Memoryful Geometry of Interaction II

Recursion and Adequacy

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Geometry of Interaction (GoI)

semantics of { linear logic proofs [Girard '89]
functional programs

token machine semantics

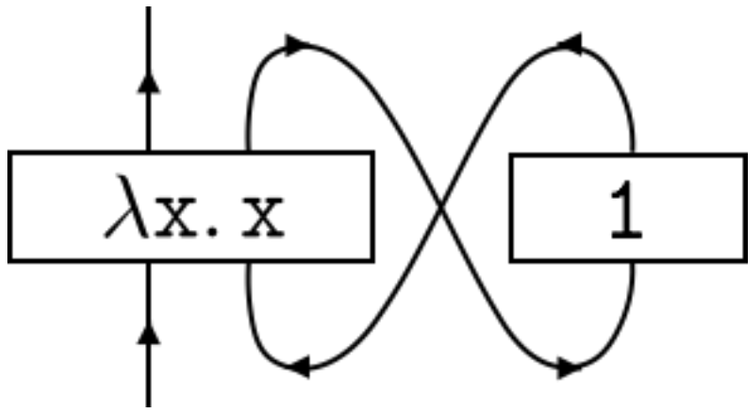
[Mackie '95] [Danos & Renier '99]

Token Machine Semantics

PCF terms

$(\lambda \mathrm{x}.\, \mathrm{x})\ 1$

token machines



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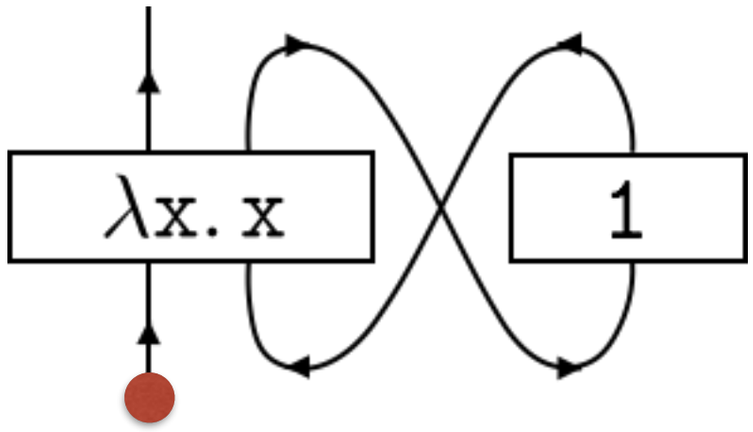

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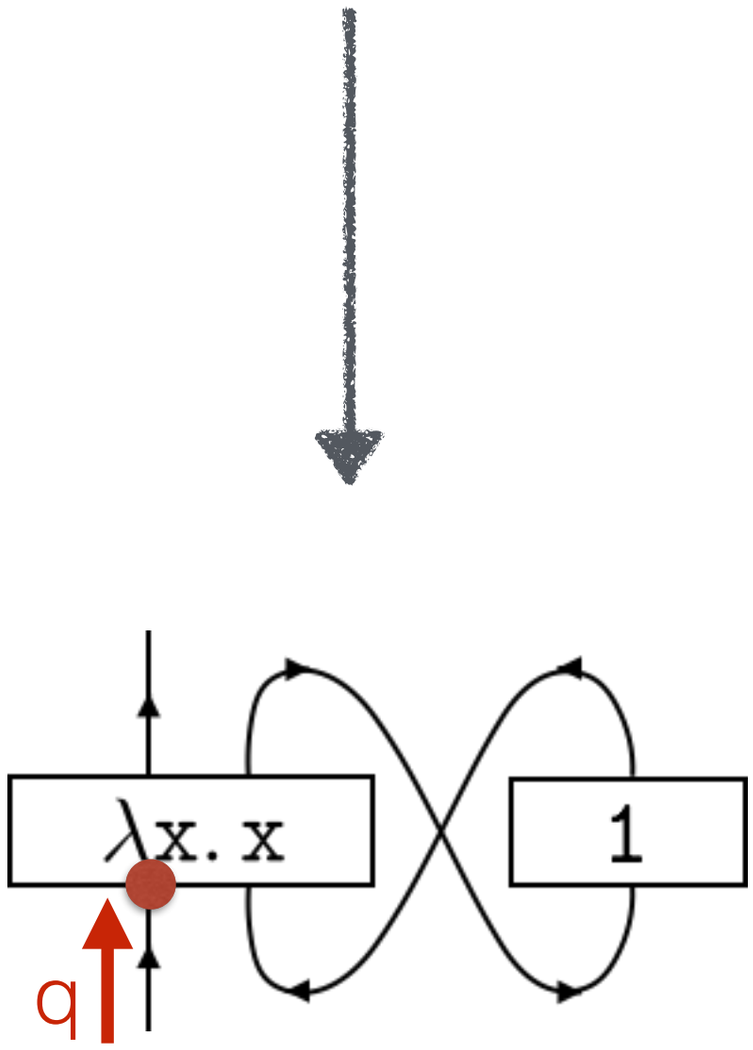

Token Machine Semantics

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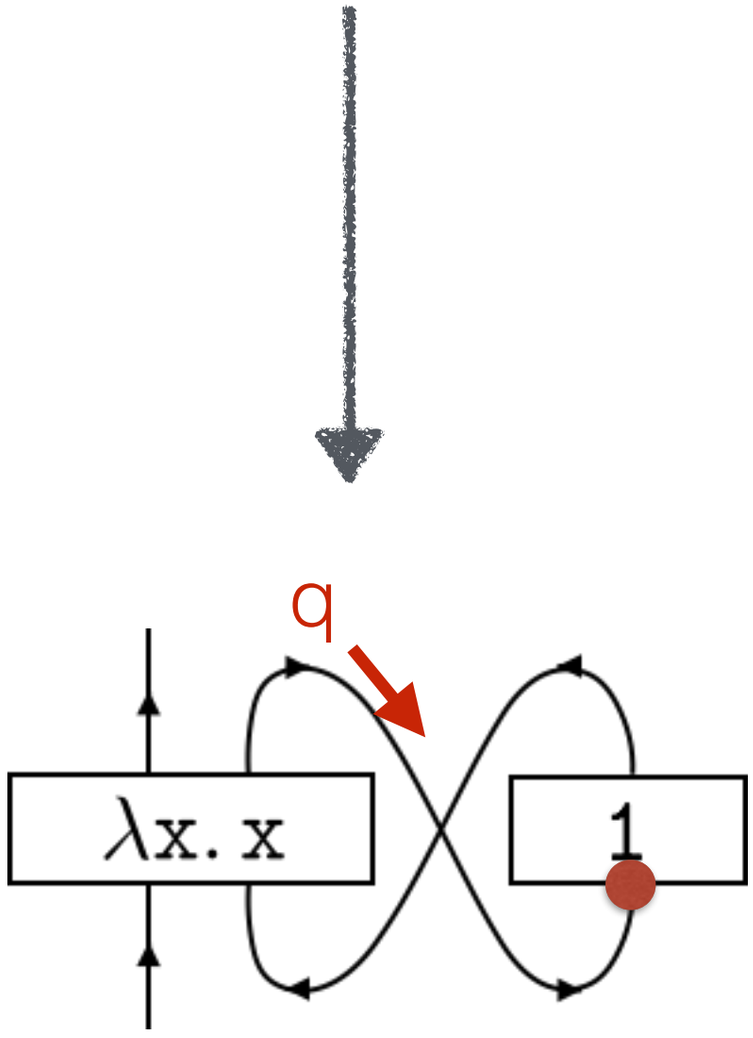

Token Machine Semantics

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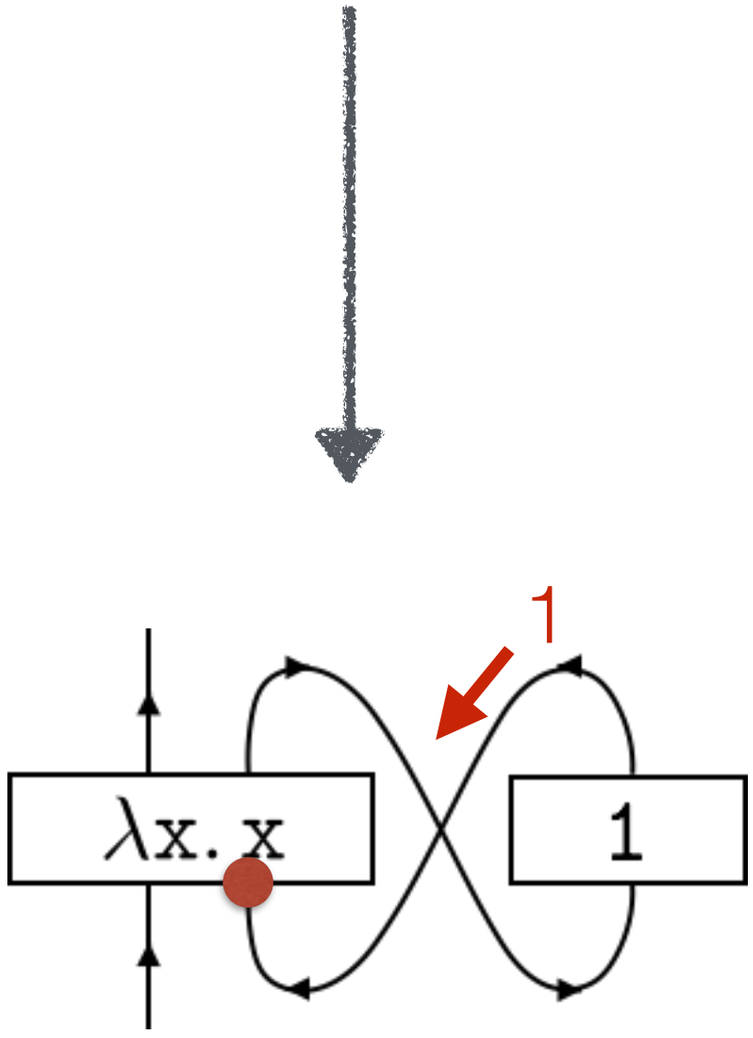

Token Machine Semantics

PCF terms

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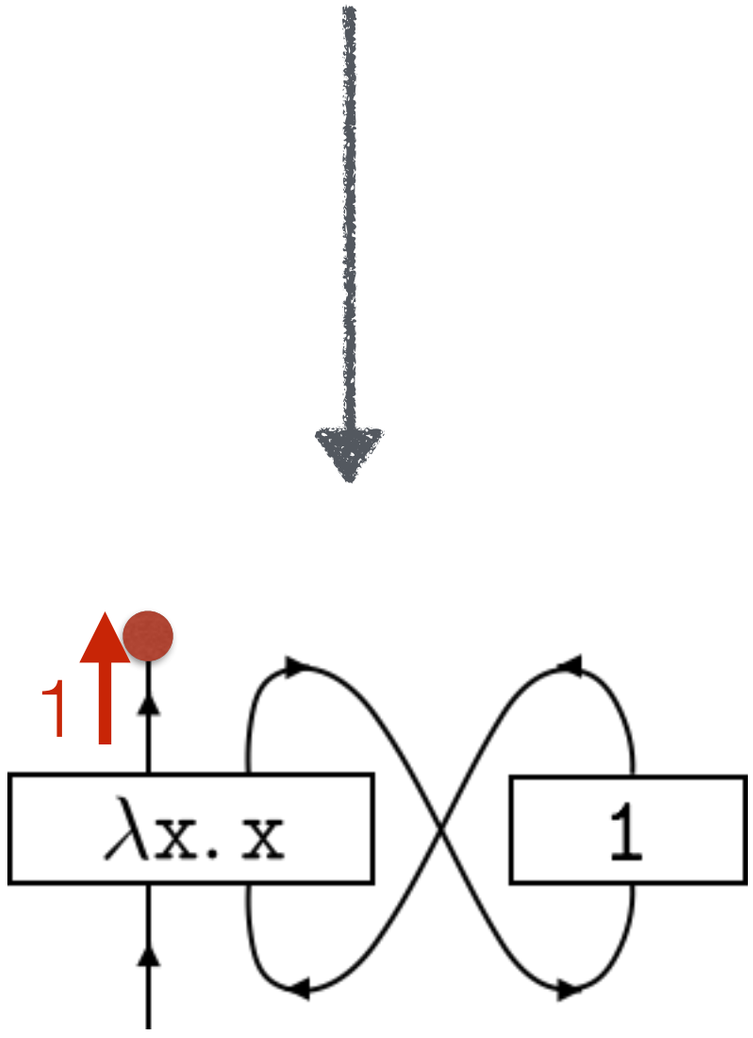
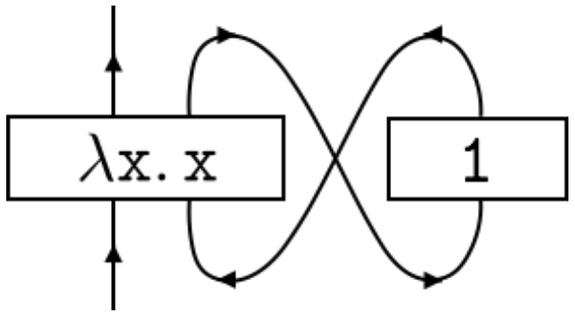


Token Machine Semantics

PCF terms

$(\lambda x.\, x)\, 1$

token machines



- applications to compilation

  [Mackie ’95] [Pinto ’01] [Ghica ’07]

- categorical formalization

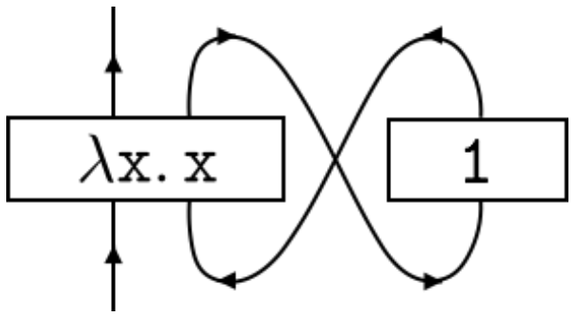
  [Abramsky et al. ’02]

Token Machine Semantics

PCF terms

$(\lambda \mathrm{x}.\, \mathrm{x})\ 1$

token machines



+ computational effects

Token Machine Semantics

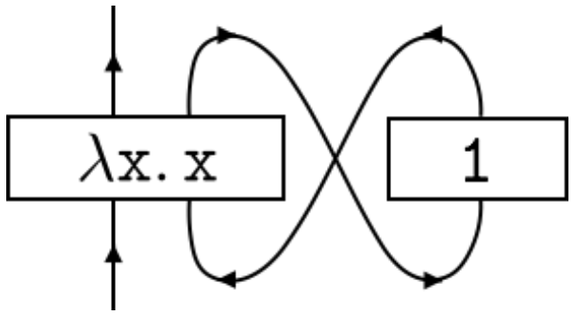
PCF terms

recursion

+ computational effects

$(\lambda \mathrm{x}.\, \mathrm{x})\ 1$

token machines



Token Machine Semantics

PCF terms

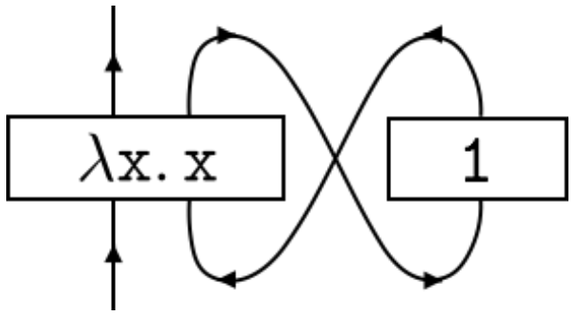
recursion

$(\lambda x.\, x)\ 1$

+ computational effects

- memoryful GoI [Hoshino, — & Hasuo ’14]

token machines



Token Machine Semantics

PCF terms

recursion

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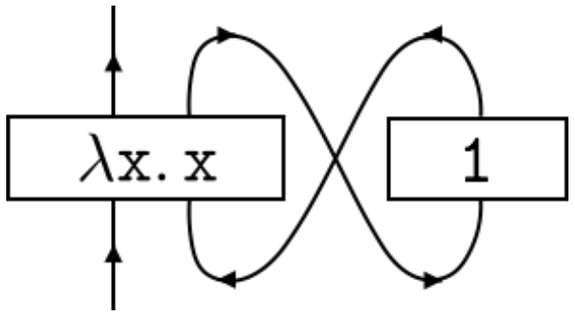
+ computational effects

