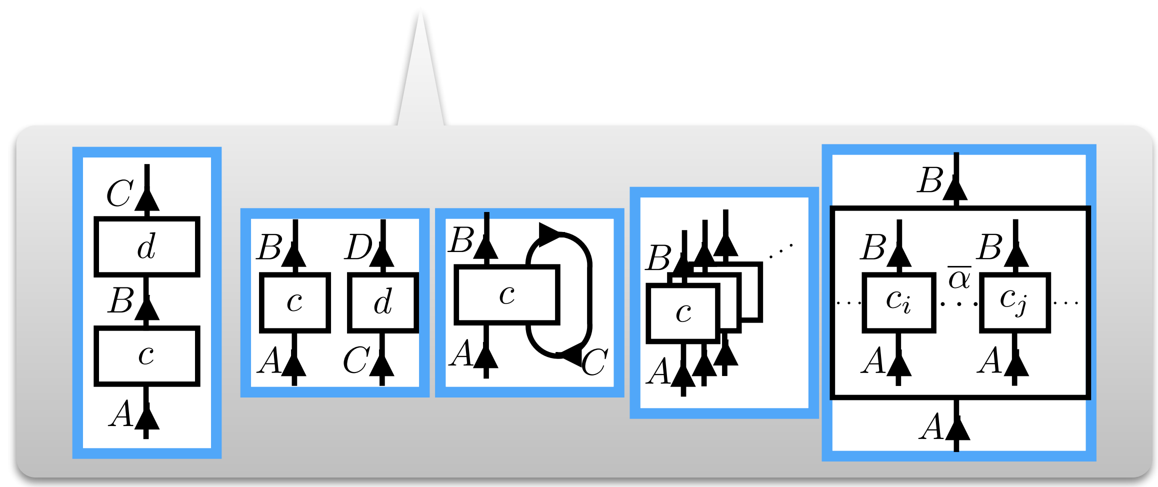
effectful terms

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$

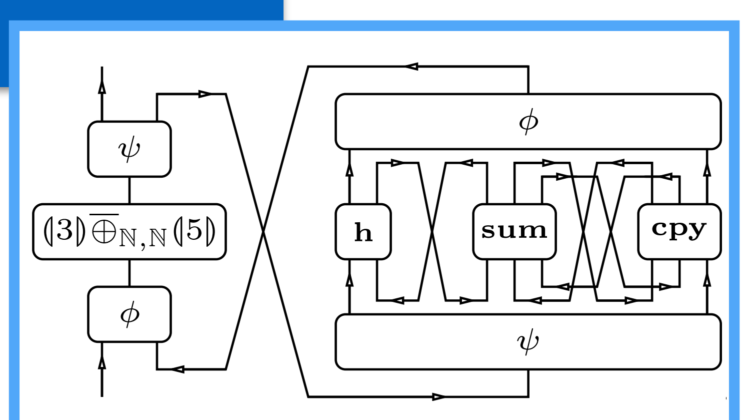


sound translation

- based on Geometry of Interaction
- via coalgebraic component calculus



transducers



Memoryful GoI with recursion

effectful terms

recursion

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\, (3 \sqcup 5) \quad : \mathtt{nat}$

translation

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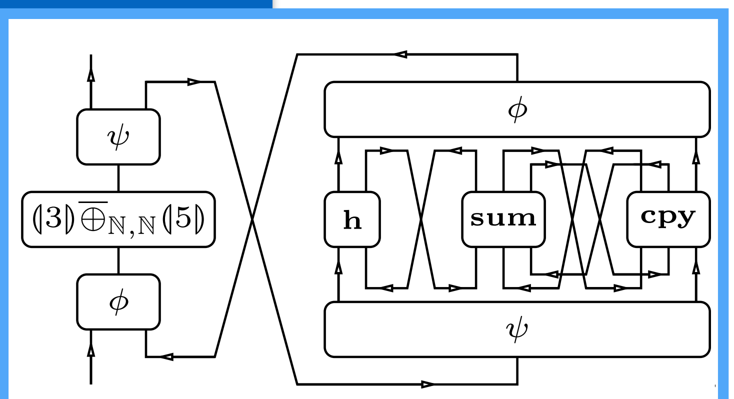

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Memoryful GoI with recursion

effectful terms

recursion

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$

translation

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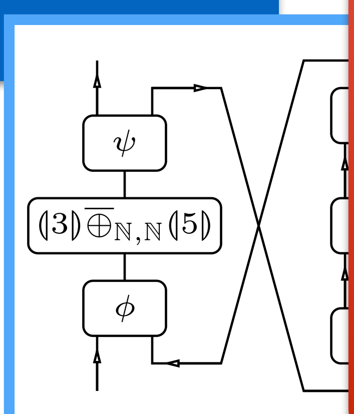


Memoryful GoI with recursion

effectful terms

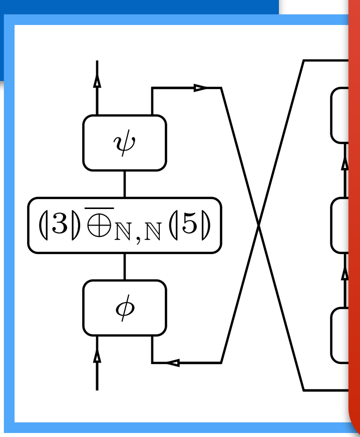
recursion

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\, (3 \sqcup 5) \quad : \mathtt{nat}$

translation
- based on Geometry of Interaction
- via coalgebraic component calculus

$$\mathrm{fix}(F) = F(F(F(\cdots)))$$

transducers



Memoryful GoI with recursion

effectful terms

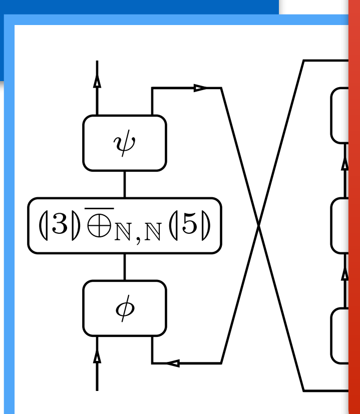
recursion

$(\lambda x : \mathtt{nat}.\, x + x)\,(3 \sqcup 5) \quad : \mathtt{nat}$

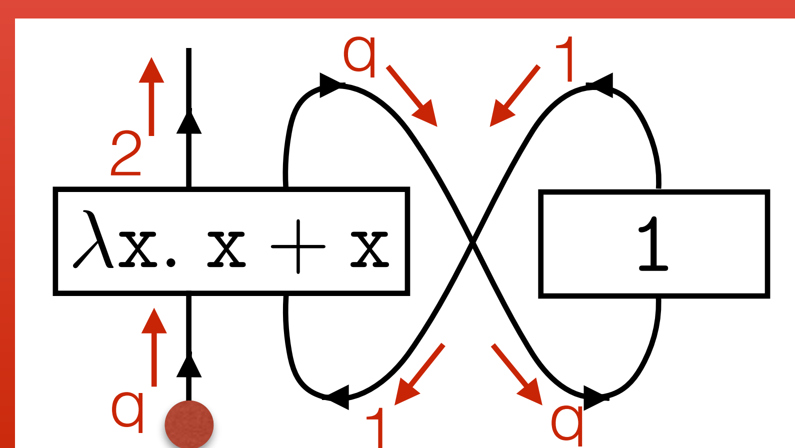
translation

- based on Geometry of Interaction
- via coalgebraic component calculus

transducers



$$\mathrm{fix}(F) = F(F(F(\cdots)))$$



Memoryful GoI with recursion

effectful terms

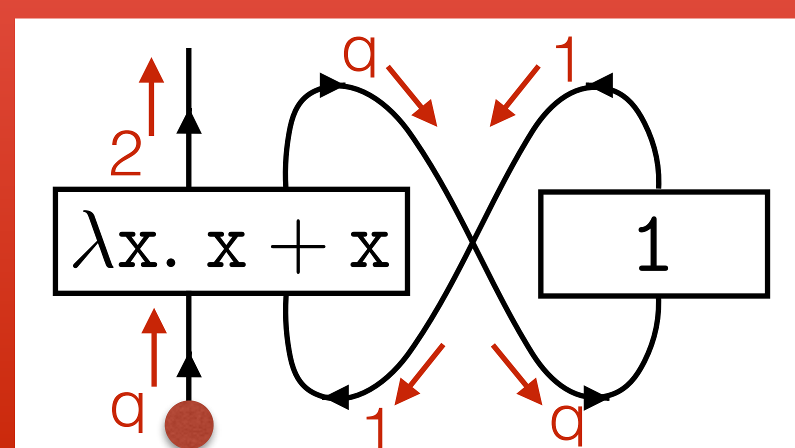
recursion

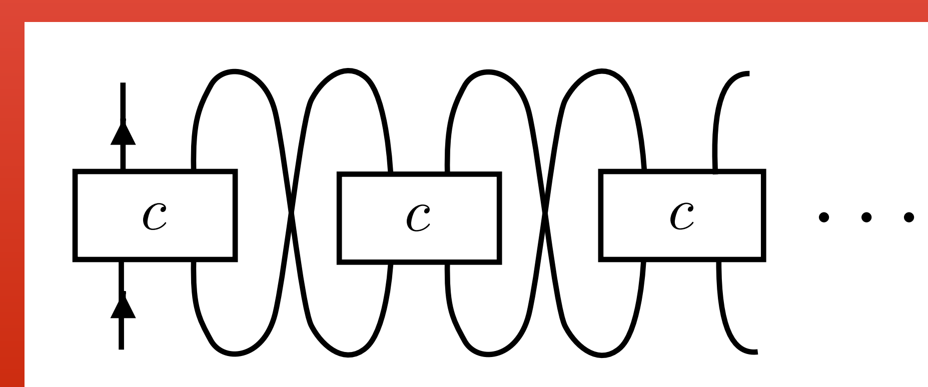
$(\lambda x : \mathtt{nat}.\, x + x)\,(3 \sqcup 5) \quad : \mathtt{nat}$

translation

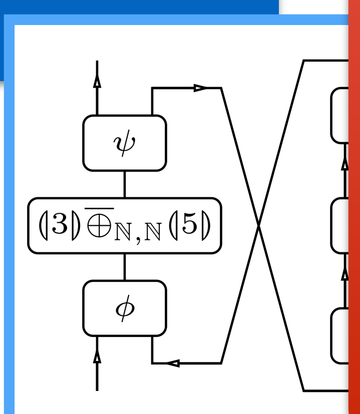
- based on Geometry of Interaction
- via coalgebraic component calculus