- memoryful GoI [Hoshino, — & Hasuo ’14]
- memoryful GoI II

token machines



Memoryful GoI

effectful PCF terms

call-by-value PCF terms

with algebraic operations [Plotkin & Power '01]

- nondeterministic choice $\texttt{choose}(\texttt{M}, \texttt{N})$
- probabilistic choice $\texttt{choose}_p(\texttt{M}, \texttt{N})$
- actions on global states $\texttt{lookup}_\ell(\texttt{M}_{v_0}, \ldots, \texttt{M}_{v_n})$

$\texttt{update}_{\ell,v}(\texttt{M})$

transducers

Memoryful GoI

effectful PCF terms

call-by-value PCF terms

with algebraic operations [Plotkin & Power '01]

$$(\lambda \mathtt{x}.\, \mathtt{M})\, \mathtt{V} \to \mathtt{M}[\mathtt{V}/\mathtt{x}]$$

$$\mathtt{rec}(\mathtt{f},\mathtt{x}).\,\mathtt{M} \to (\lambda \mathtt{x}.\,\mathtt{M})\,[\mathtt{rec}(\mathtt{f},\mathtt{x}).\,\mathtt{M}/\mathtt{f}]$$

$$\mathtt{op}(\mathtt{M}_1,\ldots,\mathtt{M}_n) \xrightarrow{\mathtt{op}_i} \mathtt{M}_i$$

$$\mathtt{op}() \downarrow_{\mathtt{op}}$$

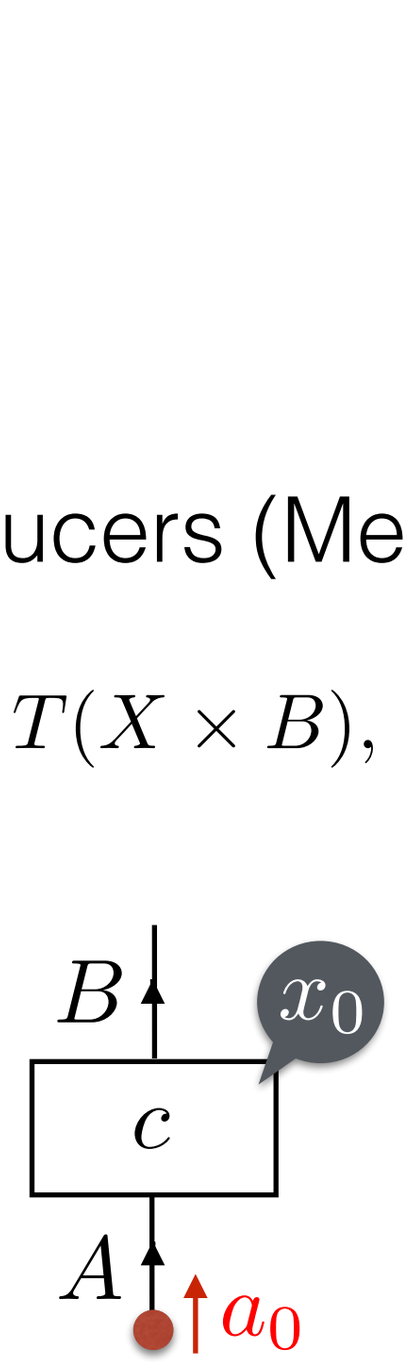
transducers

Memoryful GoI

effectful PCF terms

stream transducers (Mealy machines)

$$(X,\ c\colon X \times A \to T(X \times B),\ x_0 \in X)\colon A \nrightarrow B$$



transducers

Memoryful GoI

effectful PCF terms

transducers

stream transducers (Mealy machines)

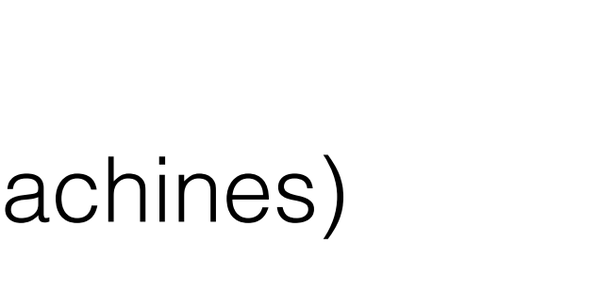
$$(X,\ c\colon X \times A \to T(X \times B),\ x_0 \in X)\colon A \nrightarrow B$$

- nondeterministic transitions

$$TX = \mathcal{P}(X)$$

- probabilistic transitions

$$TX = \mathcal{D}_{\leq 1}(X)$$

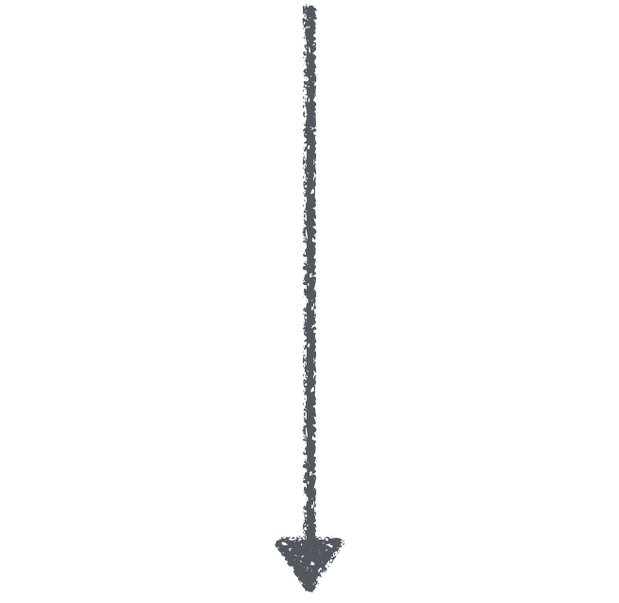


- transitions with global states

$$TX = (1 + X \times S)^S$$

Memoryful GoI

effectful PCF terms



transducers

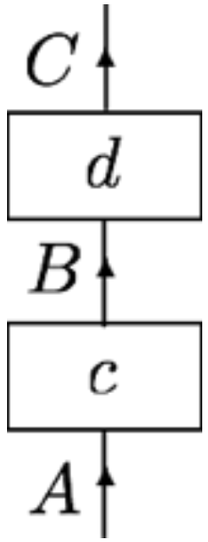
compositional translation

via coalgebraic component calculus

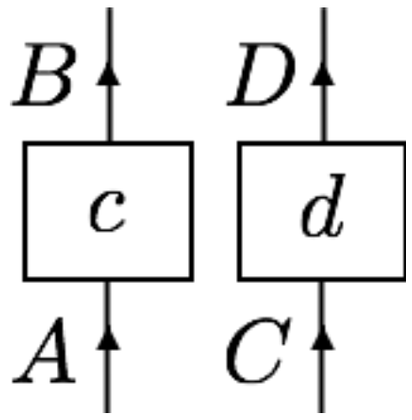
[Barbosa ’03] [Hasuo, Jacobs ’11]

Component Calculus

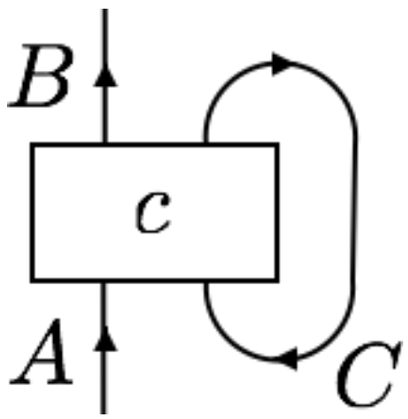
sequential composition



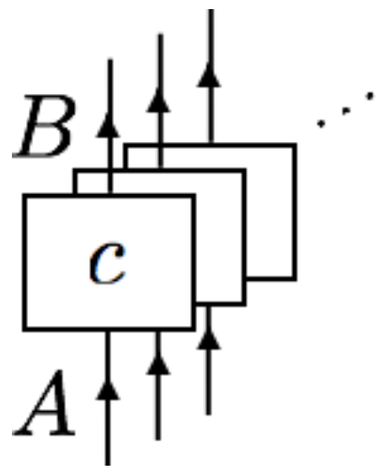
parallel composition



trace operator

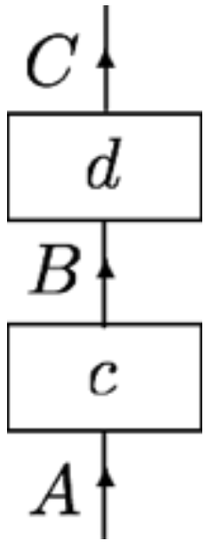


countable copy operator

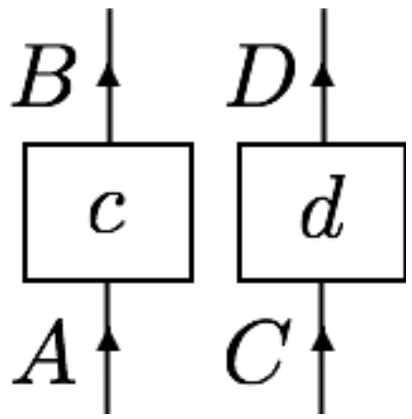


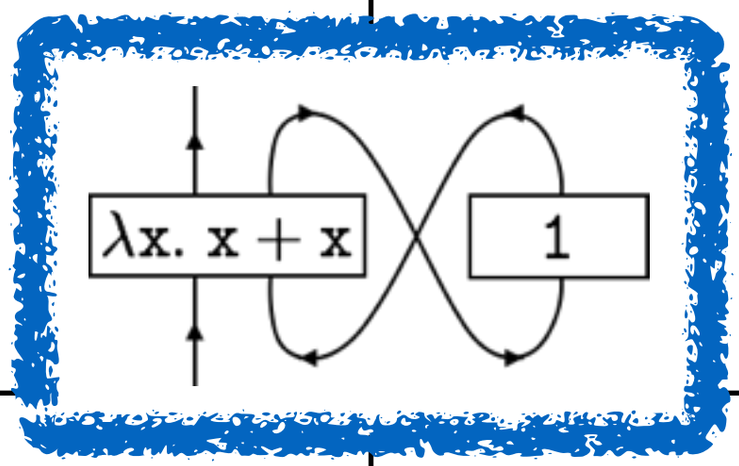
Component Calculus

sequential composition

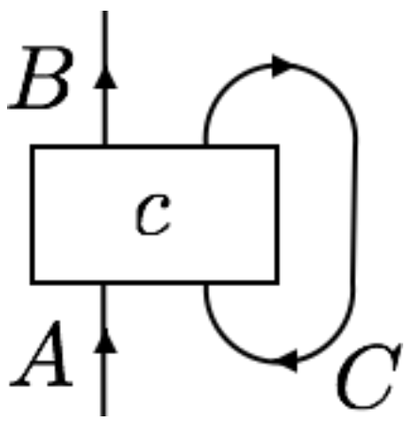


parallel composition





trace operator



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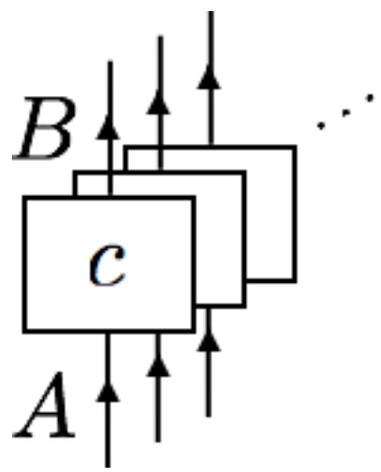
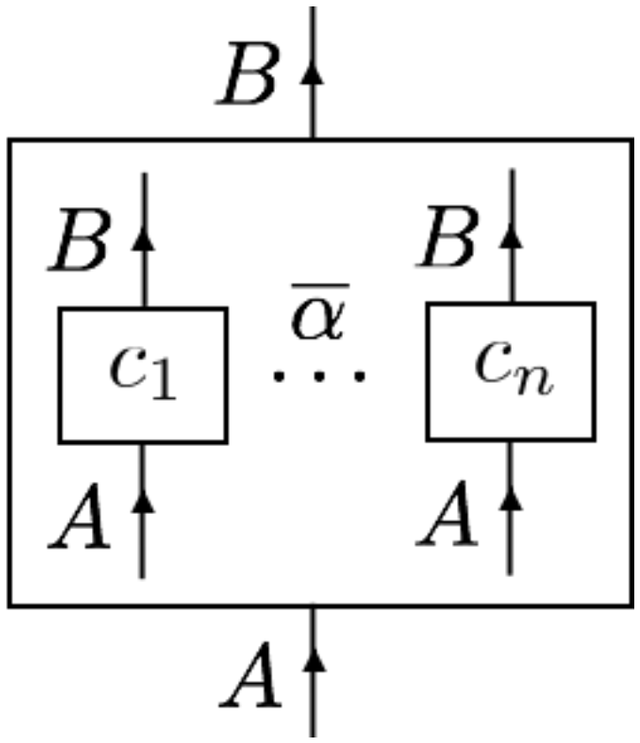
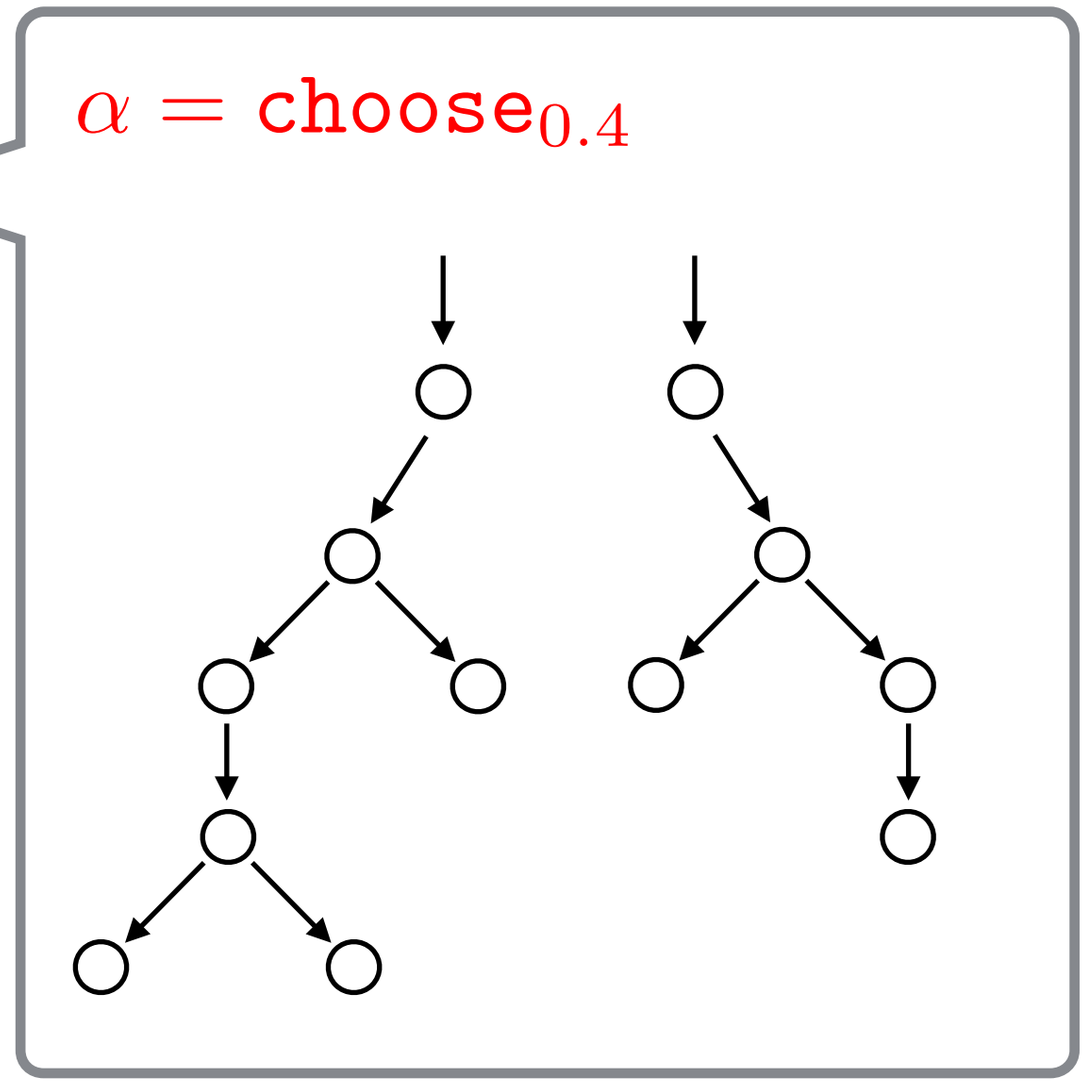


Component Calculus for Effects

lifted algebraic operations





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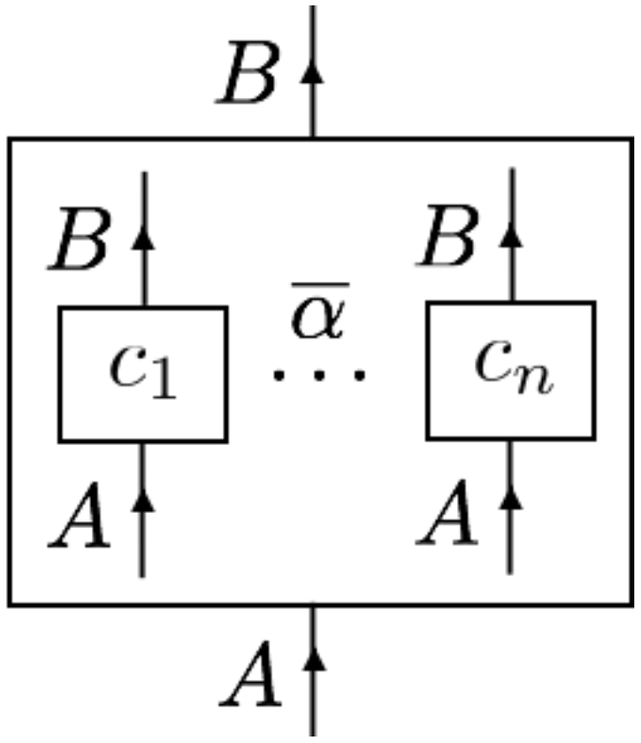
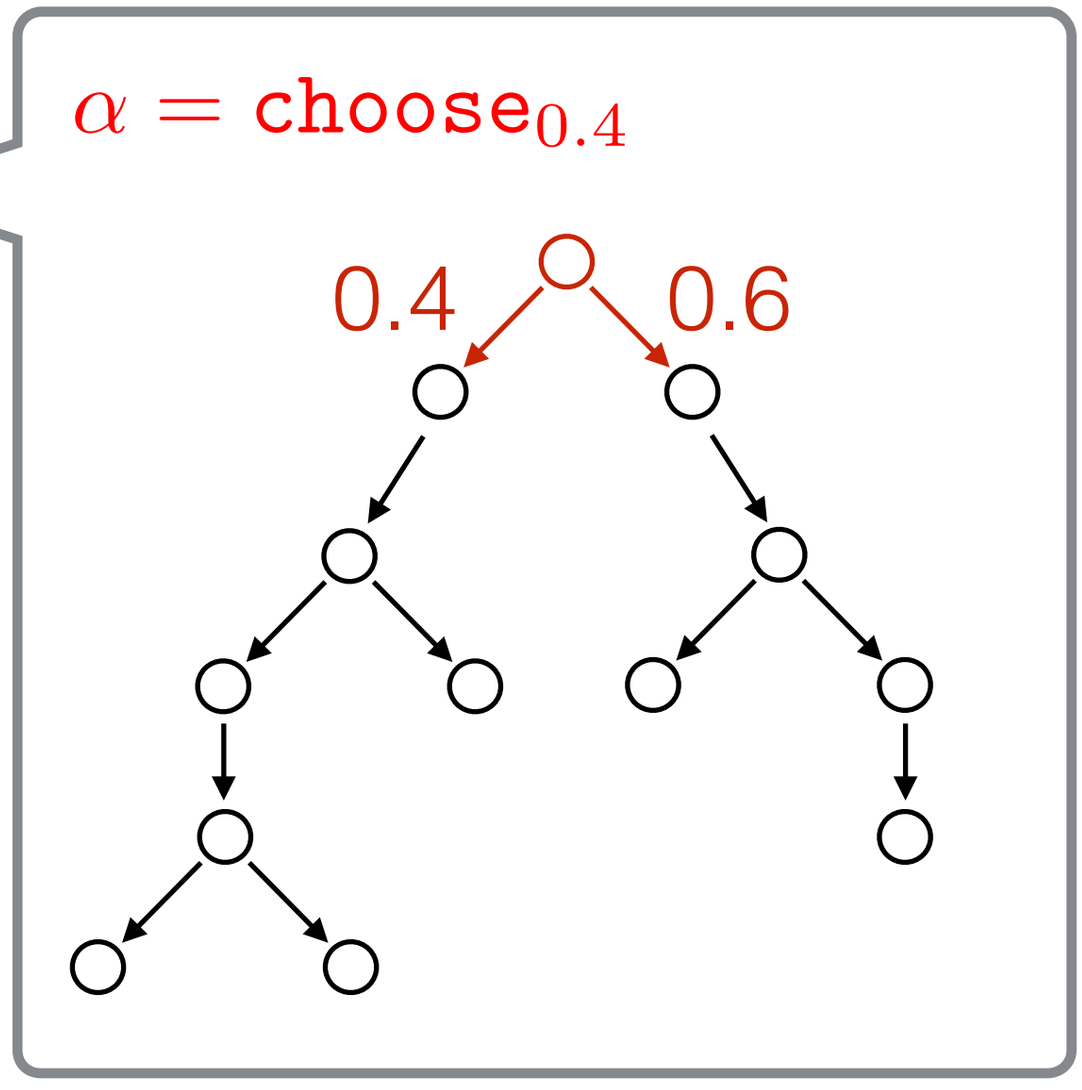
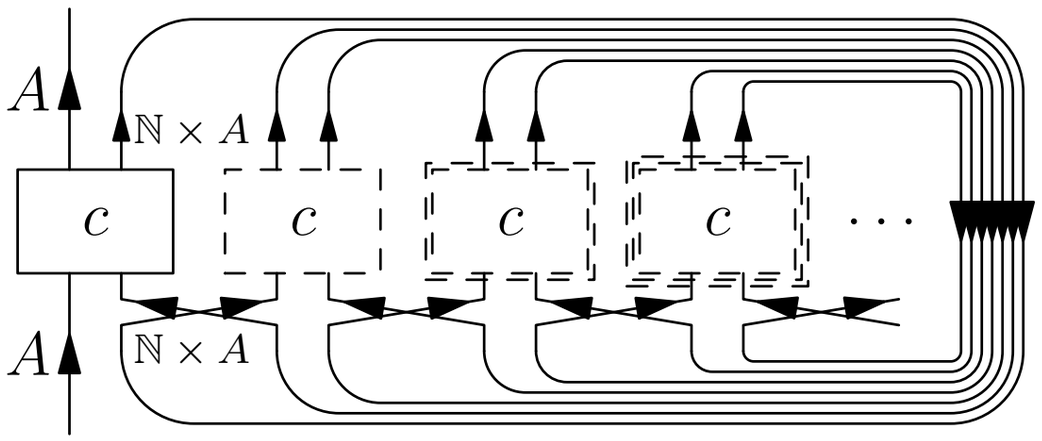


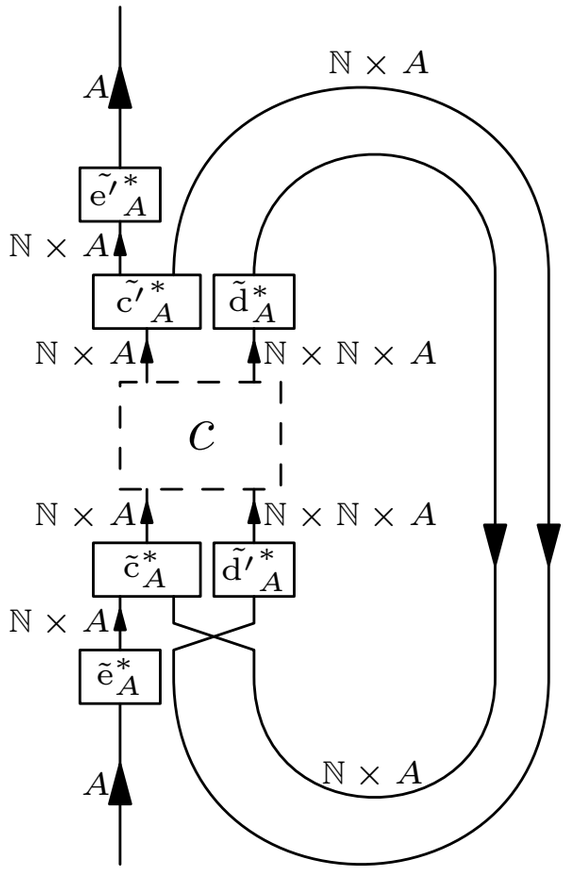


Component Calculus for Recursion

Girard style fixed point operator



Mackie style fixed point operator



Component Calculus for Recursion

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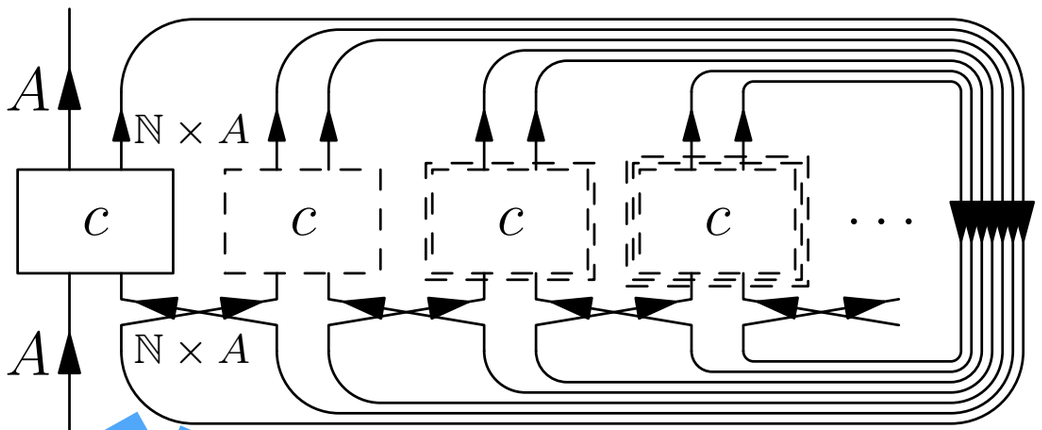
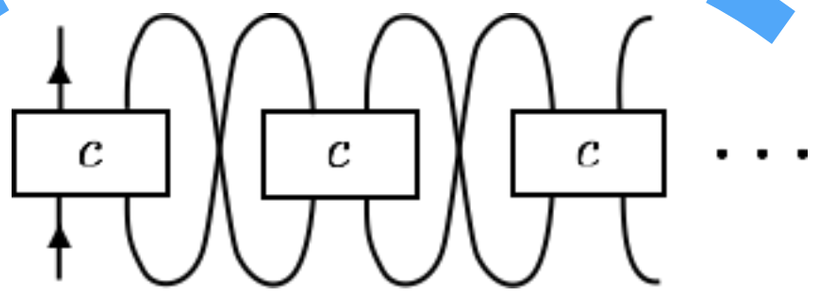
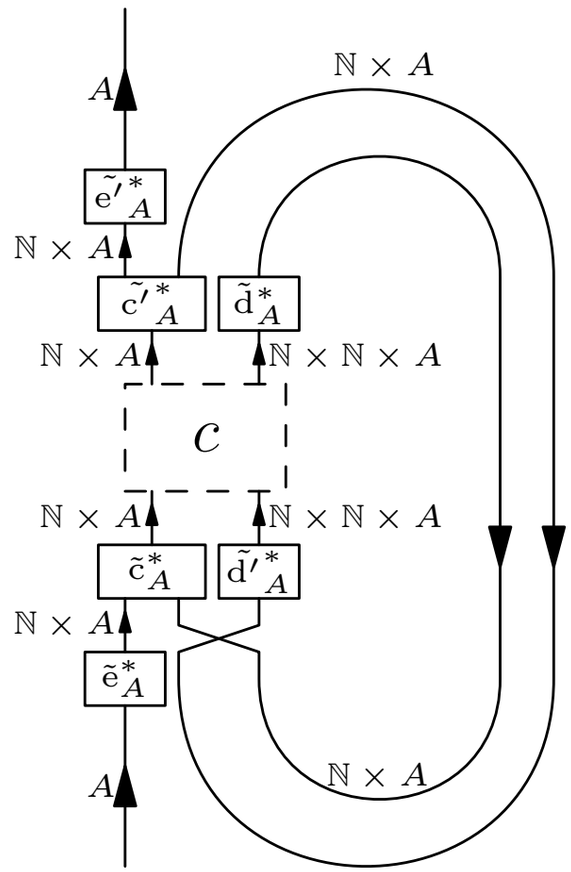




$$\mathrm{fix}(F) = F(F(F(\cdots)))$$

Mackie style fixed point operator



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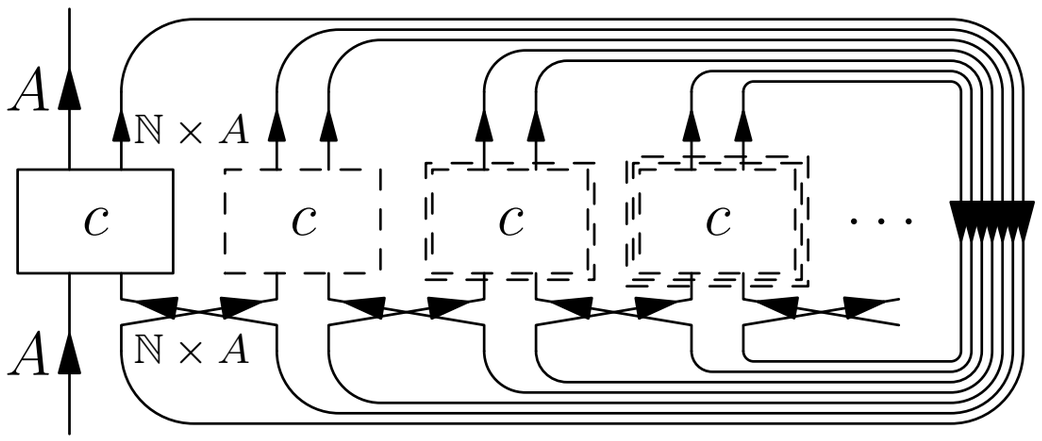
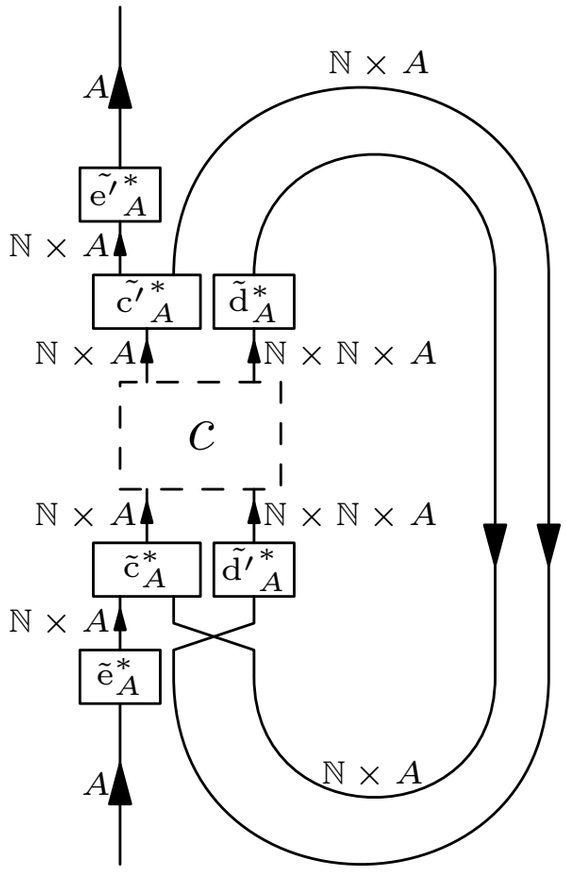
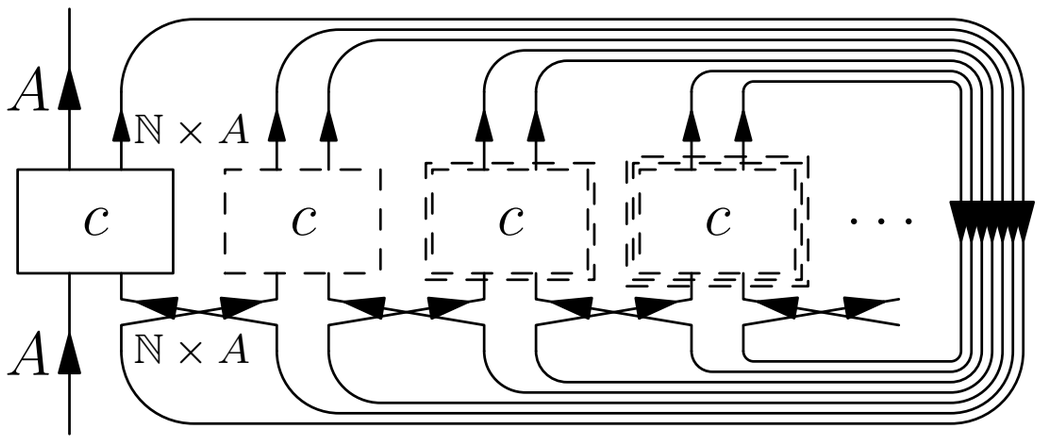