$$\mathrm{fix}(F) = F(F(F(\cdots)))$$





transducers



Memoryful GoI with recursion

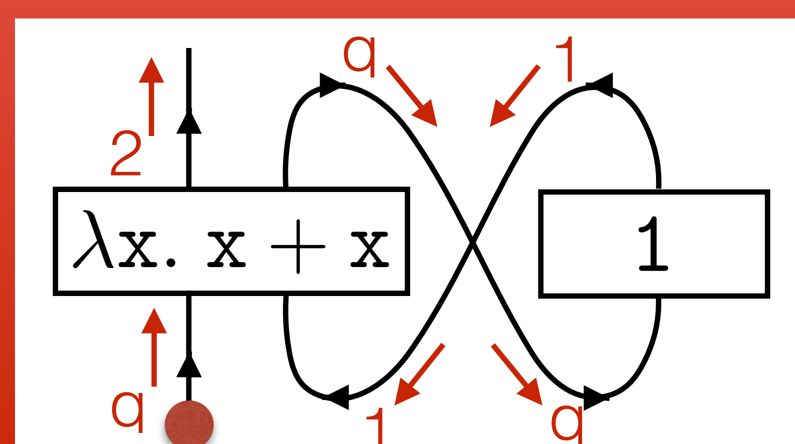
effectful terms

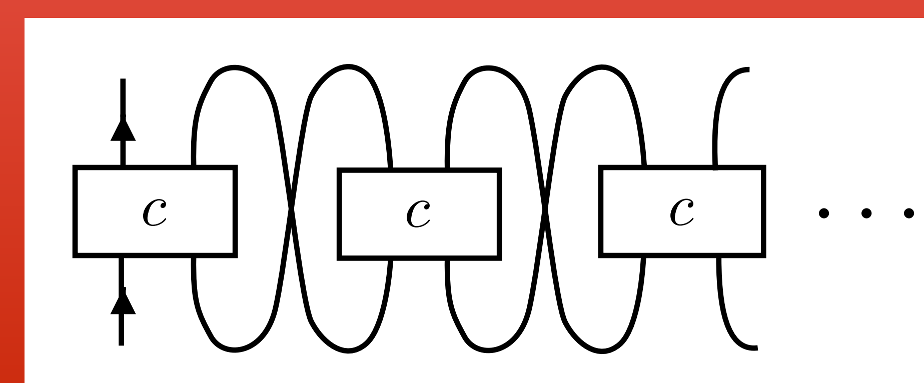
recursion

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$

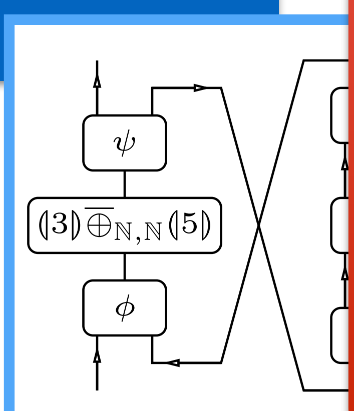
translation
- based on Geometry of Interaction
- via **extended** coalgebraic component calculus

$$\mathrm{fix}(F) = F(F(F(\cdots)))$$



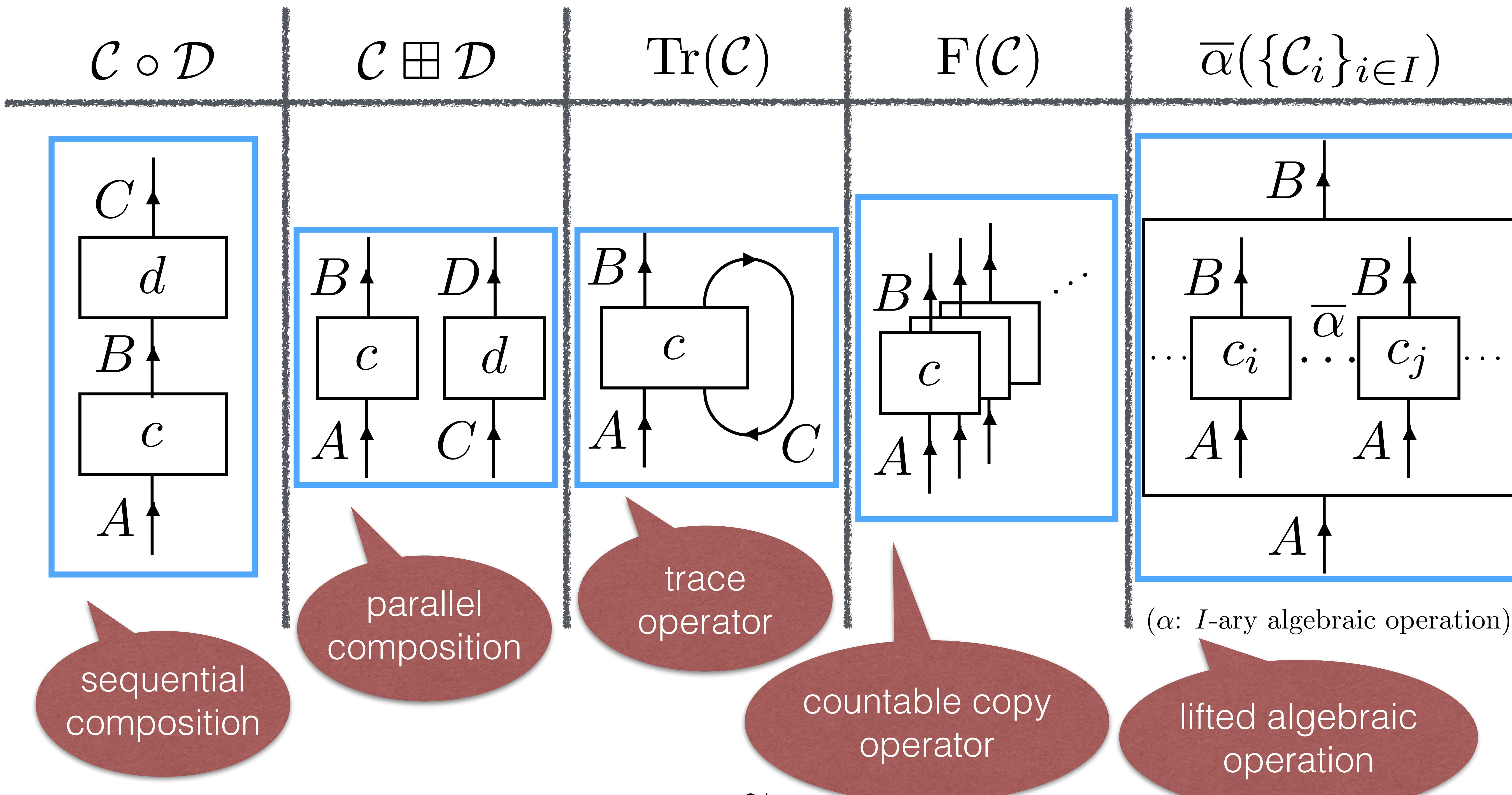


transducers



Memoryful GoI with recursion

<u>Def. (component calculus over transducers)</u>

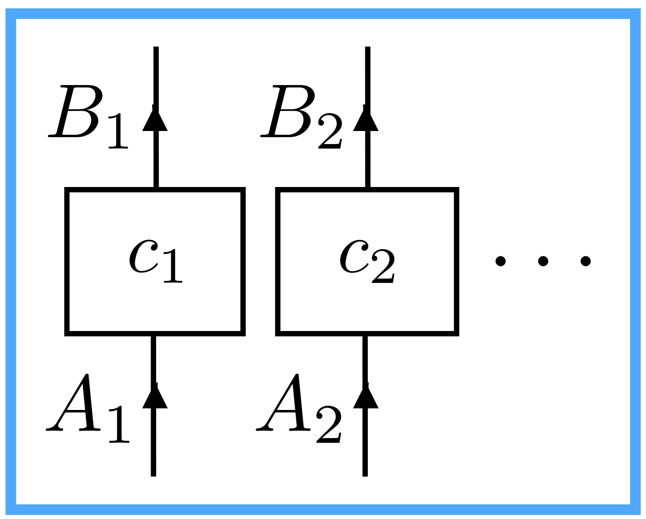
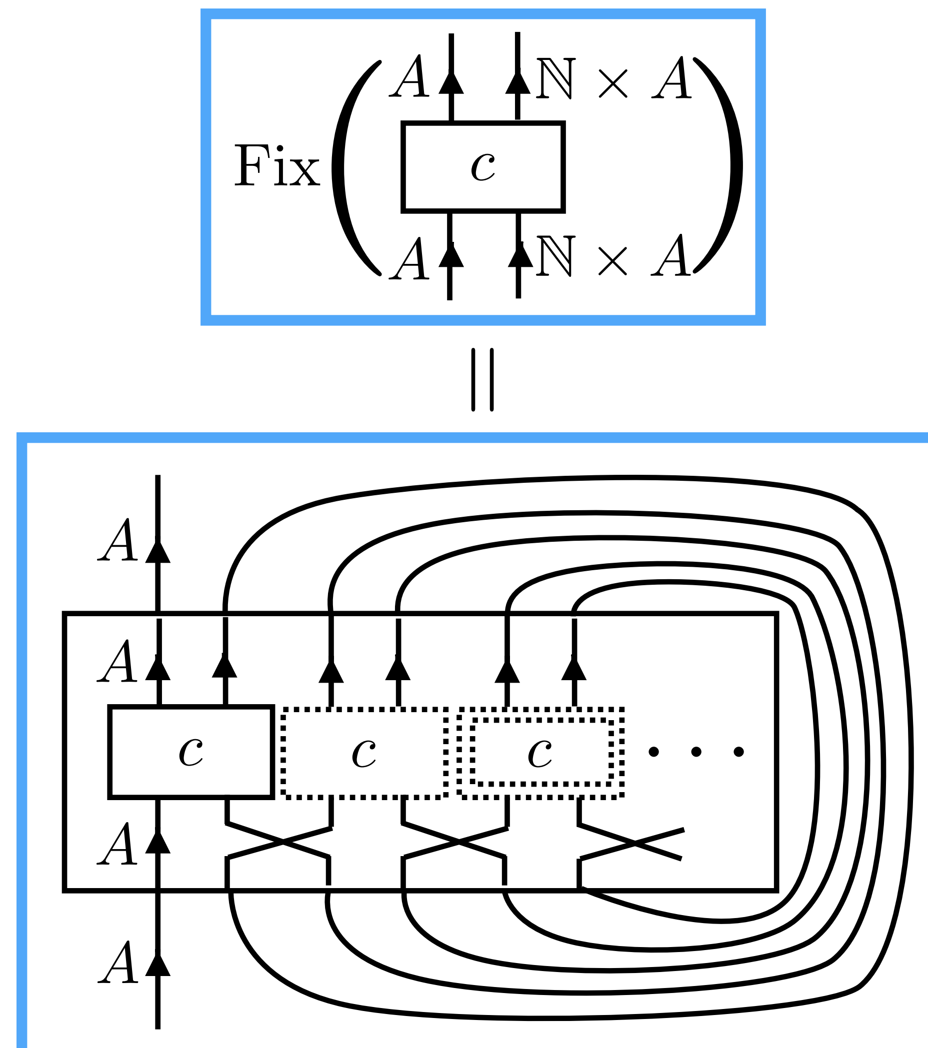