Mackie style fixed point operator



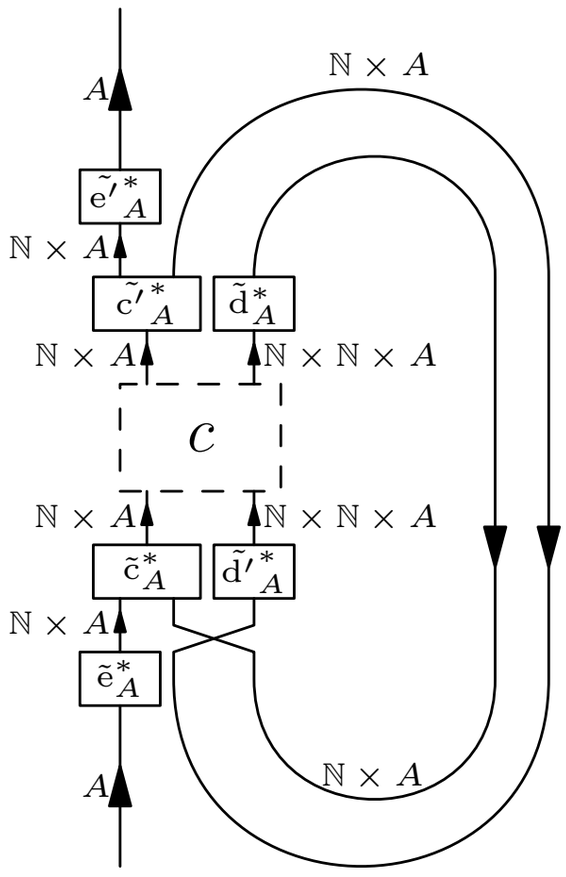
Component Calculus for Recursion

Girard style fixed point operator



- fixed point wrt. “cross connection”
- supremum of finite approximations

Mackie style fixed point operator

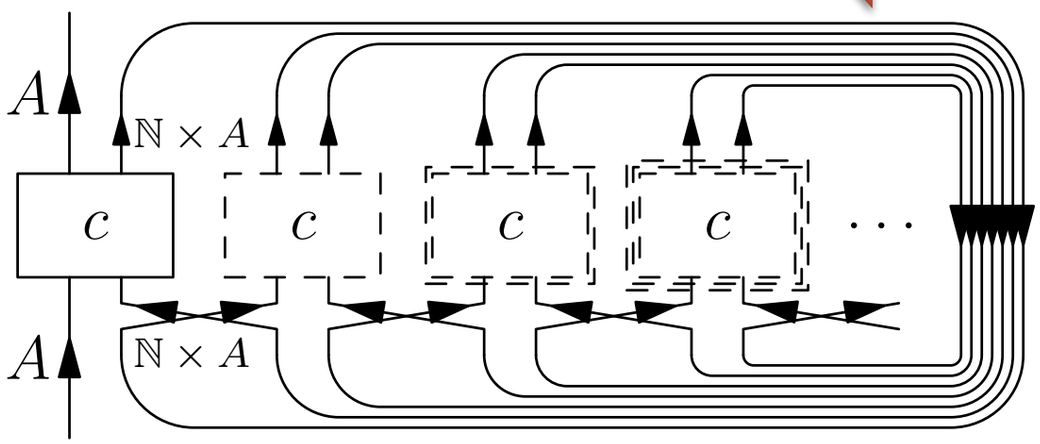


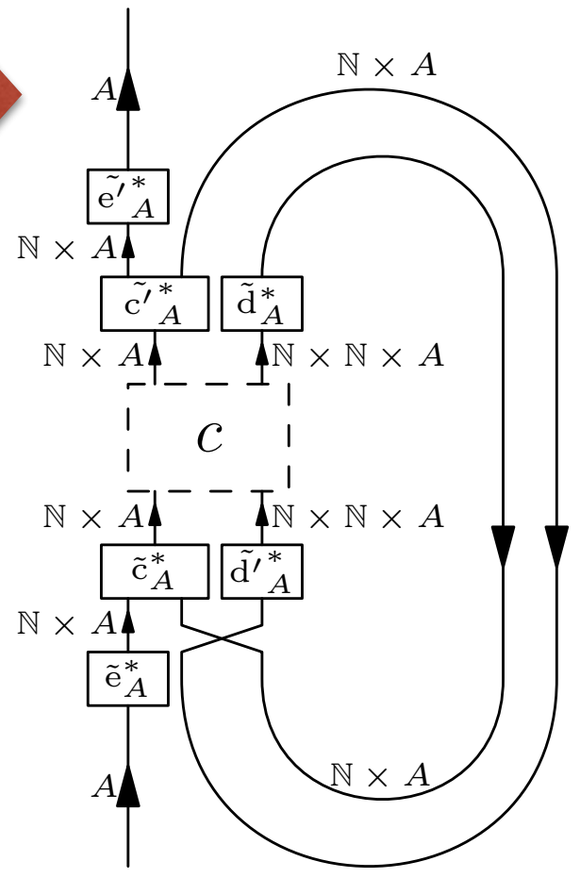
Component Calculus for Recursion

Girard style
fixed point operator

coincidence

Mackie style
fixed point operator

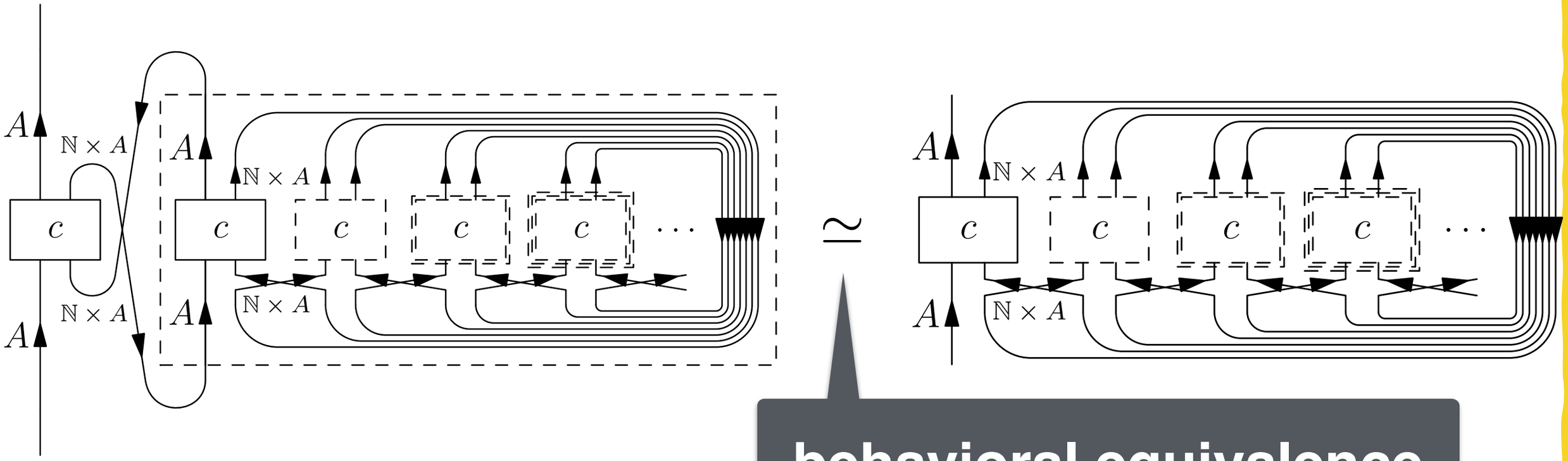




- fixed point wrt. “cross connection”
- supremum of finite approximations

Component Calculus for Recursion

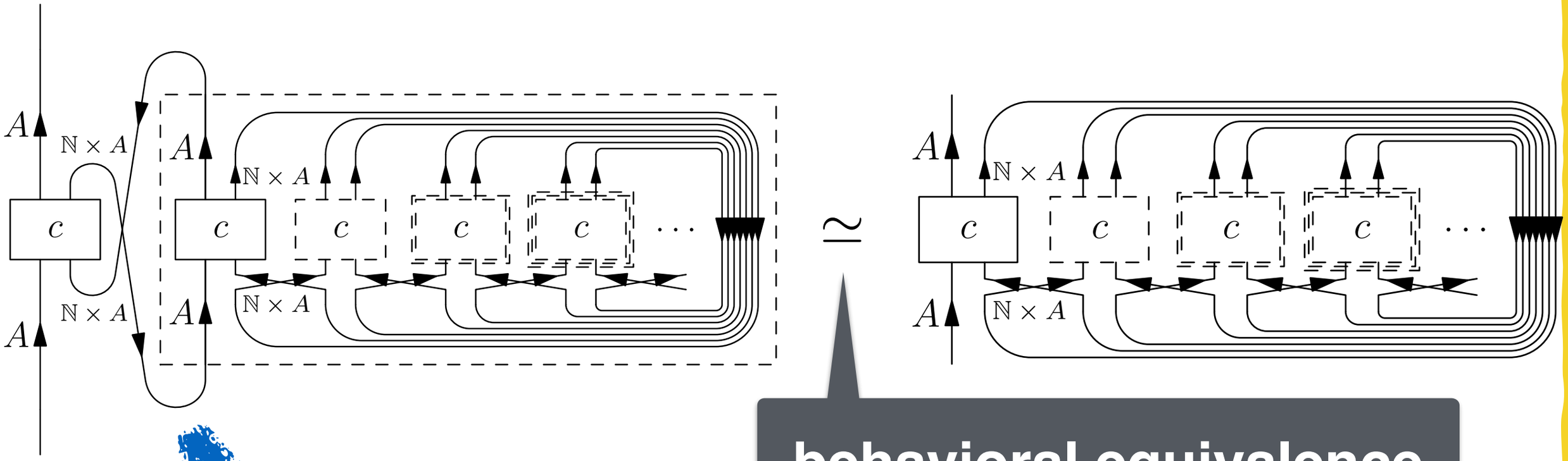
Prop. (Girard style as **fixed point** wrt. “cross connection”)



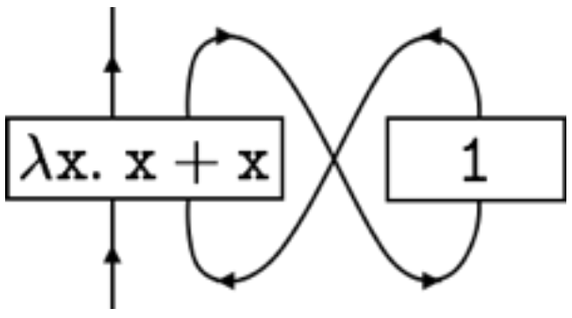
behavioral equivalence

Component Calculus for Recursion

Prop. (Girard style as **fixed point** wrt. “cross connection”)

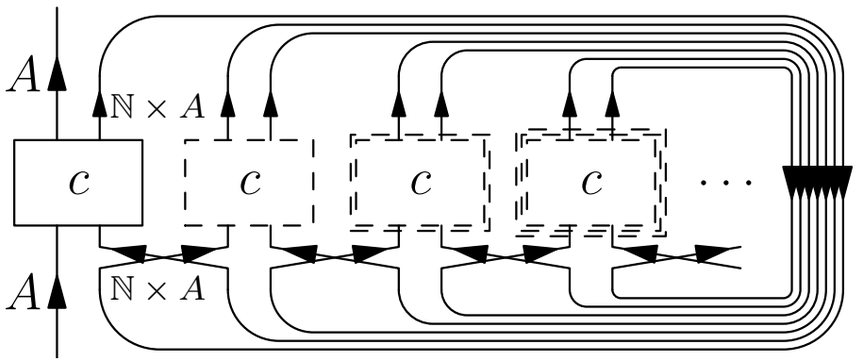


behavioral equivalence

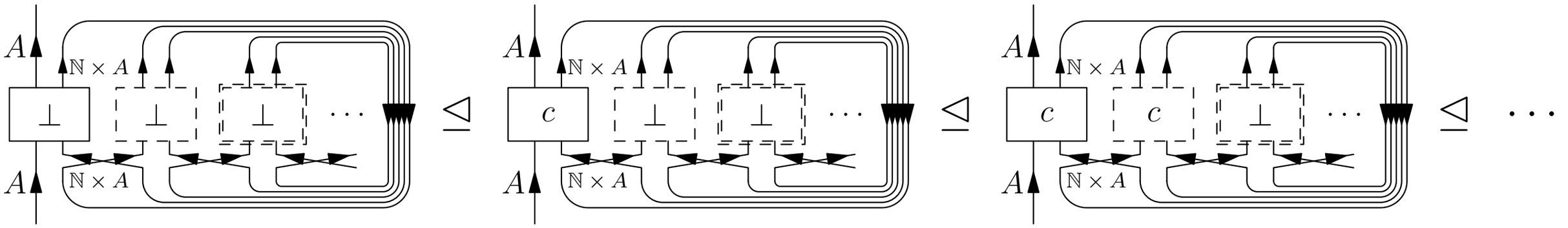


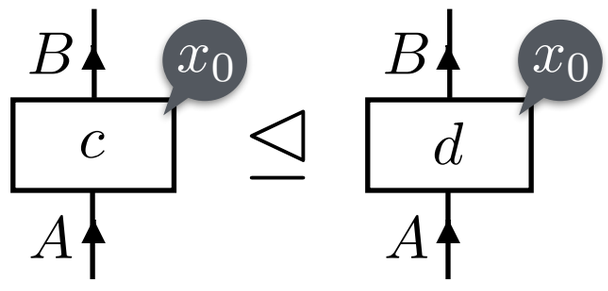
Component Calculus for Recursion

Prop. (Girard style as **supremum** of finite approximations)