Memoryful GoI with recursion

Def. (**extended** component calculus)

countable parallel composition

"fixed point" operator

$\boxplus_{i\in I} \mathcal{C}_i$	$\mathrm{Fix}(\mathcal{C})$
	

Memoryful GoI with recursion

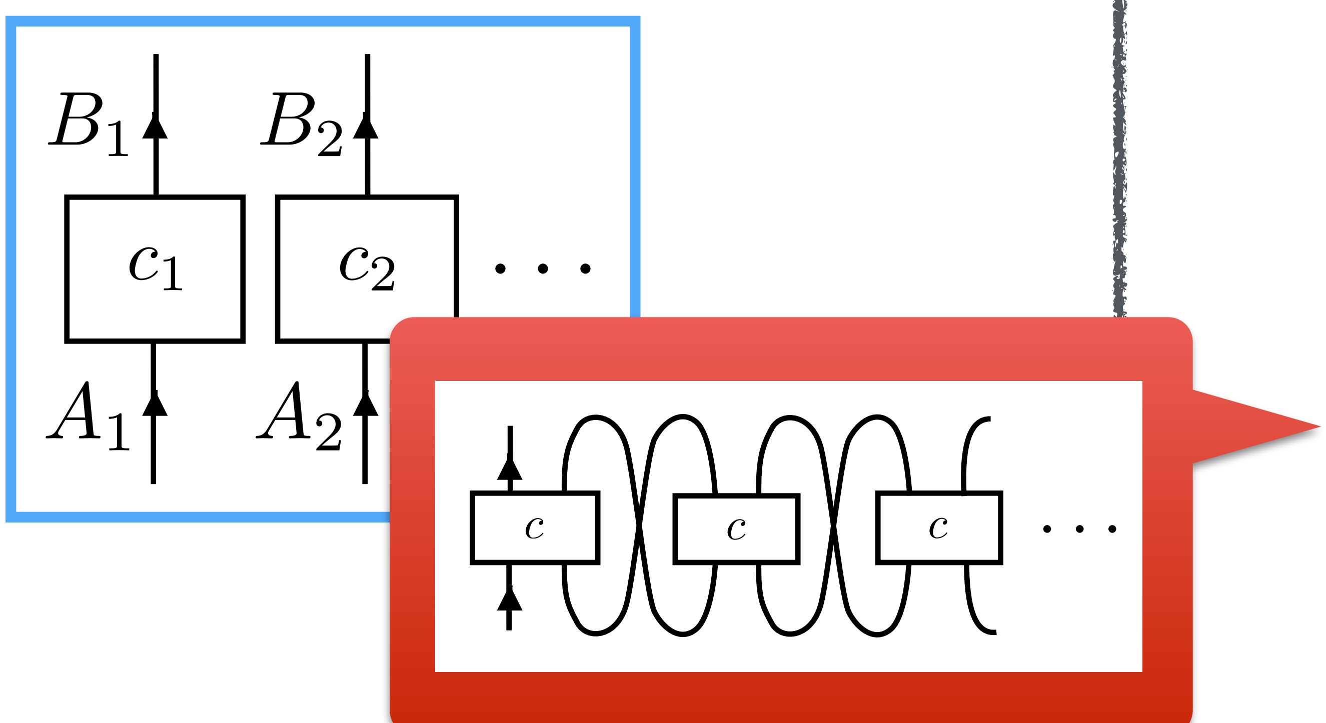
<u>Def. (**extended** component calculus)</u>

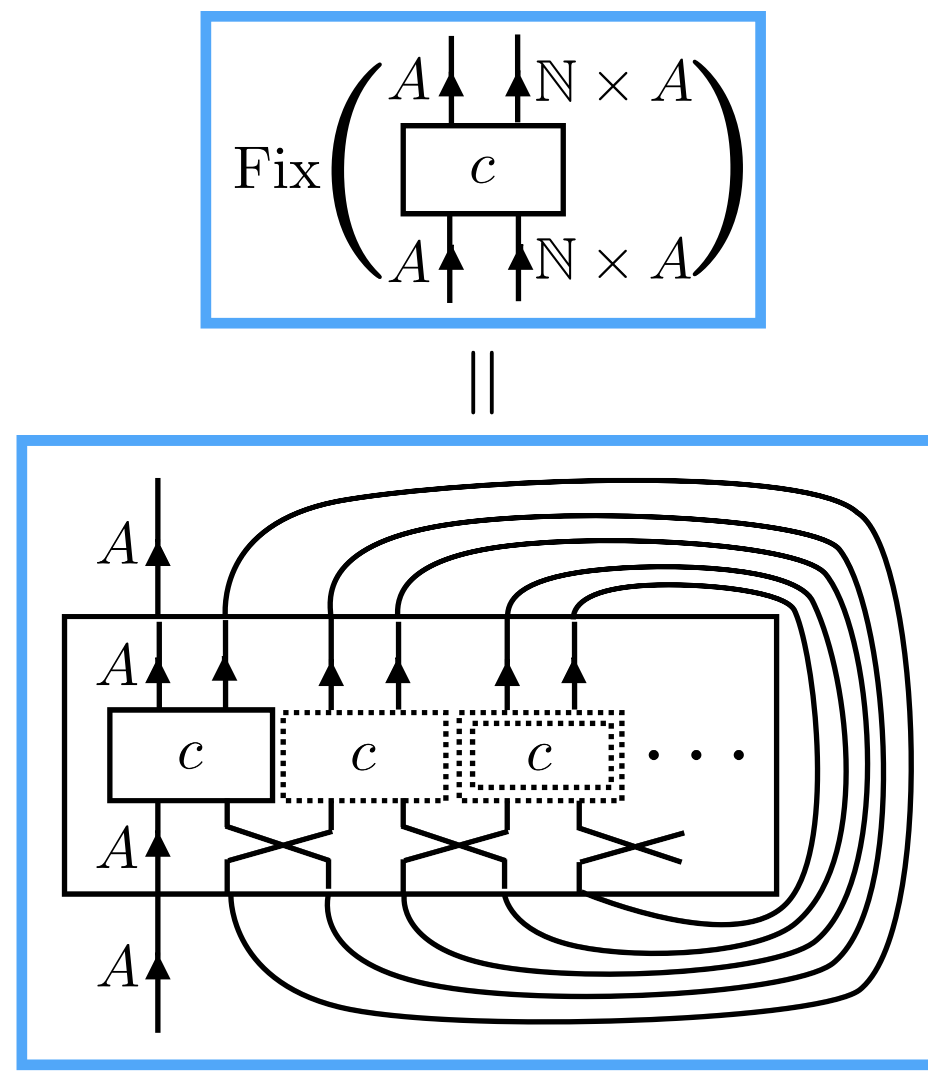
$\boxplus_{i \in I} \mathcal{C}_i$

countable parallel composition

$\mathrm{Fix}(\mathcal{C})$

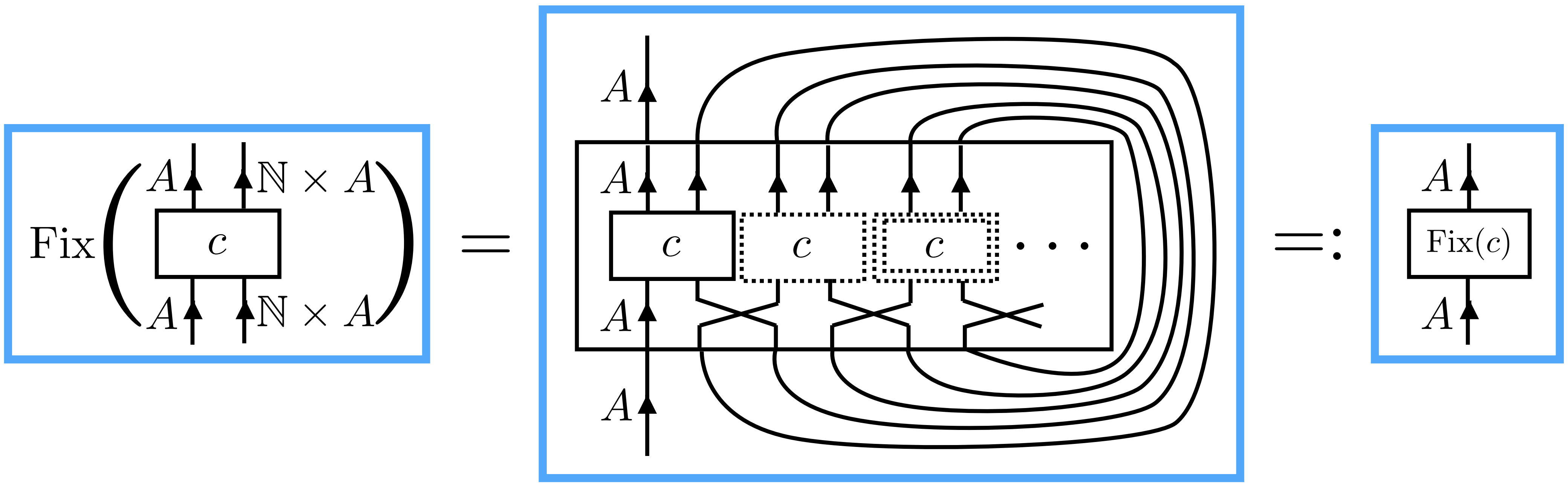
"fixed point" operator



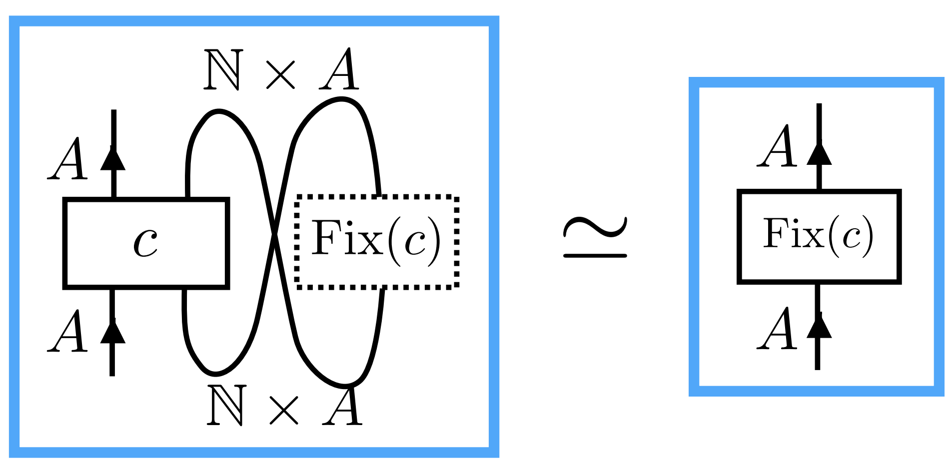


“Fixed point” operator

Lem. (Fix as a fixed point operator)



satisfies



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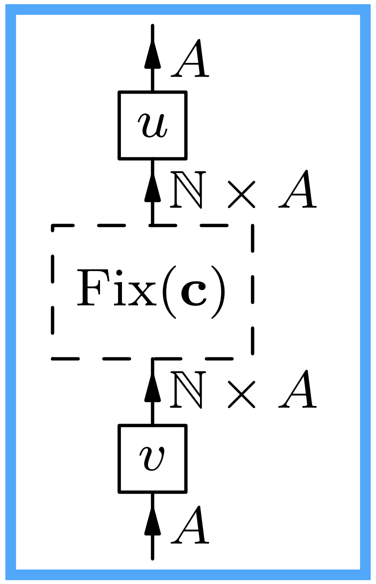
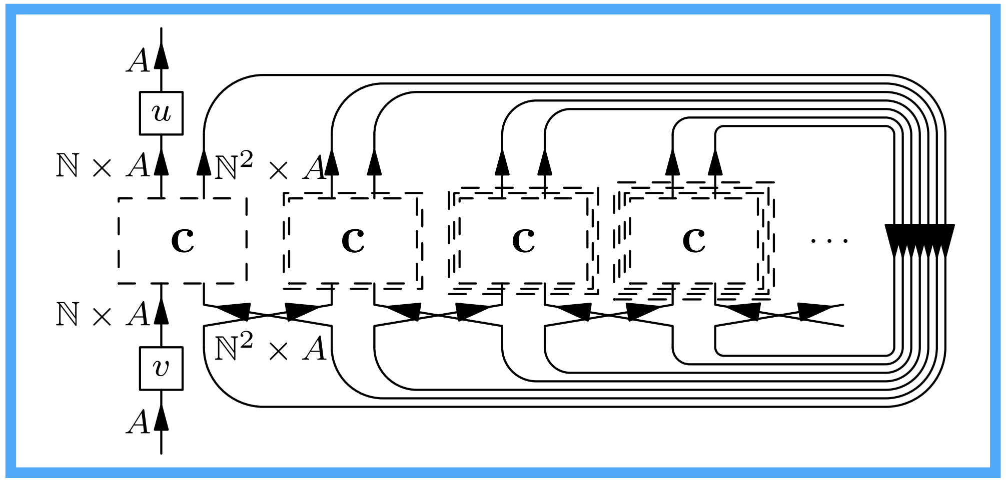
Muroya (U. Tokyo)

“Fixed point” operator

Lem. (two styles of “implementation”)

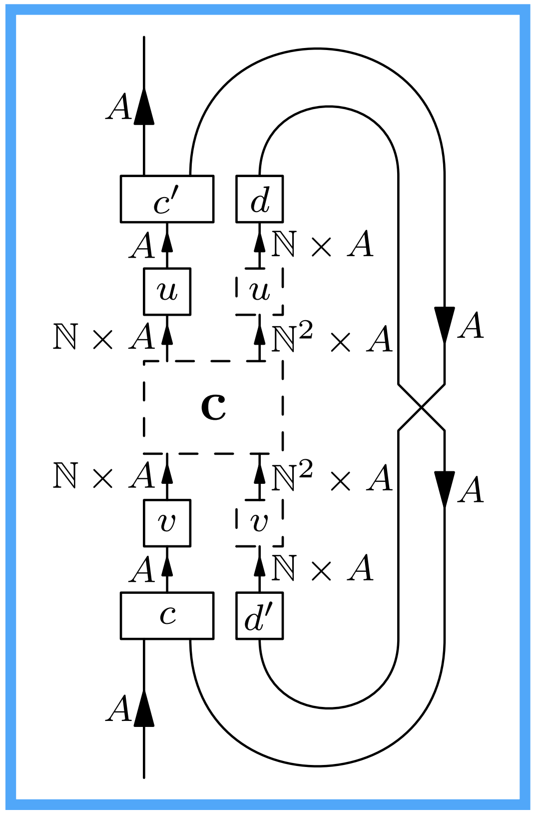


≃



Girard style

≃



Mackie style

“Fixed point” operator

Lem. (domain-theoretic characterization of Fix)

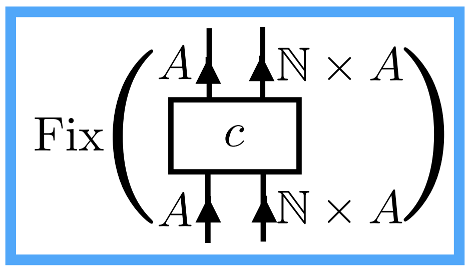
Under the assumption that

- $\mathbf{Set}_T$ is a $\mathbf{Cppo}$-enriched category with $\mathbf{Cppo}$-enriched (countable) cotuplings
- compositions $\circ_T$ of $\mathbf{Set}_T$ is strict in the restricted form: $f \circ_T \bot = \bot$ and $\bot \circ_T (\eta_Y \circ g) = \bot$ hold for any $f\colon X \to TY$ and $g\colon X \to Y$ in $\mathbf{Set}$
- premonoidal structures $X \otimes -, - \otimes X$ of $\mathbf{Set}_T$ is locally continuous and strict for any X in $\mathbf{Set}$

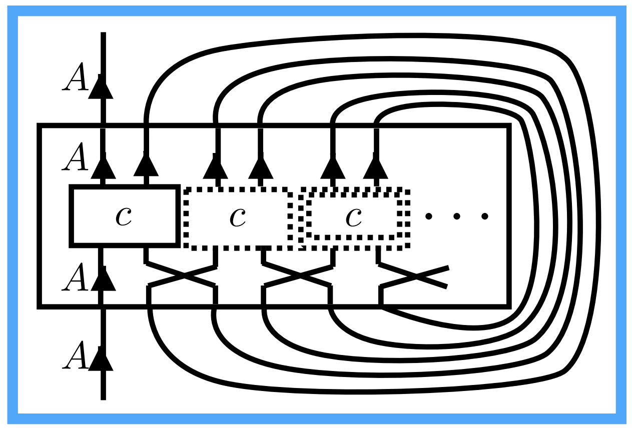
it holds that:

“Fixed point” operator

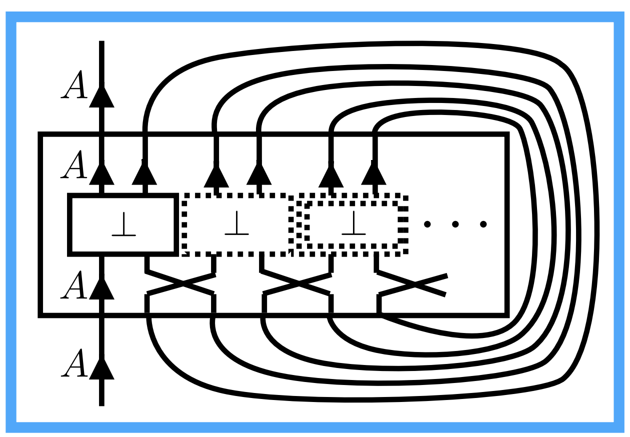
<u>Lem. (domain-theoretic characterization of Fix)</u>



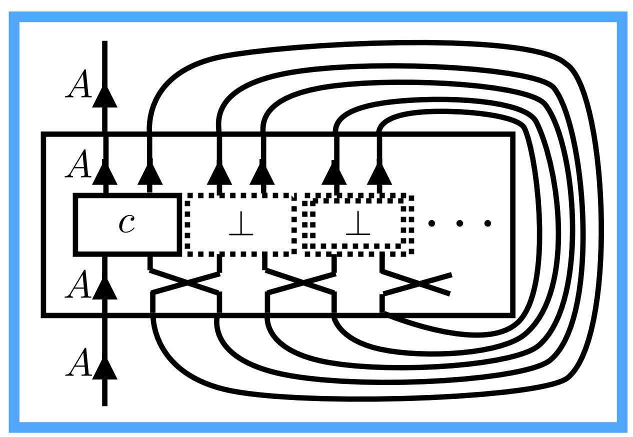
$=$



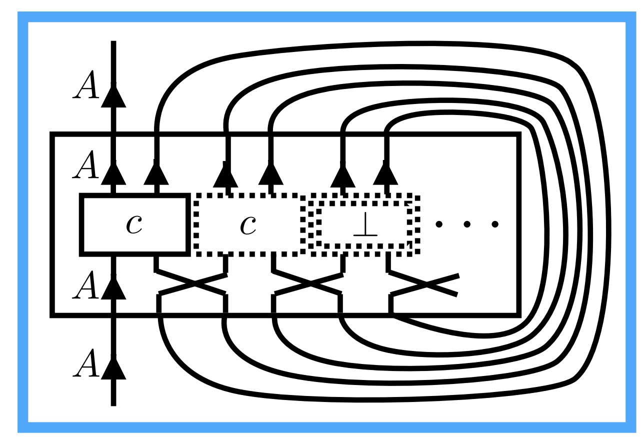
is a supremum of an ω-chain



$\trianglelefteq$



$\trianglelefteq$



$\trianglelefteq \quad \cdots$

where $\quad (X,\ c\colon X \times A \to T(X \times B),\ x_0 \in X) \trianglelefteq (Y,\ c\colon Y \times A \to T(Y \times B),\ y_0 \in Y)$

$\stackrel{\text{def.}}{\iff} X = Y \ \wedge\ x = y \ \wedge\ c \sqsubseteq d \ \text{in}\ \mathbf{Set}_T(X \times A, X \times B)$

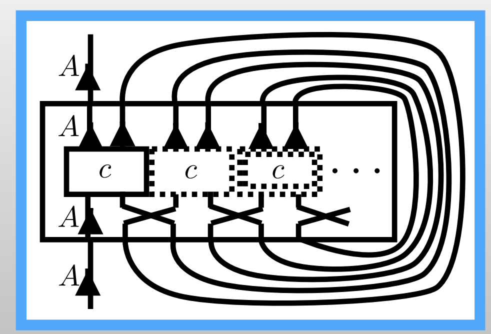
Memoryful GoI with recursion

effectful terms

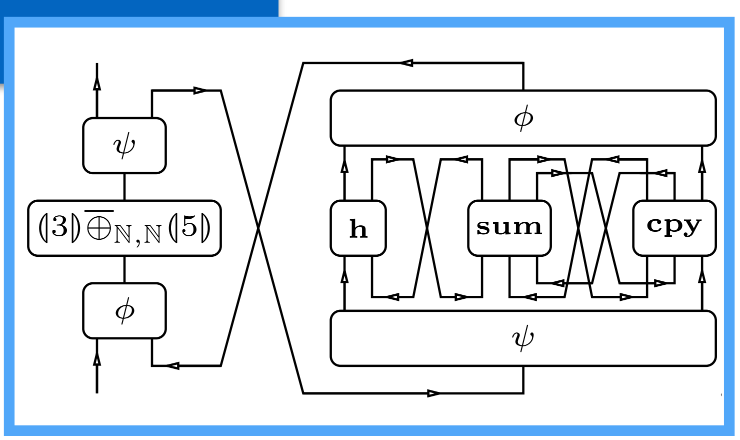
recursion

$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$

translation
- based on Geometry of Interaction
- via **extended** coalgebraic component calculus



transducers



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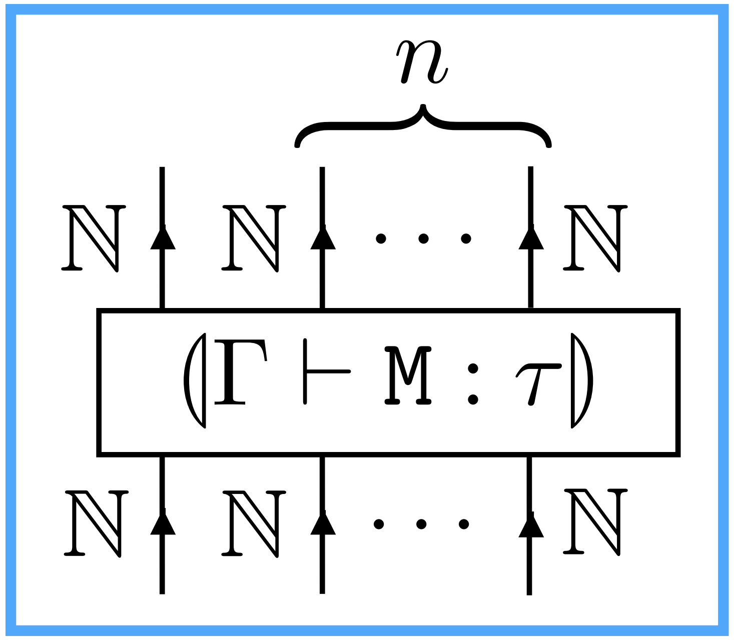
Memoryful GoI — Translation

Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

For a type judgement $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$ $(\Gamma = \mathtt{x}_1 : \tau_1, \ldots, \mathtt{x}_n : \tau_n)$

we inductively define

$(\!|\Gamma \vdash \mathtt{M} : \tau|\!) =$ 

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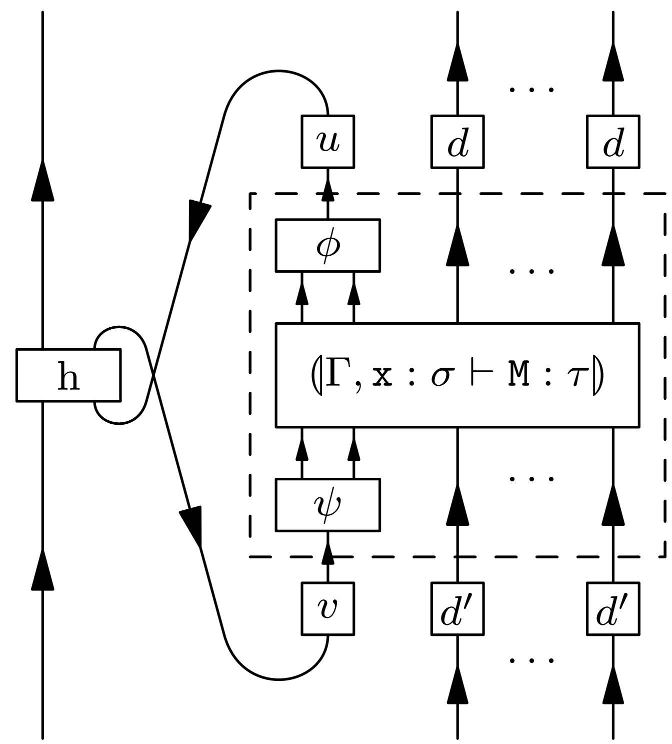
Muroya (U. Tokyo)

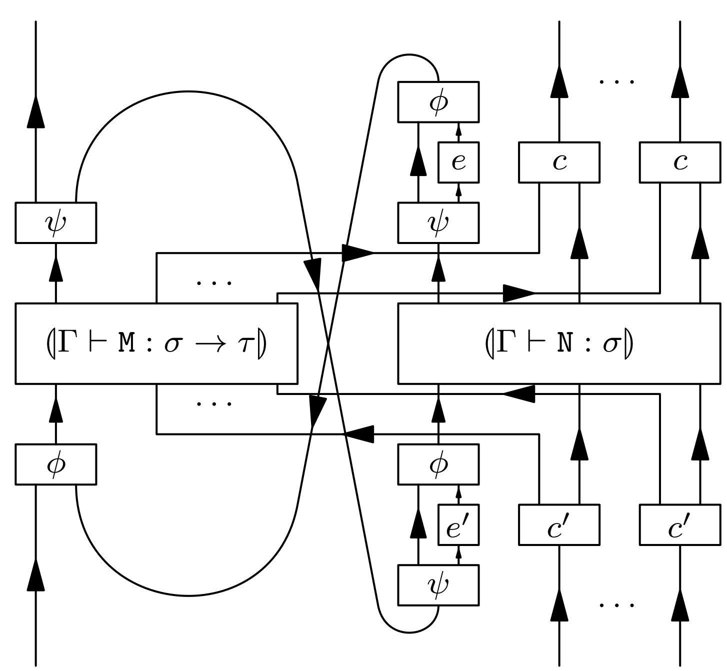
Memoryful GoI — Translation

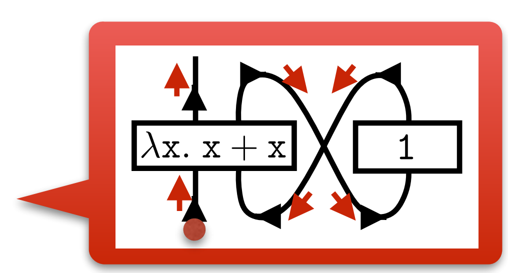
Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

$(\!|\Gamma \vdash \lambda\mathtt{x} : \sigma.\,\mathtt{M} : \sigma \to \tau|\!) =$



$(\!|\Gamma \vdash \mathtt{M}\ \mathtt{N} : \tau|\!) =$

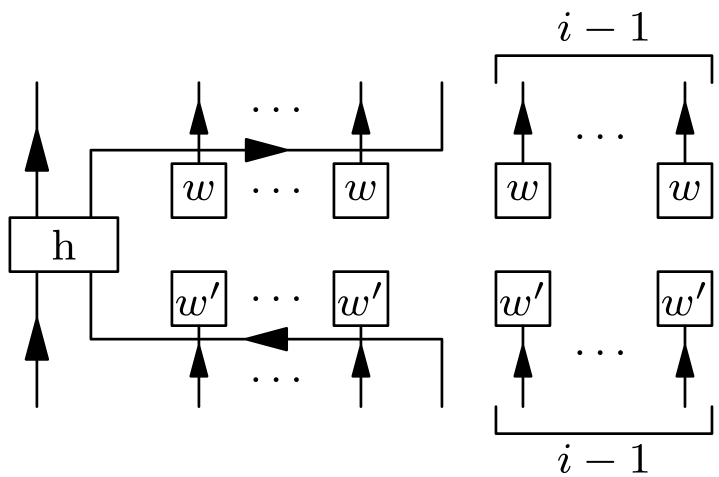


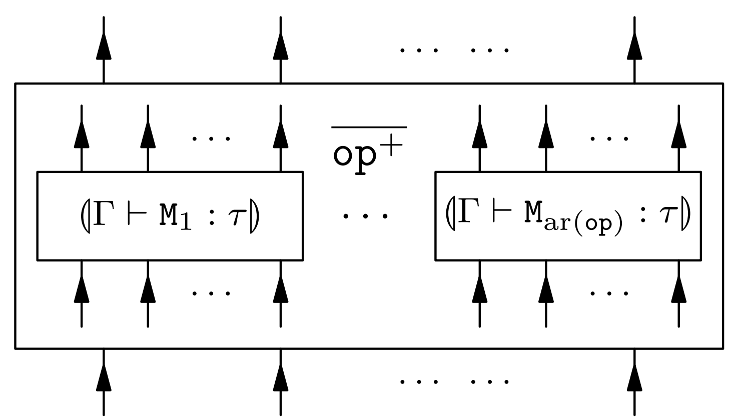


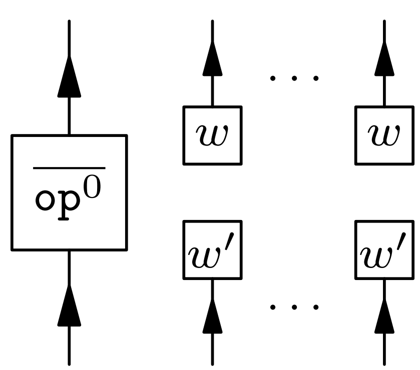
Muroya (U. Tokyo)

Memoryful GoI — Translation

<u>Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)</u>

$(\!|\Gamma \vdash \mathtt{x}_i : \tau_i|\!) =$ 

$(\!|\Gamma \vdash \mathtt{op}^{+}(\mathtt{M}_1, \ldots, \mathtt{M}_{\mathrm{ar}(\mathtt{op})}) : \tau|\!) =$ 

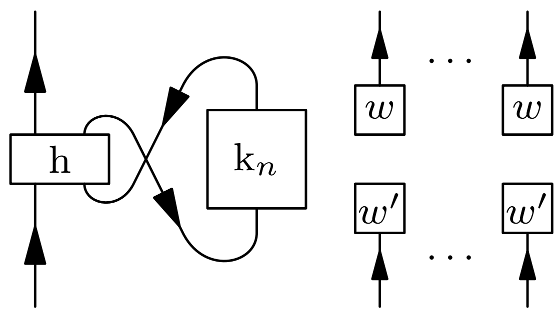
$(\!|\Gamma \vdash \mathtt{op}^{0}() : \tau|\!) =$ 

Muroya (U. Tokyo)

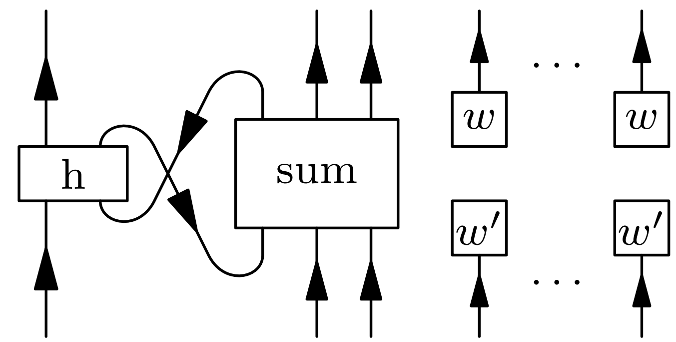
Memoryful GoI — Translation

Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

$(\!|\Gamma \vdash \underline{\mathtt{n}} : \mathtt{nat}|\!) =$

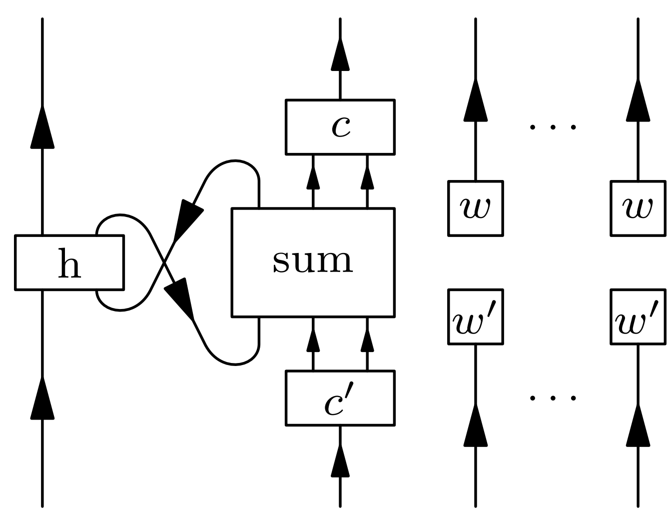


$(\!|\Gamma, \mathtt{x} : \mathtt{nat}, \mathtt{y} : \mathtt{nat} \vdash \mathtt{x} + \mathtt{y} : \mathtt{nat}|\!) =$