is the supremum of the ω-chain

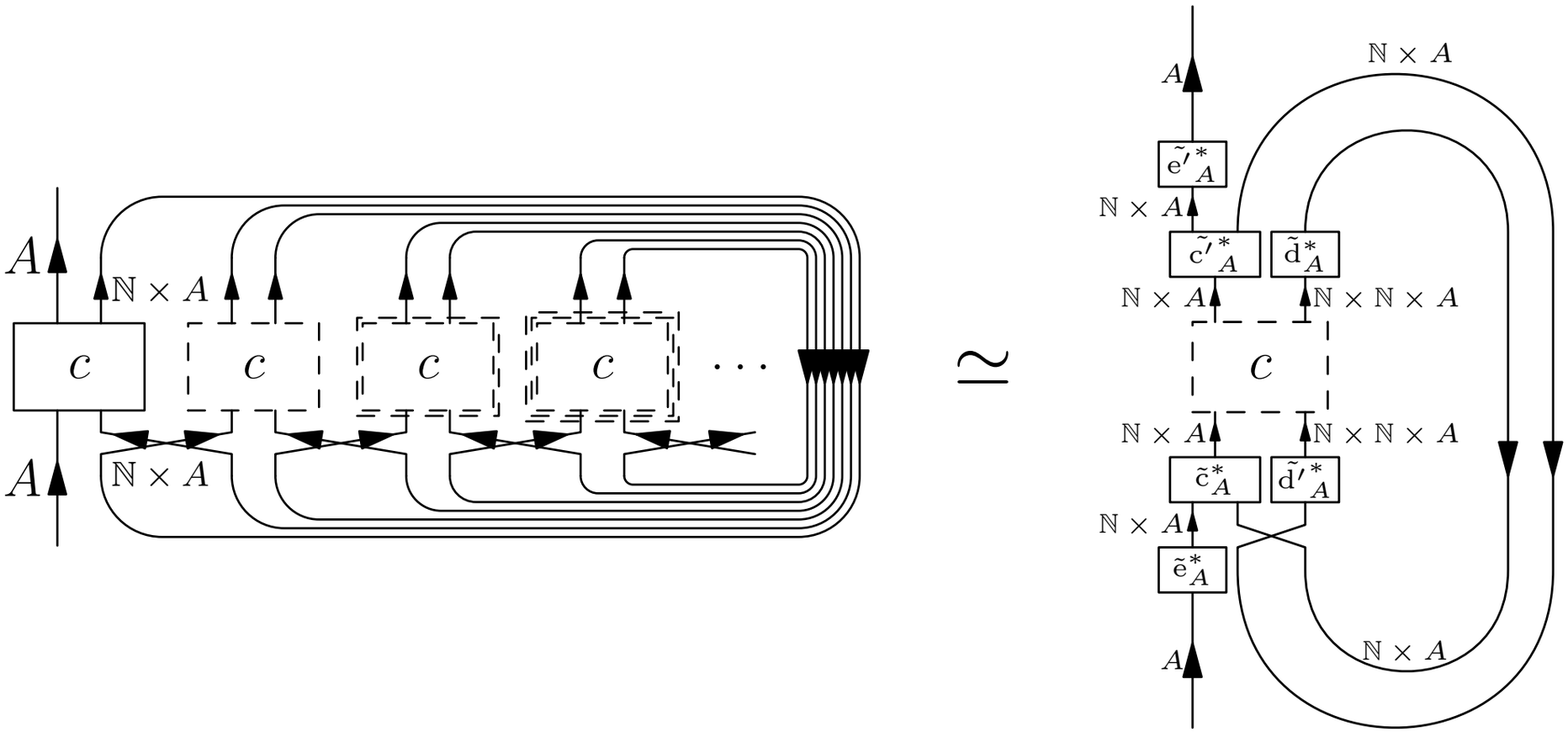


where 

$\overset{\text{def.}}{\Longleftrightarrow}$ $T(X \times B)$ with $c \sqsubseteq d$ over $X \times A$

Component Calculus for Recursion

Thm. (**coincidence** of Girard style & Mackie style)



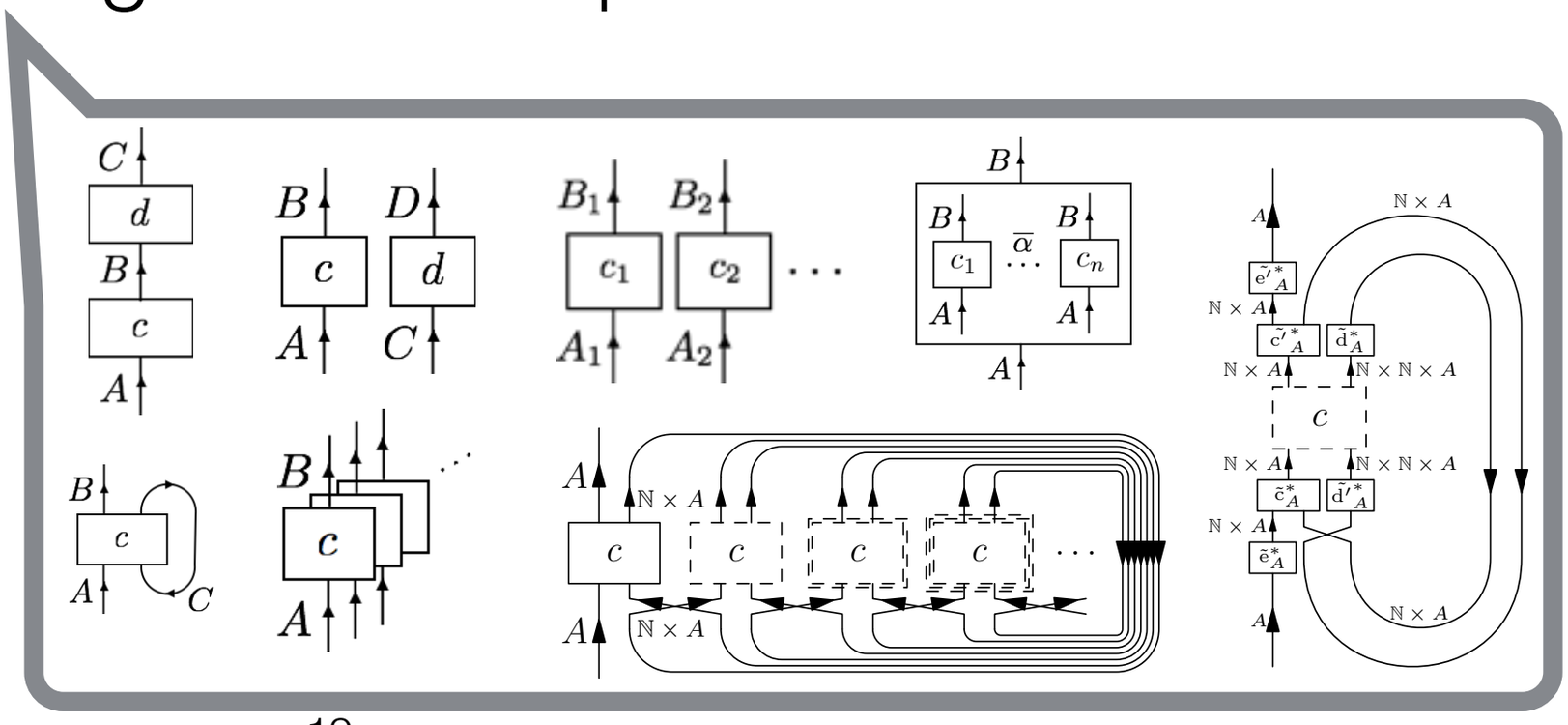
Memoryful GoI

effectful PCF terms

compositional translation

transducers

via coalgebraic component calculus



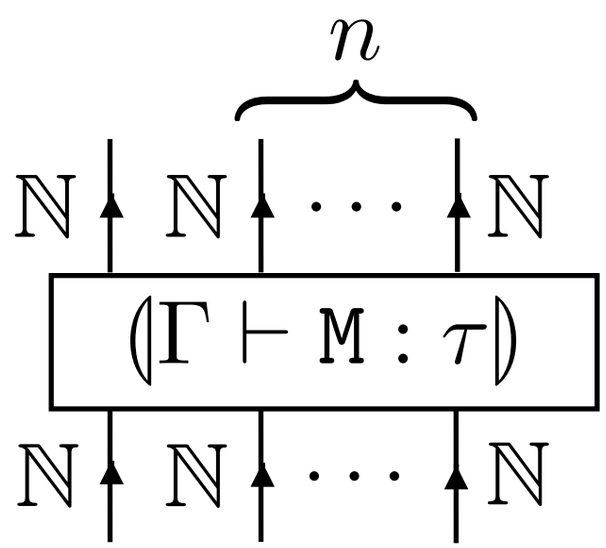
Translation

Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

For a type judgement $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$ $(\Gamma = \mathtt{x}_1 : \tau_1, \ldots, \mathtt{x}_n : \tau_n)$

we inductively define

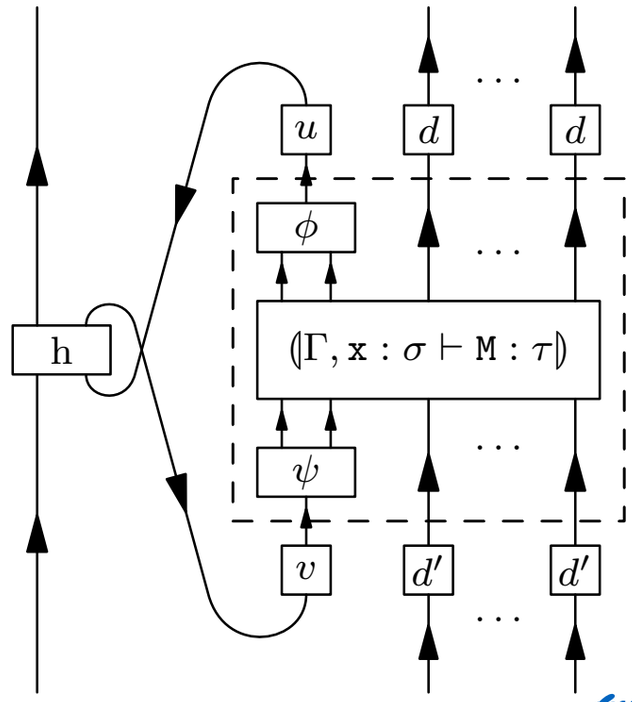
$$(\!|\Gamma \vdash \mathtt{M} : \tau|\!) \;=$$



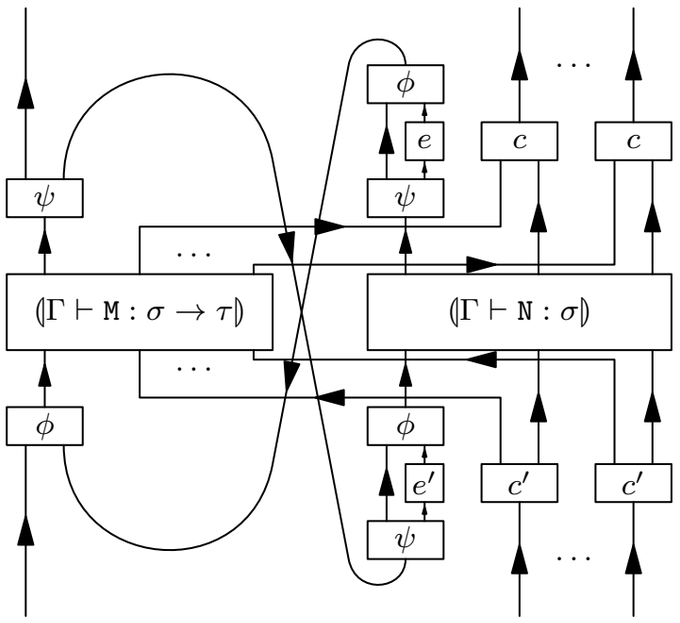
Translation

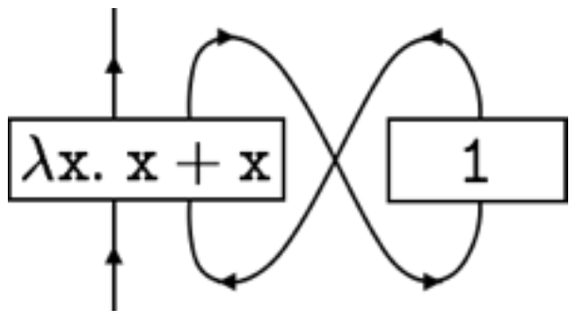
Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

$(\!|\Gamma \vdash \lambda \mathtt{x} : \sigma . \mathtt{M} : \sigma \to \tau|\!) =$



$(\!|\Gamma \vdash \mathtt{M}\ \mathtt{N} : \tau|\!) =$

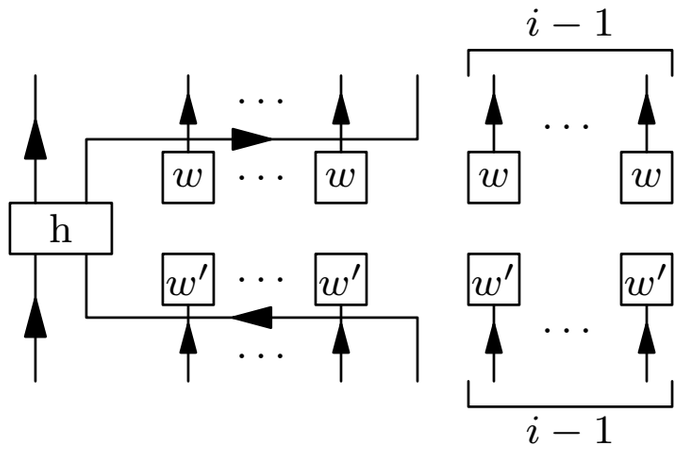




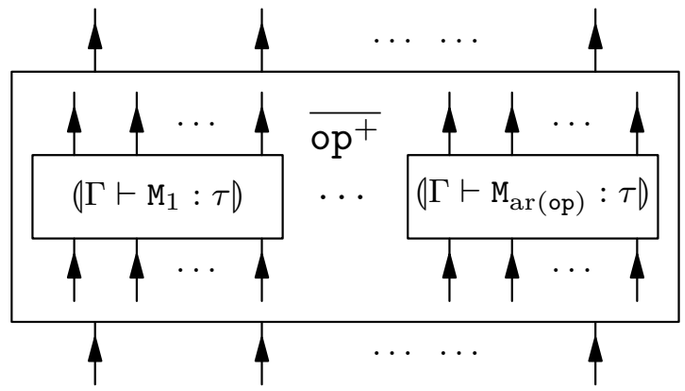
Translation

Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

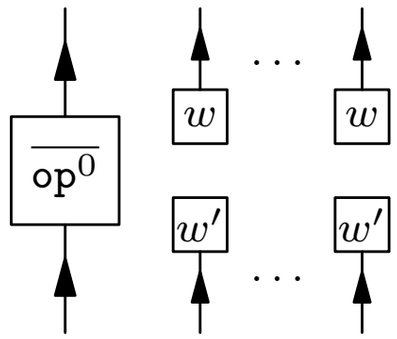
$$(\!|\Gamma \vdash \mathbf{x}_i : \tau_i|\!) =$$



$$(\!|\Gamma \vdash \mathtt{op}^{+}(\mathtt{M}_1, \ldots, \mathtt{M}_{\mathrm{ar}(\mathtt{op})}) : \tau|\!) =$$



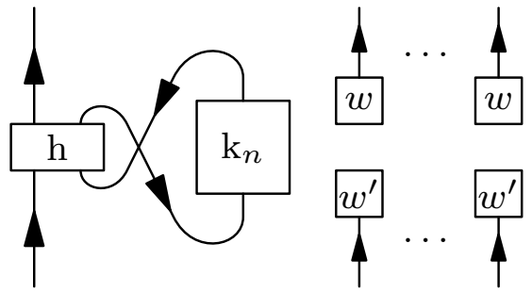
$$(\!|\Gamma \vdash \mathtt{op}^{0}() : \tau|\!) =$$



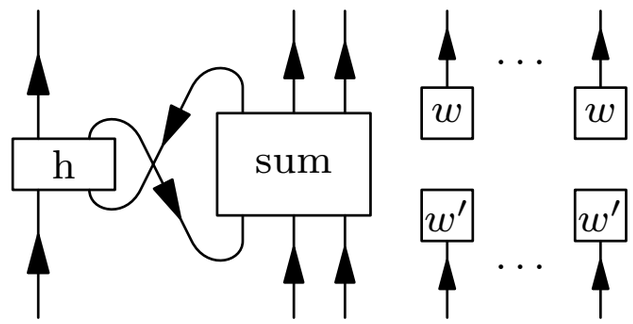
Translation

Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

$(\!|\Gamma \vdash \underline{\mathtt{n}} : \mathtt{nat}|\!) =$

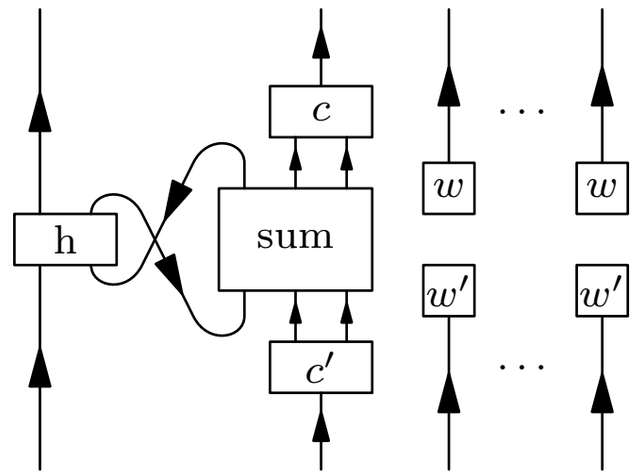


$(\!|\Gamma, \mathtt{x} : \mathtt{nat}, \mathtt{y} : \mathtt{nat} \vdash \mathtt{x} + \mathtt{y} : \mathtt{nat}|\!) =$



(if $\mathtt{x} \not\equiv \mathtt{y}$)

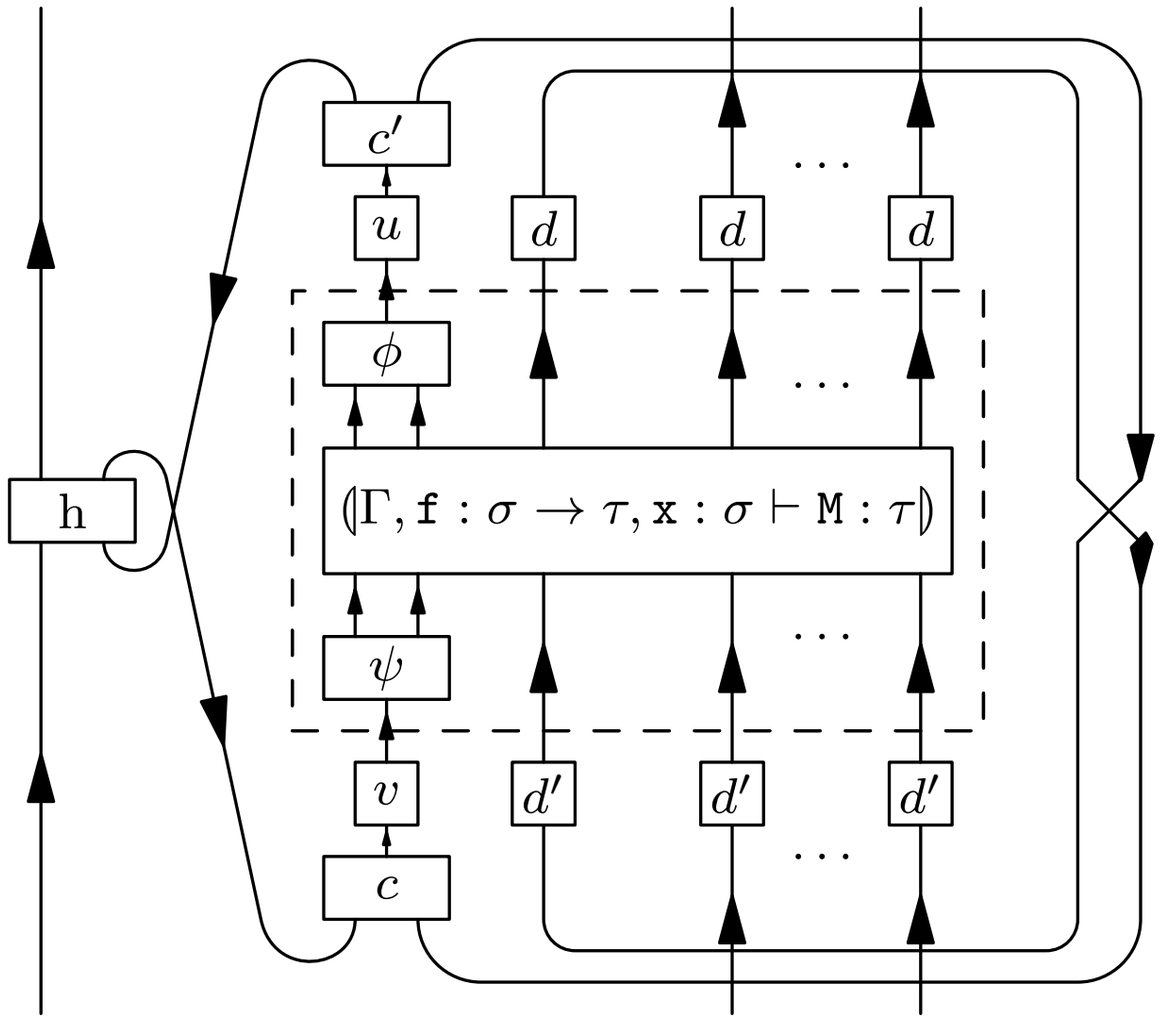
$(\!|\Gamma, \mathtt{x} : \mathtt{nat} \vdash \mathtt{x} + \mathtt{x} : \mathtt{nat}|\!) =$

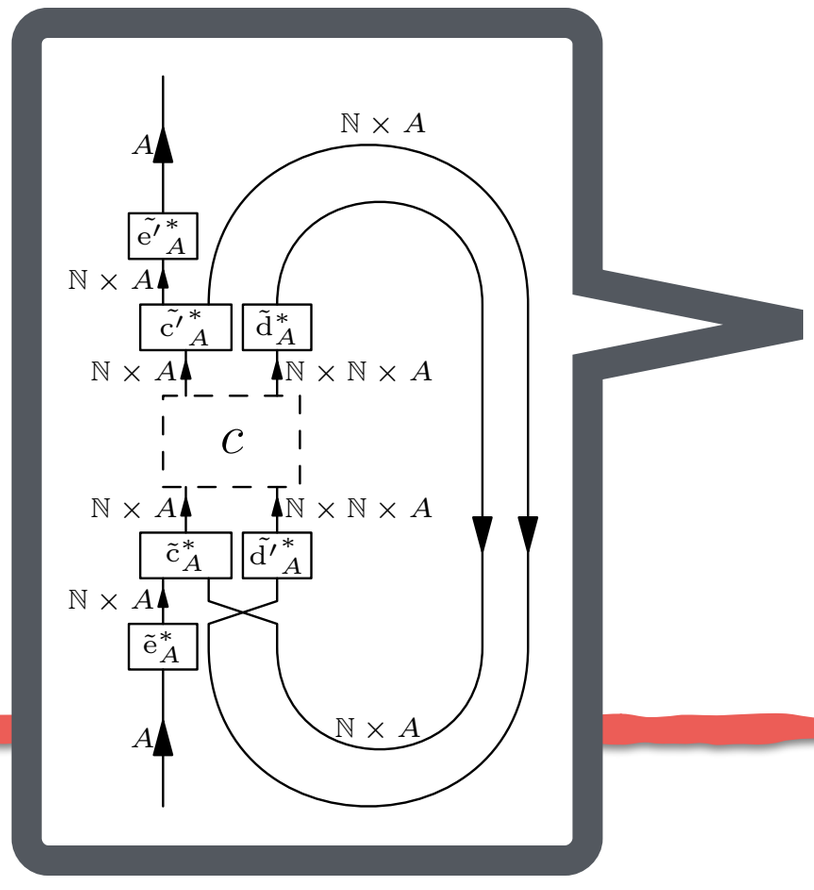


Translation

Def. (translation $(\!|\Gamma \vdash \mathtt{M} : \tau|\!)$)

$$(\!|\Gamma \vdash \mathtt{rec}(\mathtt{f} : \sigma \to \tau, \mathtt{x} : \sigma).\mathtt{M} : \sigma \to \tau|\!) =$$



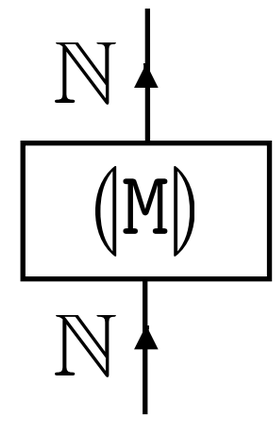


Translation

Theorem 6.4 (adequacy)**.** Any closed term $\mathtt{M}$ of base type $\mathtt{nat}$ satisfies $[\![|\mathtt{M}|]\!] = (\!|\mathtt{M}|\!)^{\dagger}$.

evaluation result of term $\mathtt{M}$

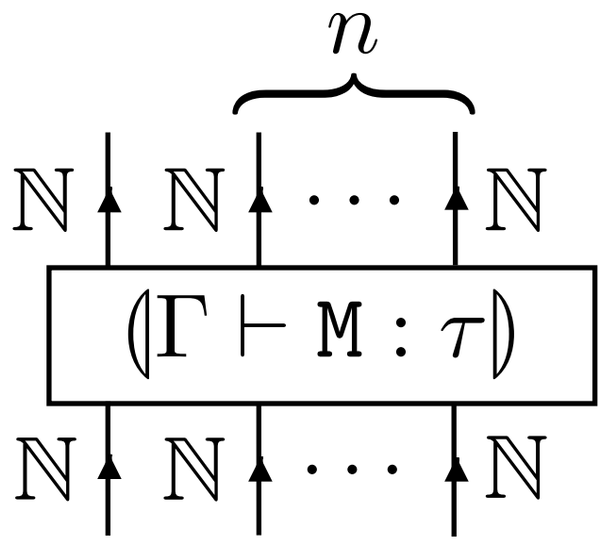
output of transducer



Memoryful GoI

effectful PCF terms

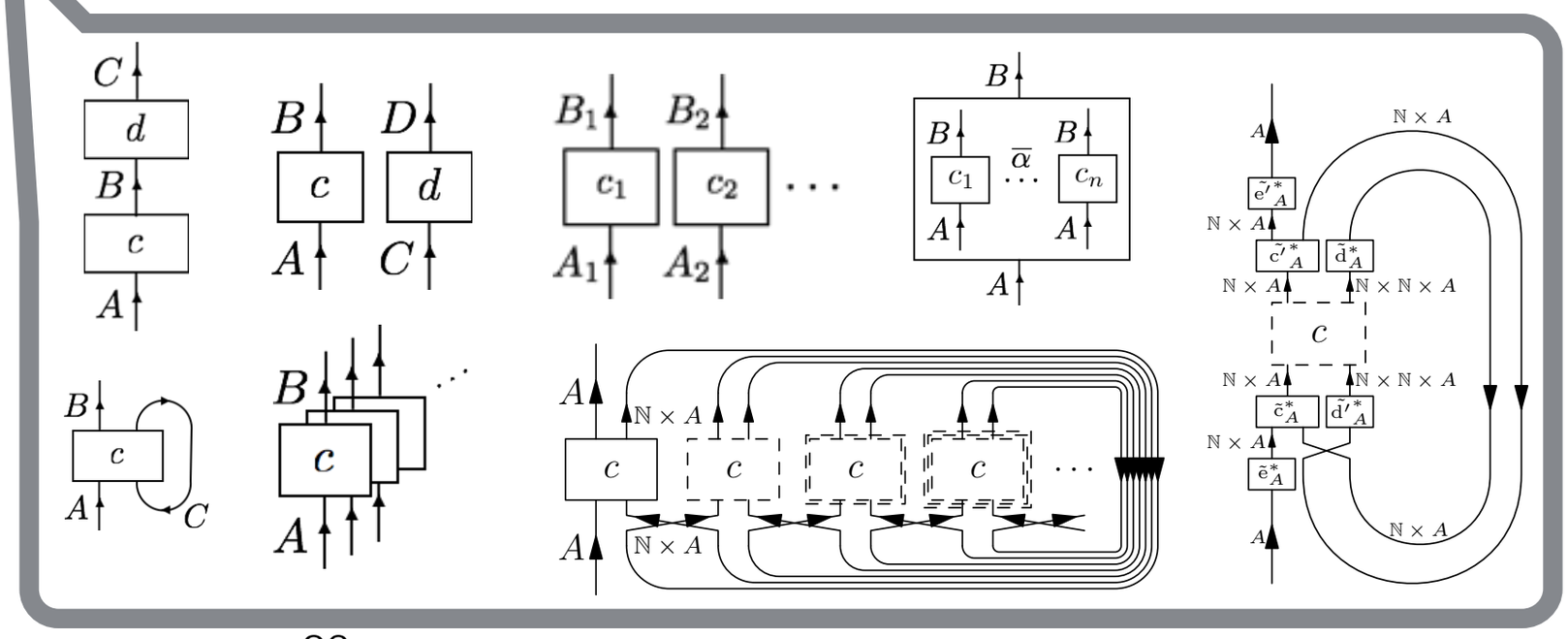
$$(\!|\Gamma \vdash \mathtt{M} : \tau|\!) \;=$$



adequate translation

transducers

via coalgebraic component calculus



Example: Recursive Probabilistic Program

$$(\texttt{rec}(\texttt{flipLoop}, \texttt{x}).\, \texttt{choose}_{0.4}(\texttt{x},\ \texttt{flipLoop}\,(\texttt{x}+1)))\ \ 0$$

coin flips	return value	probability
H	0	0.4
TH	1	$0.4 * 0.6^2$
TTH	2	$0.4 * 0.6^3$
TTTH	3	$0.4 * 0.6^4$

Example: Recursive Probabilistic Program

$$(\texttt{rec}(\texttt{flipLoop}, \texttt{x}).\, \texttt{choose}_{0.4}(\texttt{x},\ \texttt{flipLoop}\,(\texttt{x}+1)))\ \ \texttt{0}$$

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Example: Recursive Probabilistic Program

$$(\texttt{rec}(\texttt{flipLoop}, \texttt{x}).\, \texttt{choose}_{0.4}(\texttt{x},\ \texttt{flipLoop}\,(\texttt{x}+1)))\ \ \texttt{0}$$

$$\text{evaluation result of term } \texttt{M} = \begin{bmatrix} 0 & \mapsto & 0.4 \\ 1 & \mapsto & 0.4 \times 0.6 \\ 2 & \mapsto & 0.4 \times 0.6^2 \\ 3 & \mapsto & 0.4 \times 0.6^3 \\ & \vdots & \end{bmatrix} \in \mathcal{D}_{\leq 1}(\mathbb{N})$$

Example: Recursive Probabilistic Program

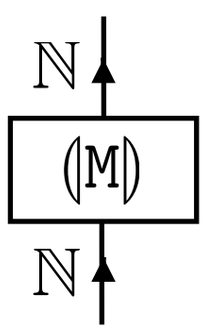
$$(\mathtt{rec}(\mathtt{flipLoop}, \mathtt{x}).\, \mathtt{choose}_{0.4}(\mathtt{x},\ \mathtt{flipLoop}\,(\mathtt{x}+\mathtt{1})))\ \mathtt{0}$$

evaluation result of term $\mathtt{M}$

$$= \begin{bmatrix} 0 & \mapsto & 0.4 \\ 1 & \mapsto & 0.4 \times 0.6 \\ 2 & \mapsto & 0.4 \times 0.6^2 \\ 3 & \mapsto & 0.4 \times 0.6^3 \\ & \vdots & \end{bmatrix} \in \mathcal{D}_{\leq 1}(\mathbb{N})$$