(if $\mathtt{x} \not\equiv \mathtt{y}$)

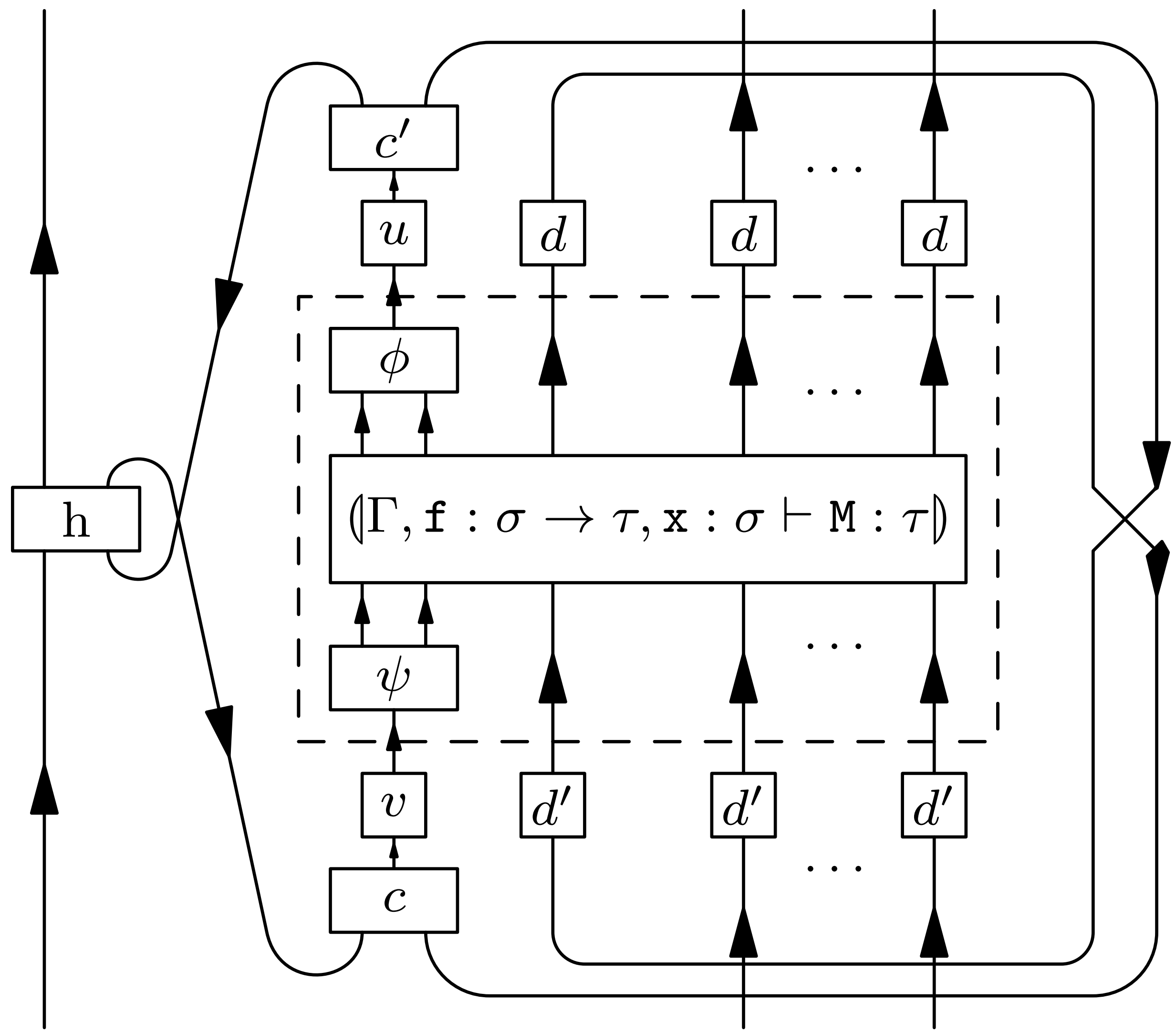
$(\!|\Gamma, \mathtt{x} : \mathtt{nat} \vdash \mathtt{x} + \mathtt{x} : \mathtt{nat}|\!) =$



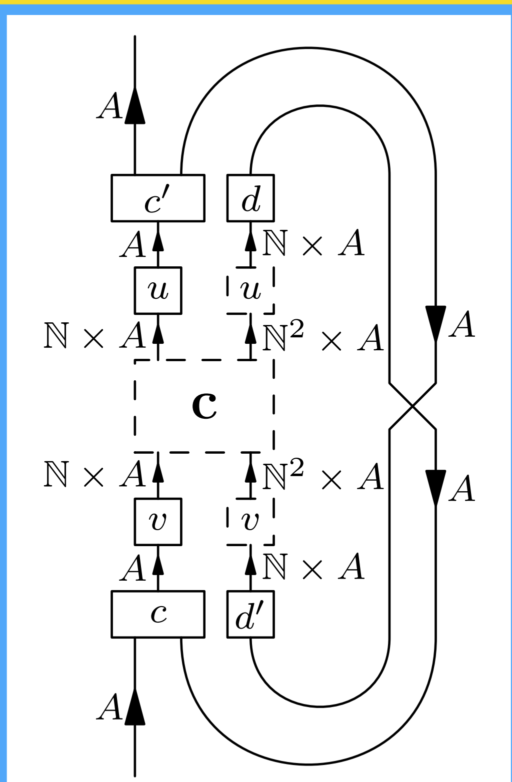
Memoryful GoI with recursion

Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

$$(\!|\Gamma \vdash \mathtt{rec}(\mathtt{f} : \sigma \to \tau, \mathtt{x} : \sigma).\mathtt{M} : \sigma \to \tau|\!) =$$



Mackie style



Memoryful GoI — Translation

Theorem III.3 (soundness of $(\!|-|\!)$). *For closed terms M and N of the base type* $\mathtt{nat}$, $\vdash M = N : \mathtt{nat}$ *implies* $(\!|\vdash M : \mathtt{nat}|\!) \simeq (\!|\vdash N : \mathtt{nat}|\!)$.

behavioral equivalence

- (almost full fragment of) Moggi's equations for computational lambda-calculus
- equations for algebraic operations

$$M \sqcup M = M$$

$$E[\mathrm{opr}(M_1, \dots, M_n)] = \mathrm{opr}(E[M_1], \dots, E[M_n])$$

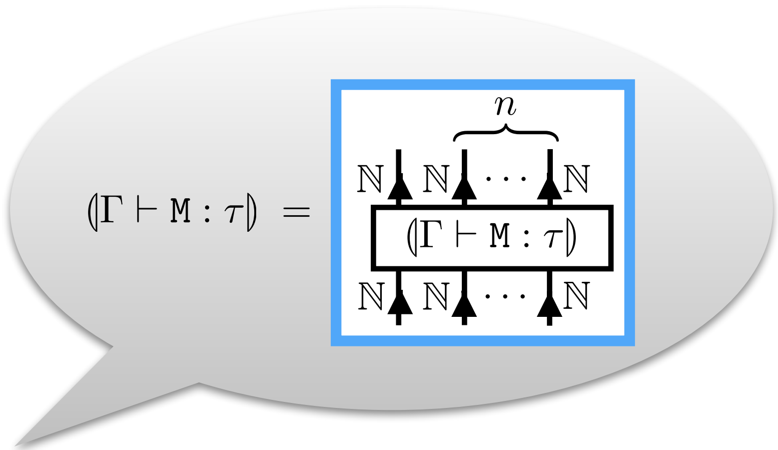
$$(\lambda x.\, M)\,(N_1 \sqcup N_2) = (\lambda x.\, M)\,N_1 \;\sqcup\; (\lambda x.\, M)\,N_2$$

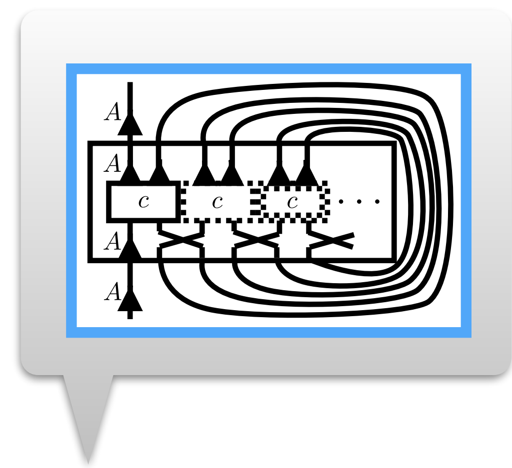
Memoryful GoI recursion

effectful terms

recursion

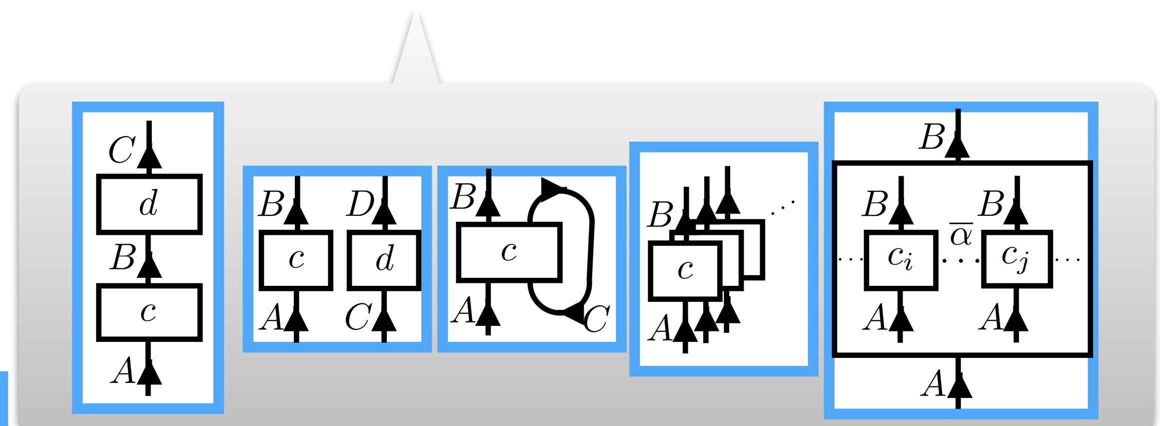
$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$