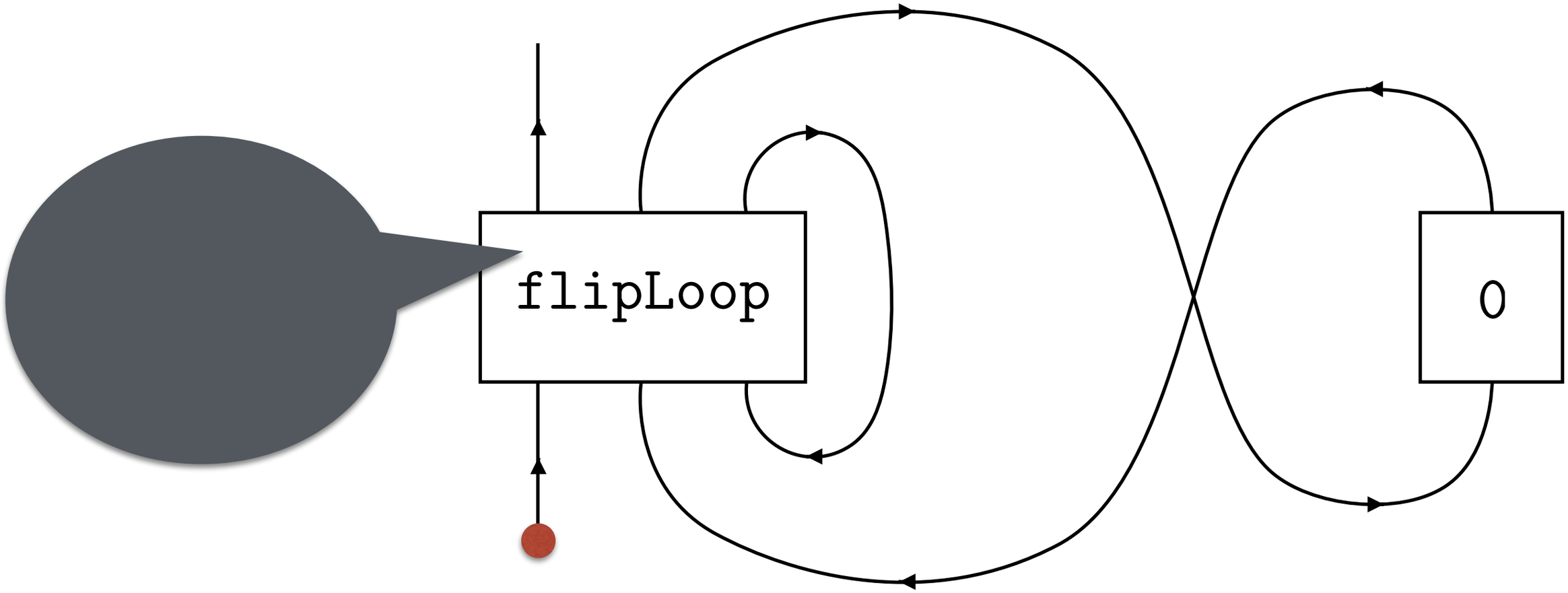
||

output of transducer



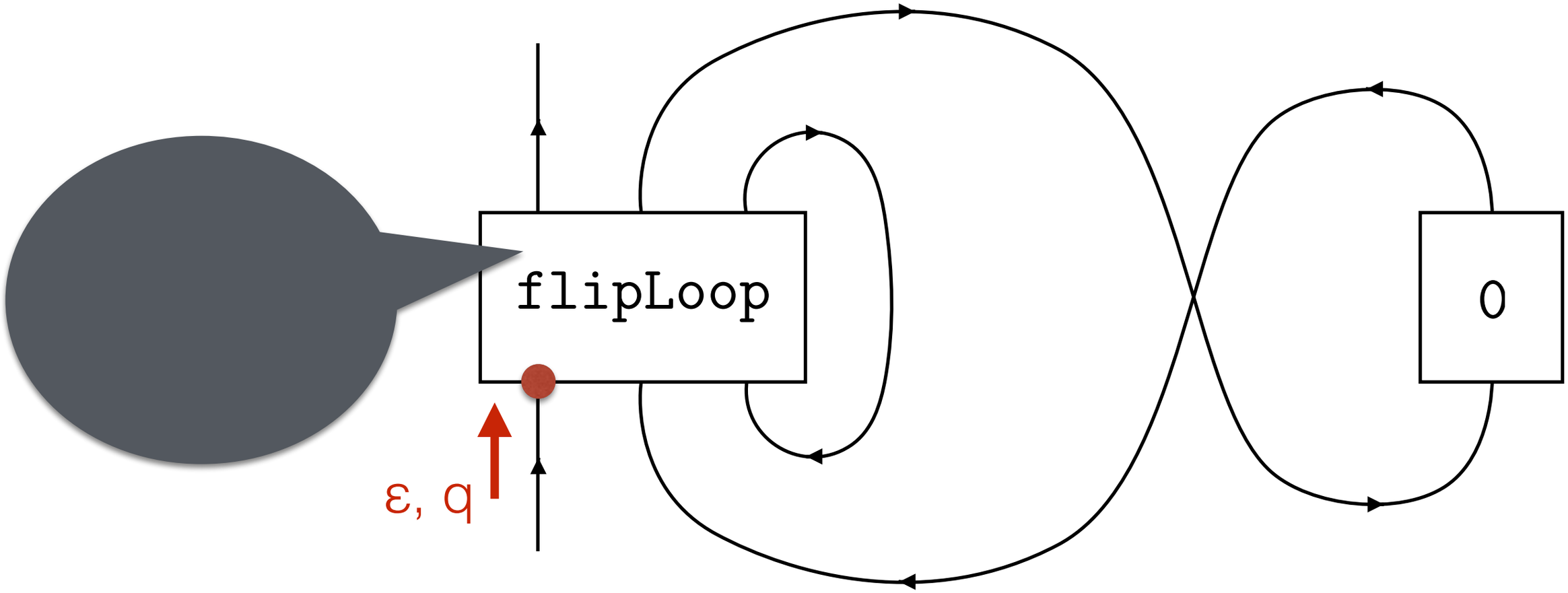
Example: Recursive Probabilistic Program

$$(\texttt{rec}(\texttt{flipLoop}, \texttt{x}).\, \texttt{choose}_{0.4}(\texttt{x},\ \texttt{flipLoop}\,(\texttt{x}+1)))\ \ \texttt{0}$$



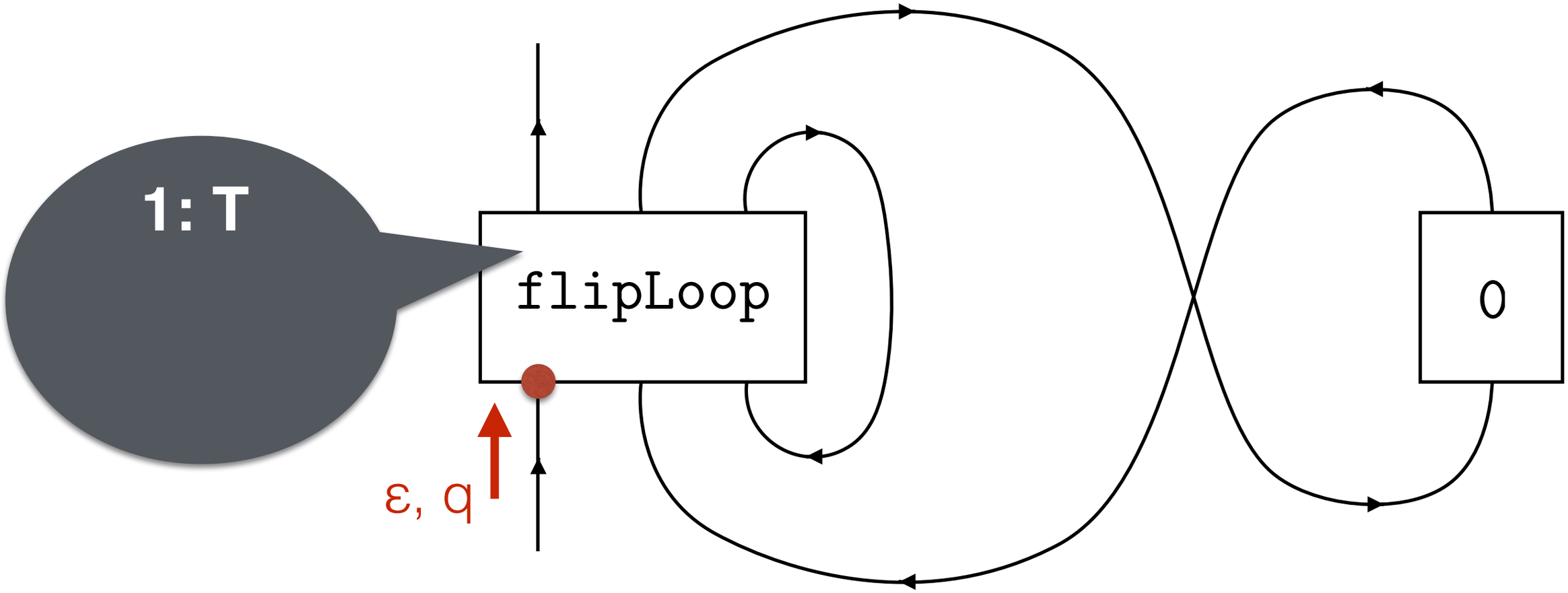
Example: Recursive Probabilistic Program

$$(\texttt{rec}(\texttt{flipLoop}, \texttt{x}).\, \texttt{choose}_{0.4}(\texttt{x},\ \texttt{flipLoop}\,(\texttt{x}+1)))\ \ \texttt{0}$$



Example: Recursive Probabilistic Program

$$(\texttt{rec}(\texttt{flipLoop}, \texttt{x}).\, \texttt{choose}_{0.4}(\texttt{x},\ \texttt{flipLoop}\,(\texttt{x}+1)))\ \ 0$$



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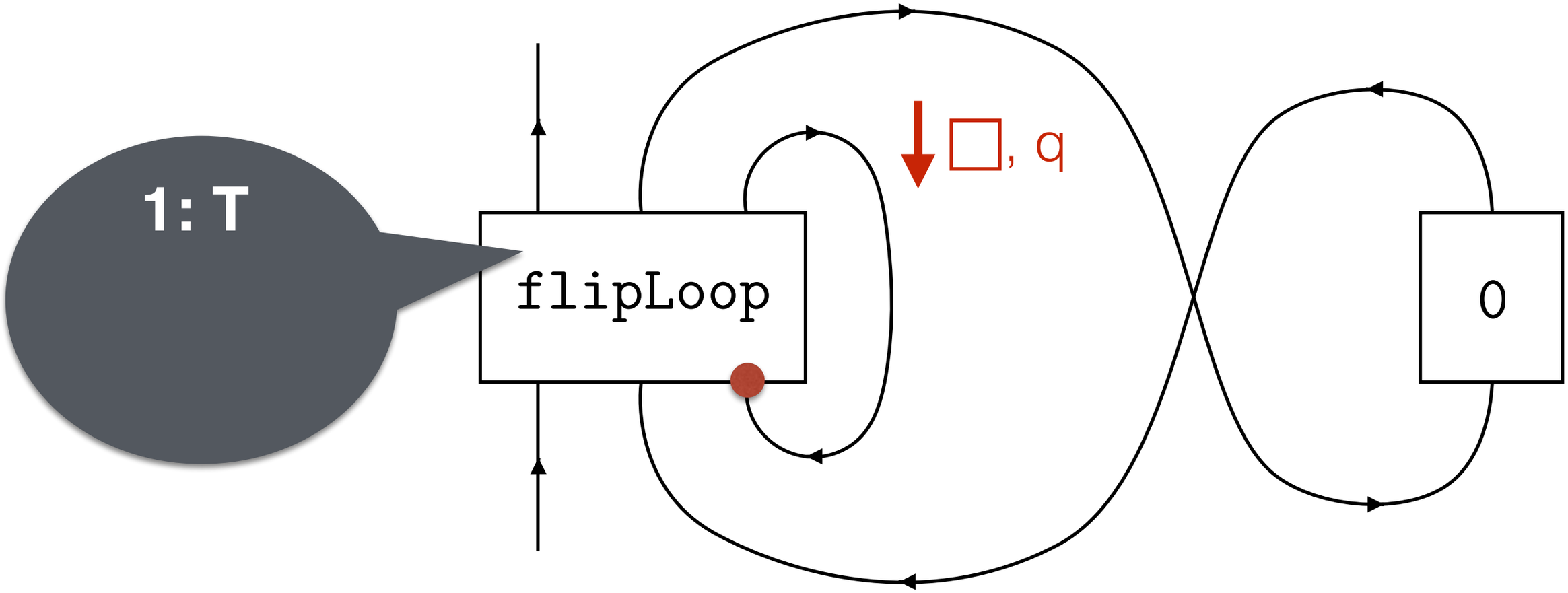
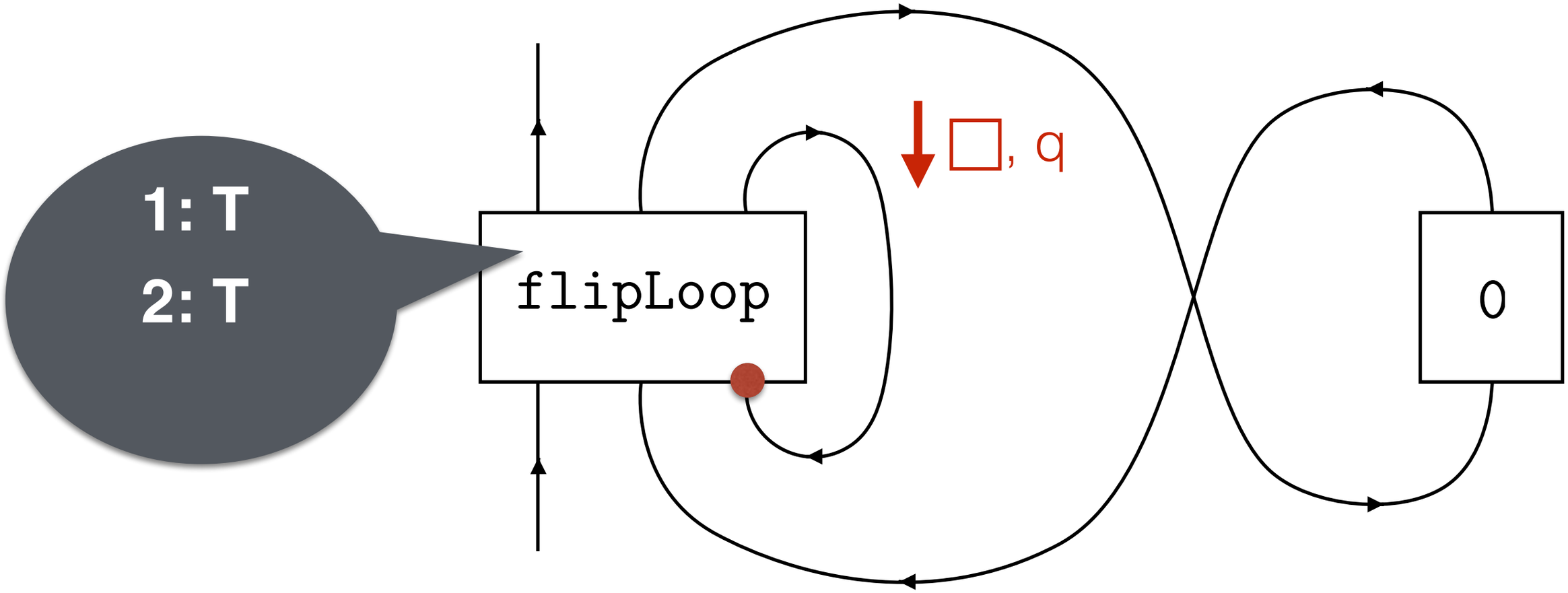


Example: Recursive Probabilistic Program

$$(\texttt{rec}(\texttt{flipLoop}, \texttt{x}).\, \texttt{choose}_{0.4}(\texttt{x},\ \texttt{flipLoop}\,(\texttt{x}+1)))\ \ 0$$