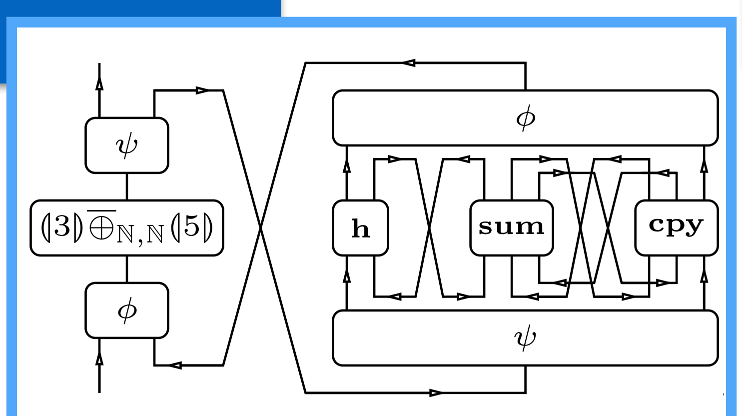


sound translation

- based on Geometry of Interaction
- via **extended** coalgebraic component calculus

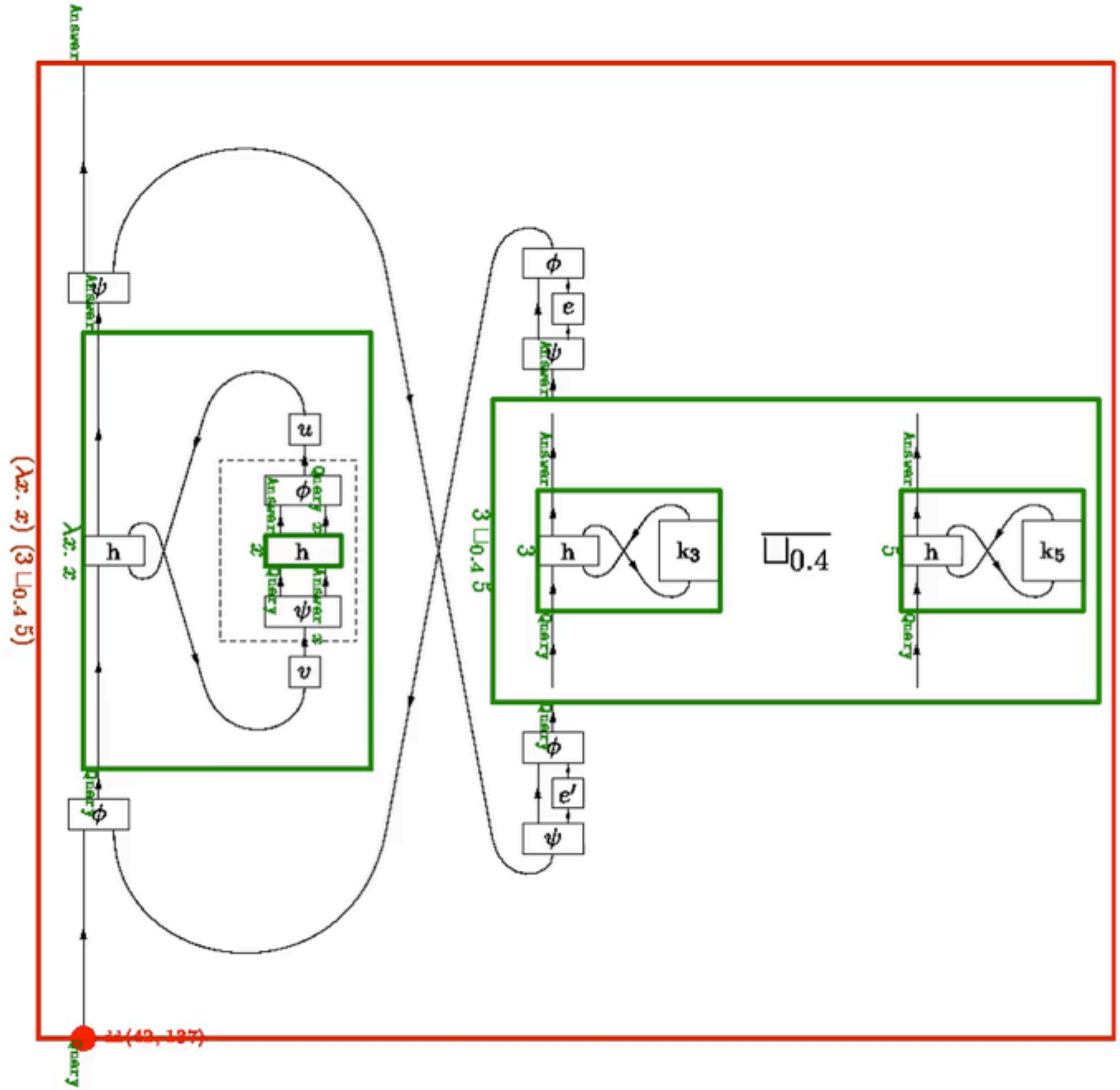


transducers



Examples

$$\llparenthesis(\lambda x.x)\,(3 \sqcup_{0.4} 5)\rrparenthesis =$$



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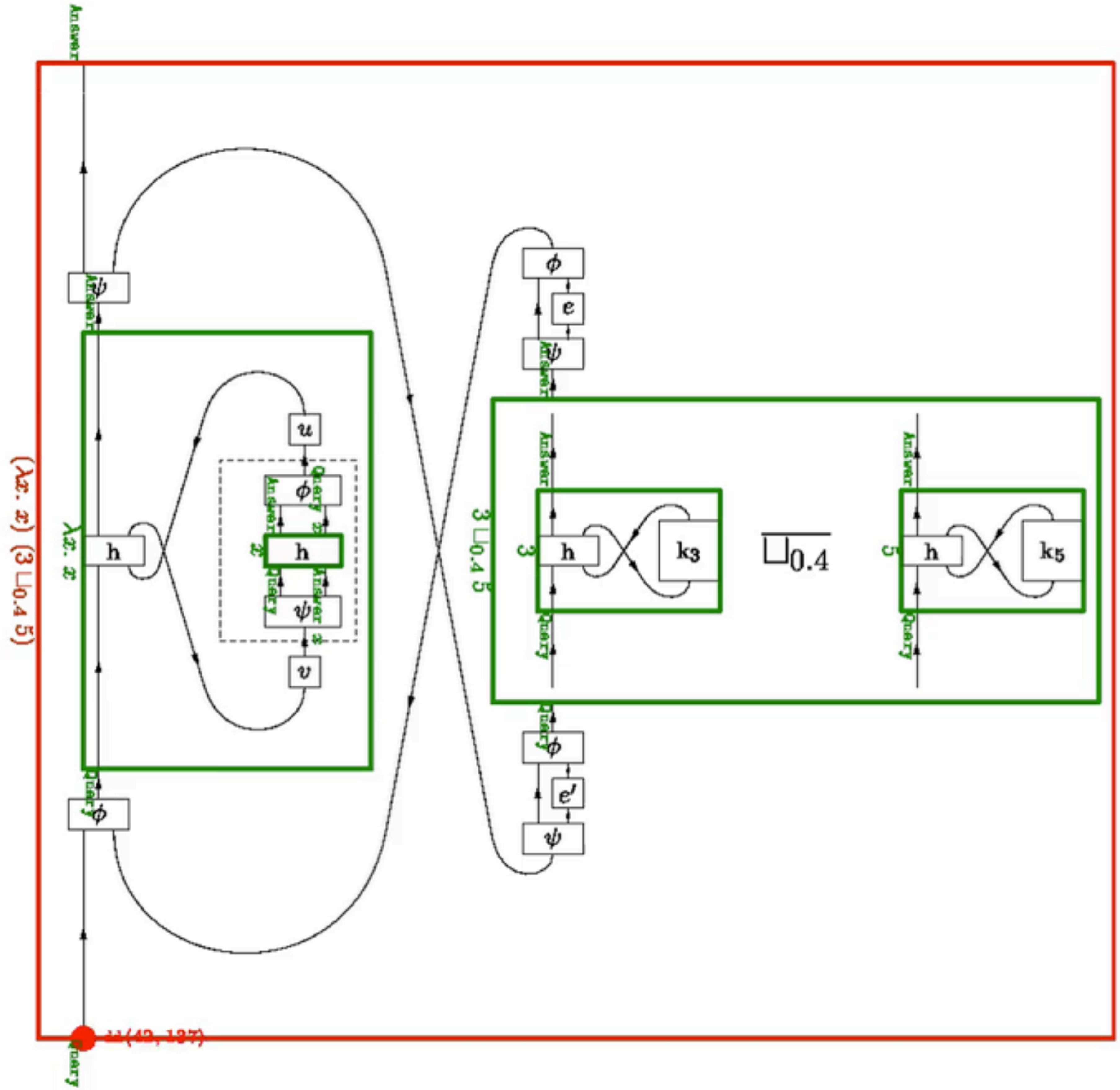
Examples

$(\!|(\lambda x.x)\,(3 \sqcup_{0.4} 5)|\!) =$



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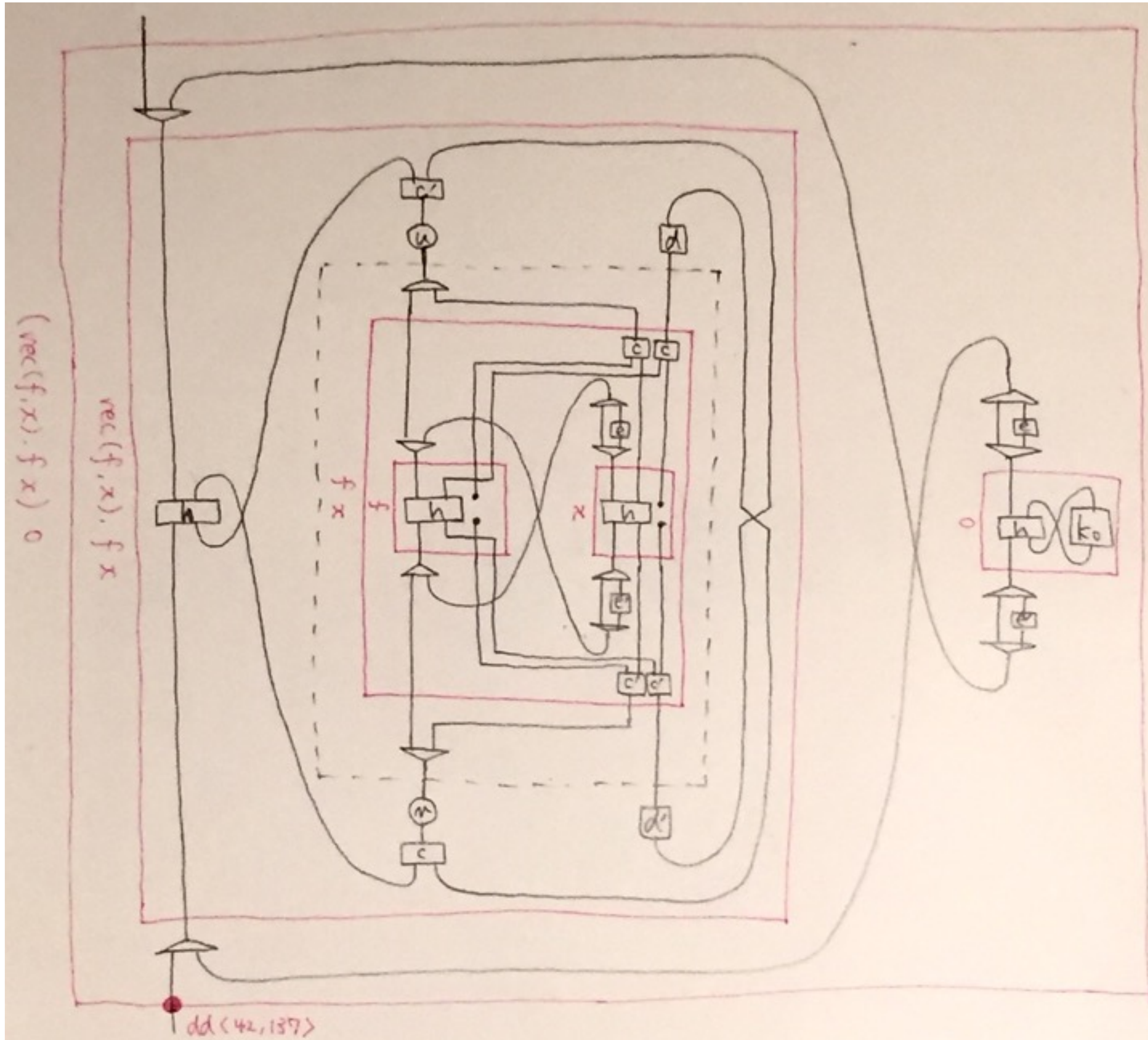
Examples

$(\!|(\texttt{rec}(\texttt{f}, \texttt{x}).\texttt{f x})\ 0|\!) =$



Examples

$\texttt{case}(\texttt{inl}_{1,1}(*) \sqcup \texttt{inr}_{1,1}(*),\ \texttt{y}.1,\ \texttt{z}.f\ x)$

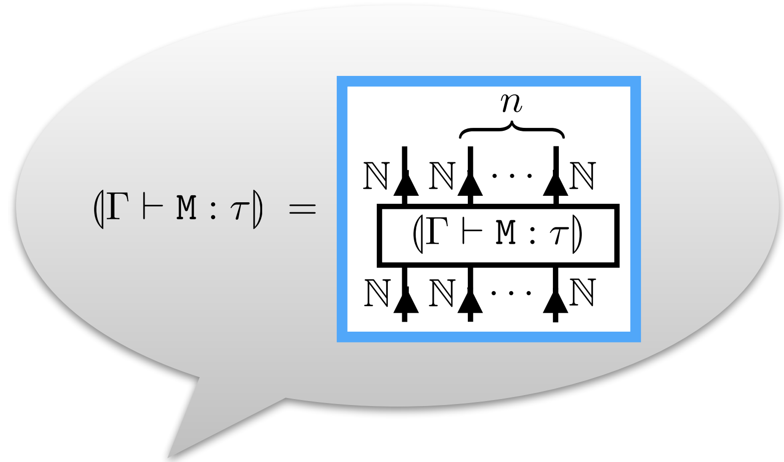
$$(\!|(\texttt{rec}(f,x).\texttt{if true} \sqcup \texttt{false then } 1 \texttt{ else } f\ x)\ 0|\!) =$$

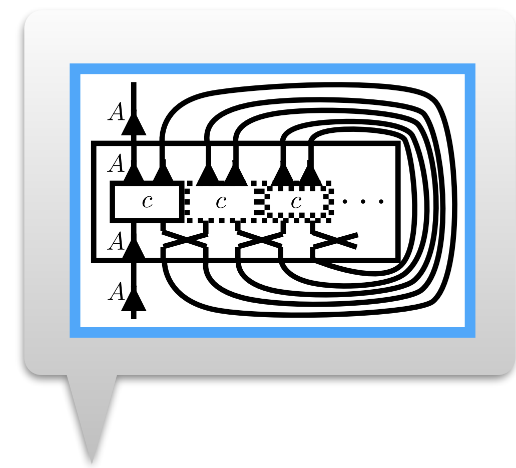
Memoryful GoI recursion

effectful terms

recursion

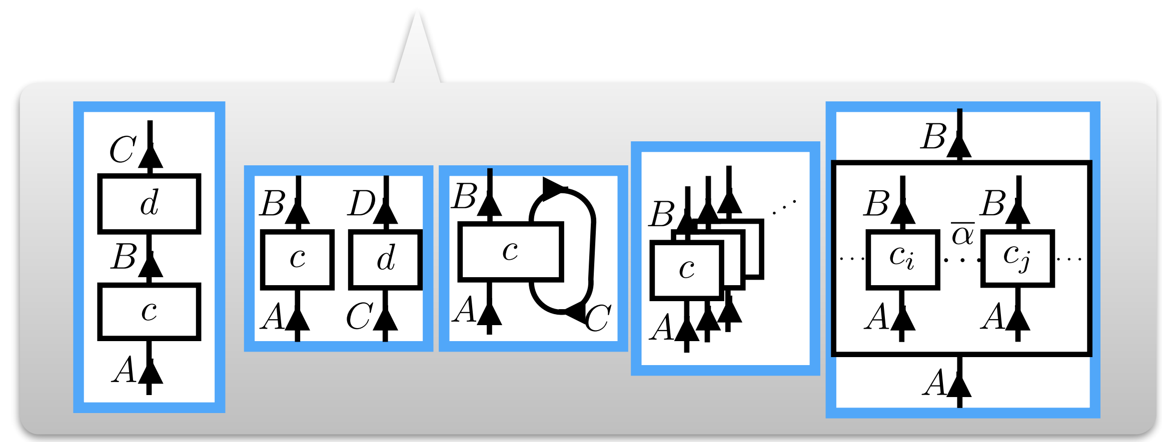
$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$



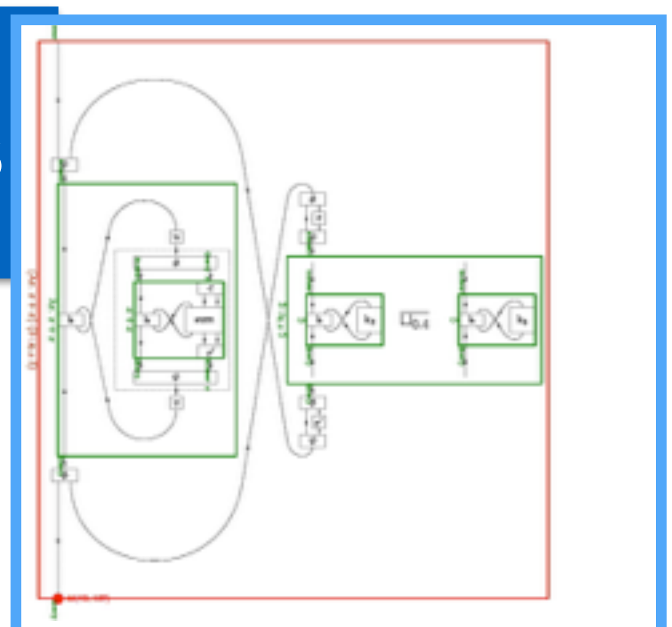


sound (& adequate) translation

- based on Geometry of Interaction
- via **extended** coalgebraic component calculus



transducers

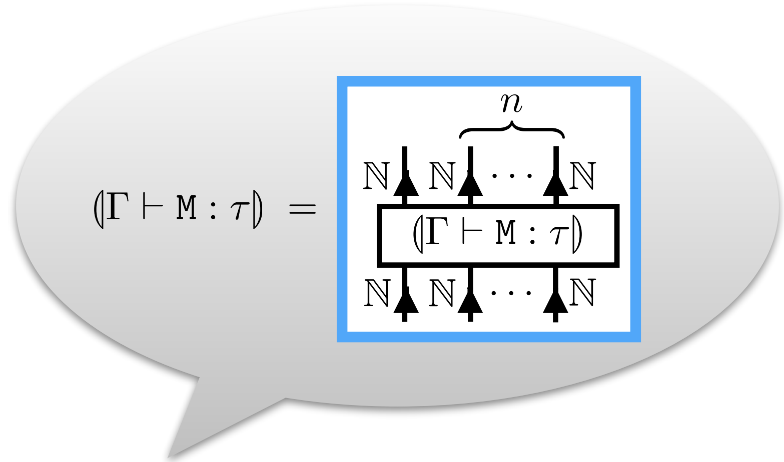


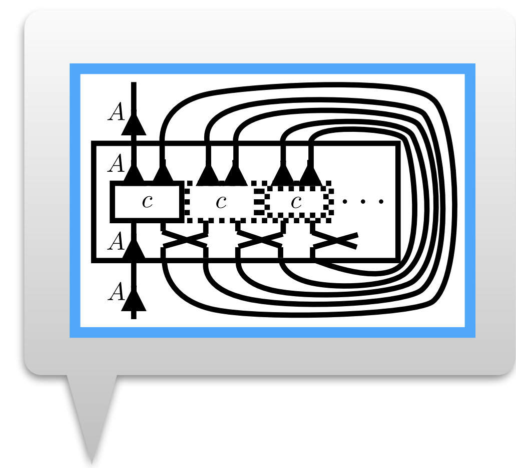
Memoryful GoI recursion

effectful terms

recursion

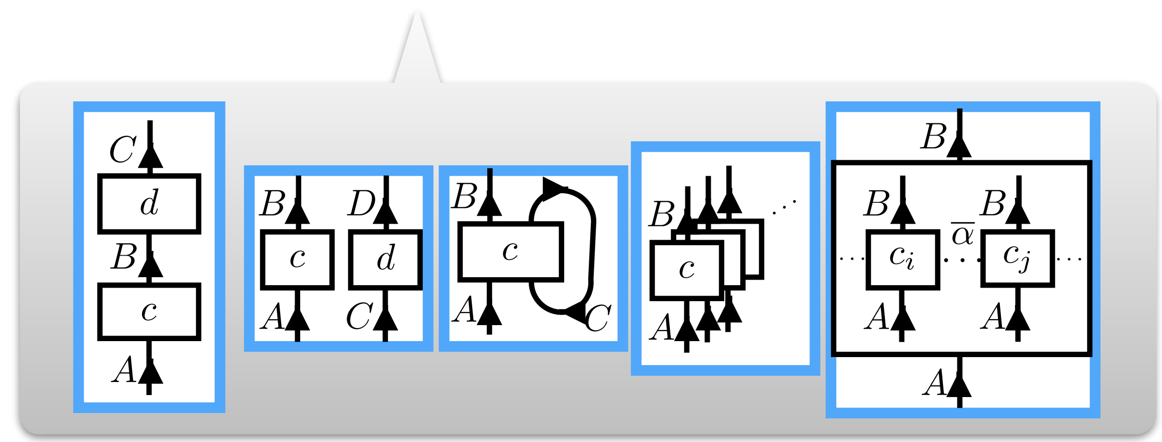
$(\lambda \mathtt{x} : \mathtt{nat}.\, \mathtt{x} + \mathtt{x})\,(3 \sqcup 5) \quad : \mathtt{nat}$





sound (& adequate) translation

- based on Geometry of Interaction
- via **extended** coalgebraic component calculus



transducers