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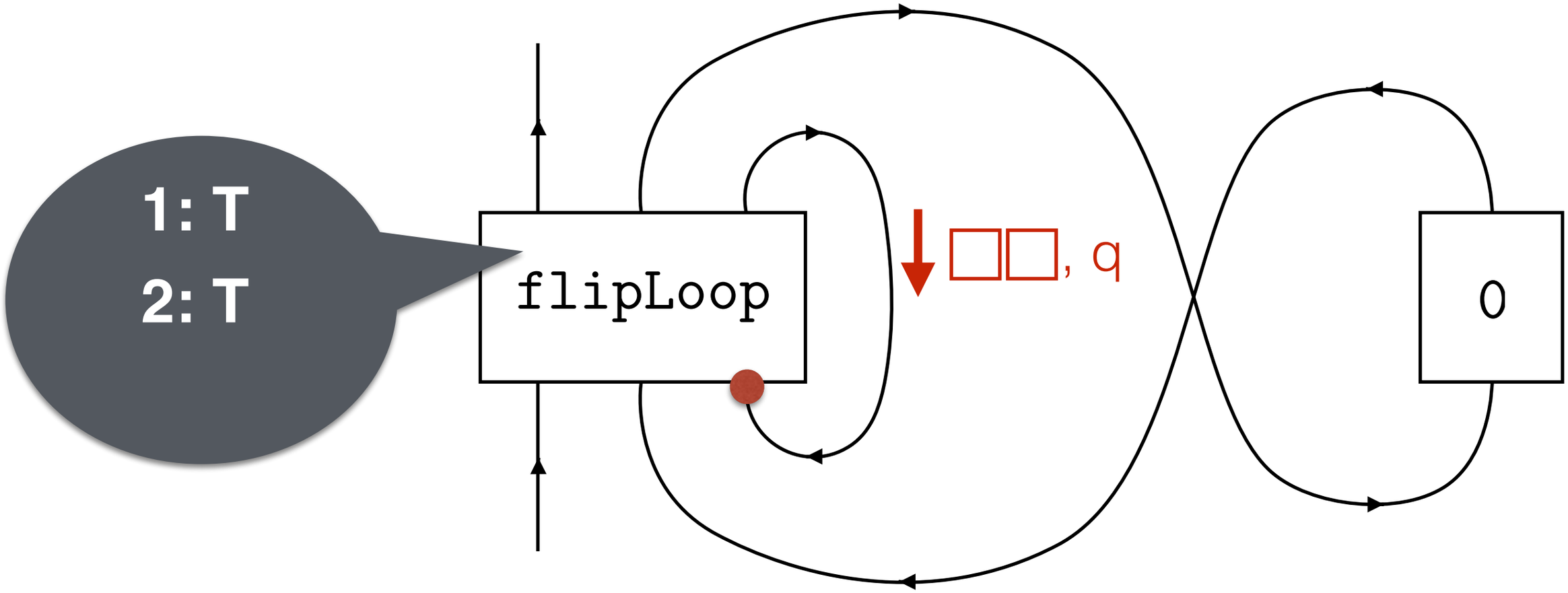

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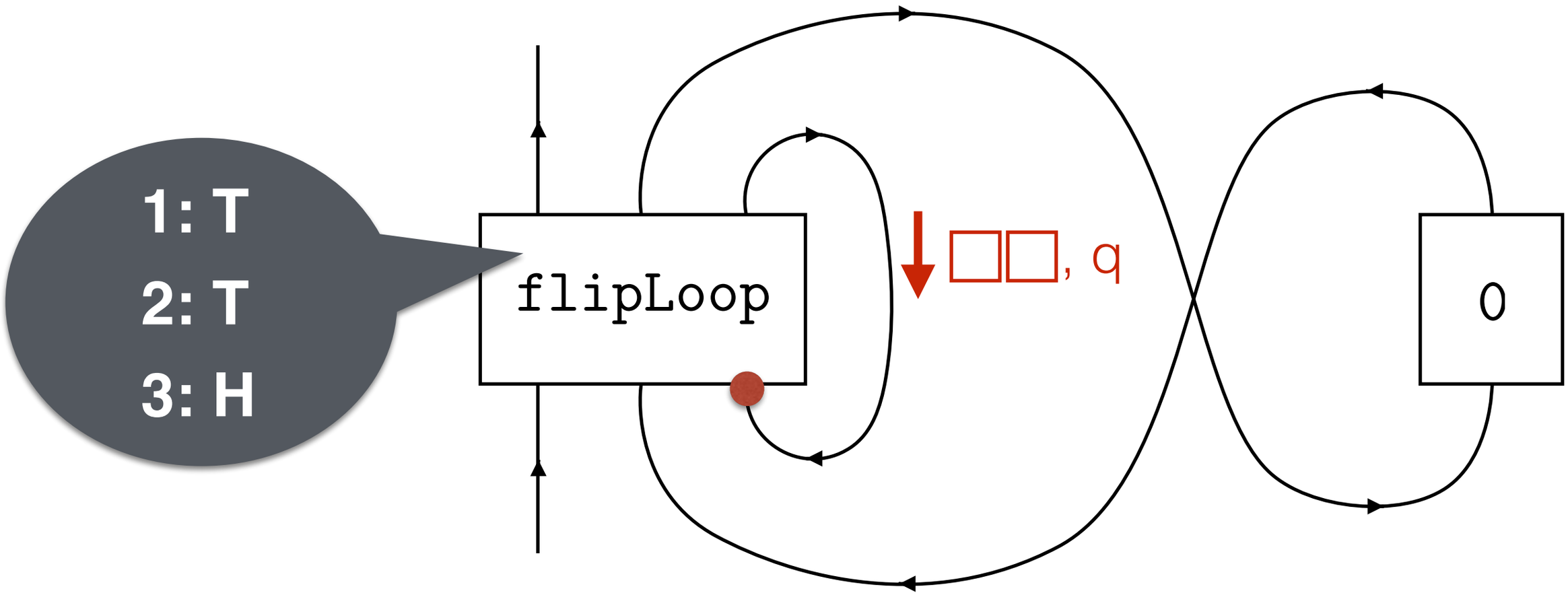

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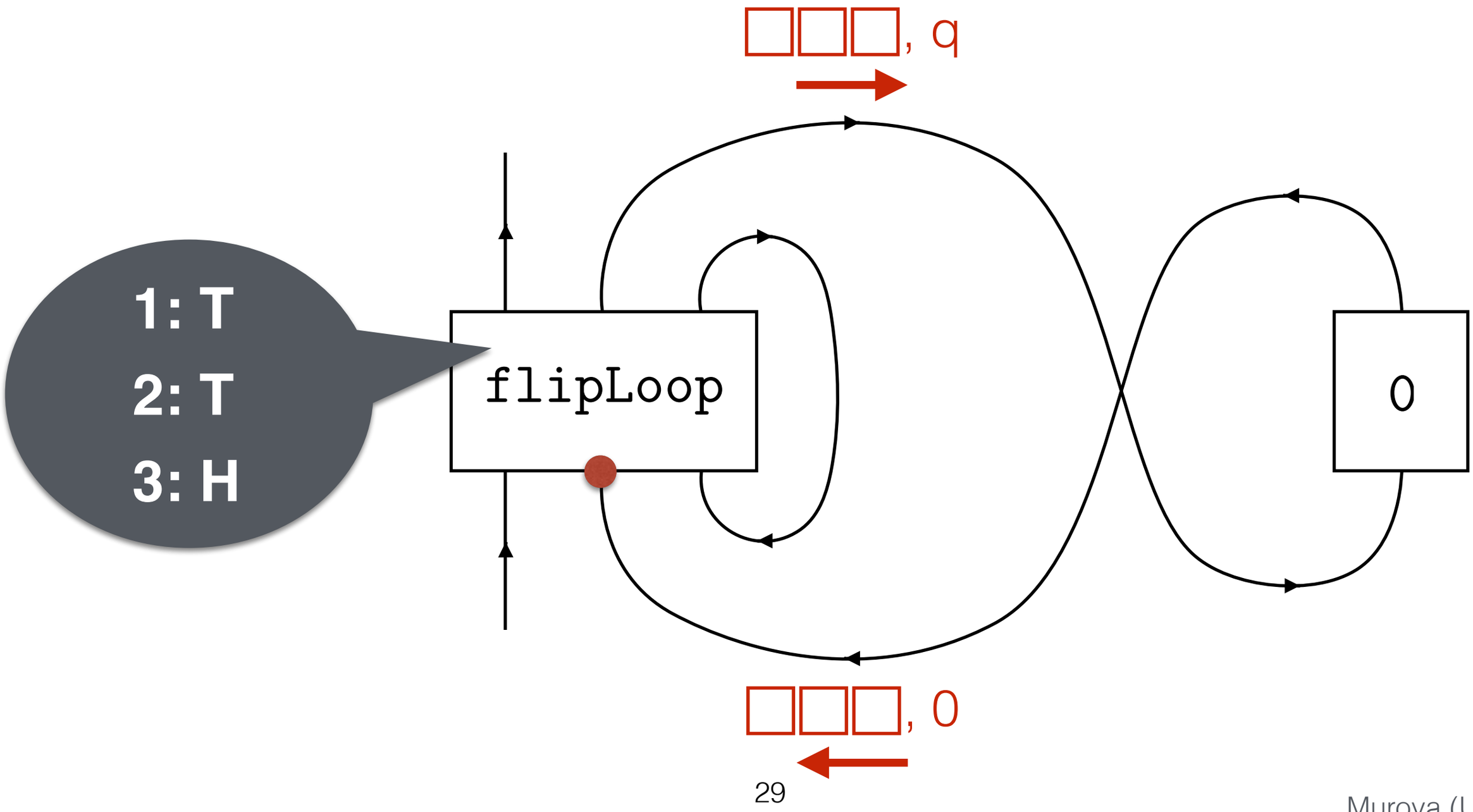
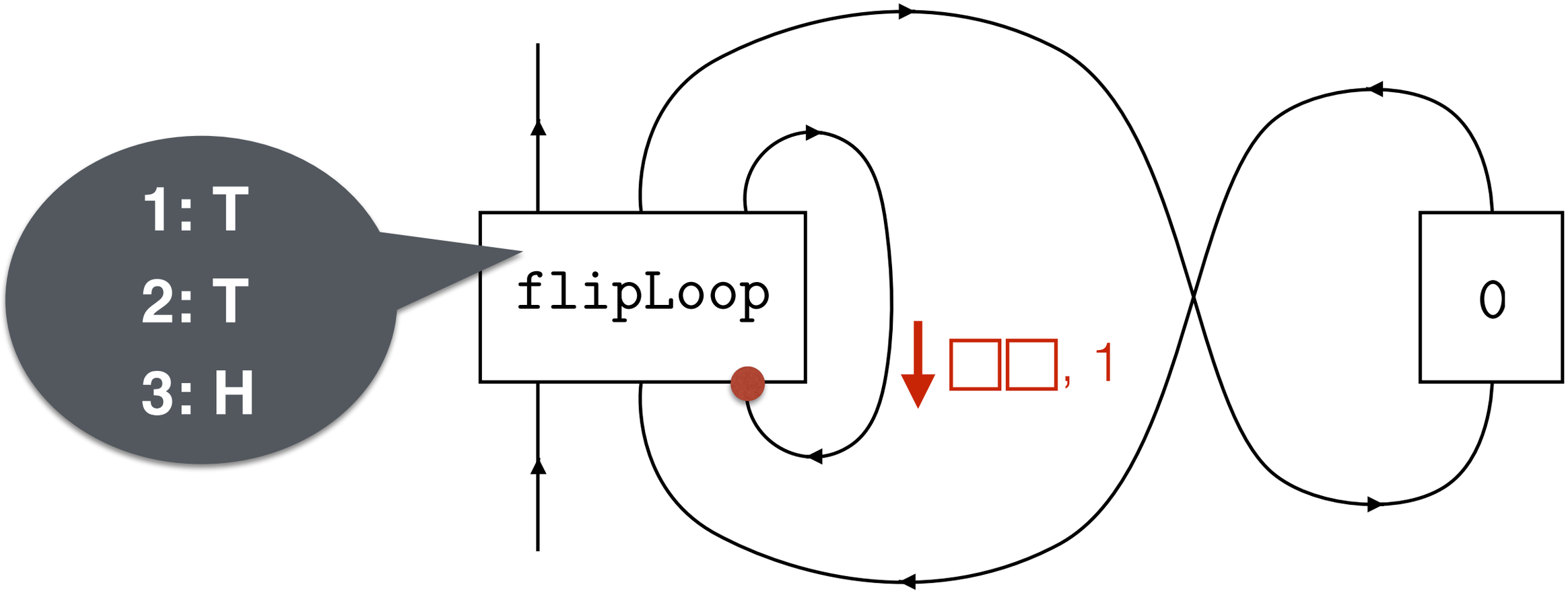


Example: Recursive Probabilistic Program

$$(\texttt{rec}(\texttt{flipLoop}, \texttt{x}).\, \texttt{choose}_{0.4}(\texttt{x},\ \texttt{flipLoop}\,(\texttt{x}+1)))\ \ \texttt{0}$$



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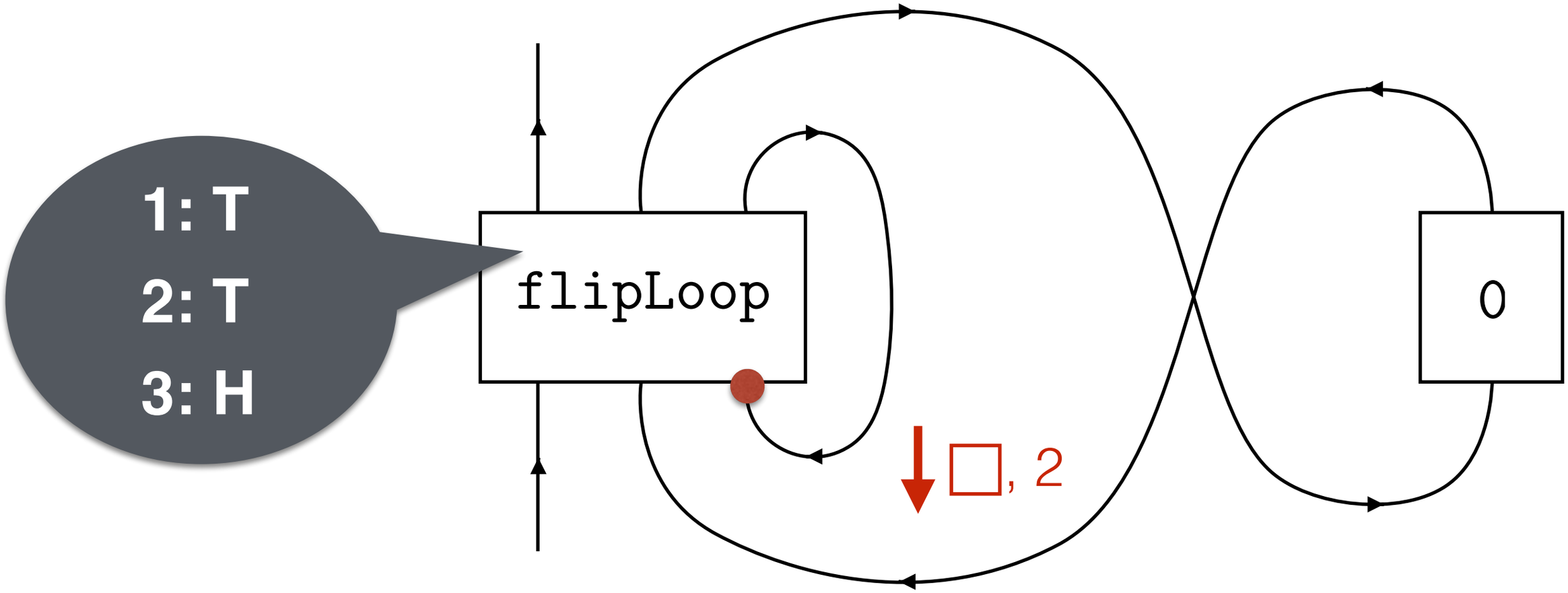

Example: Recursive Probabilistic Program

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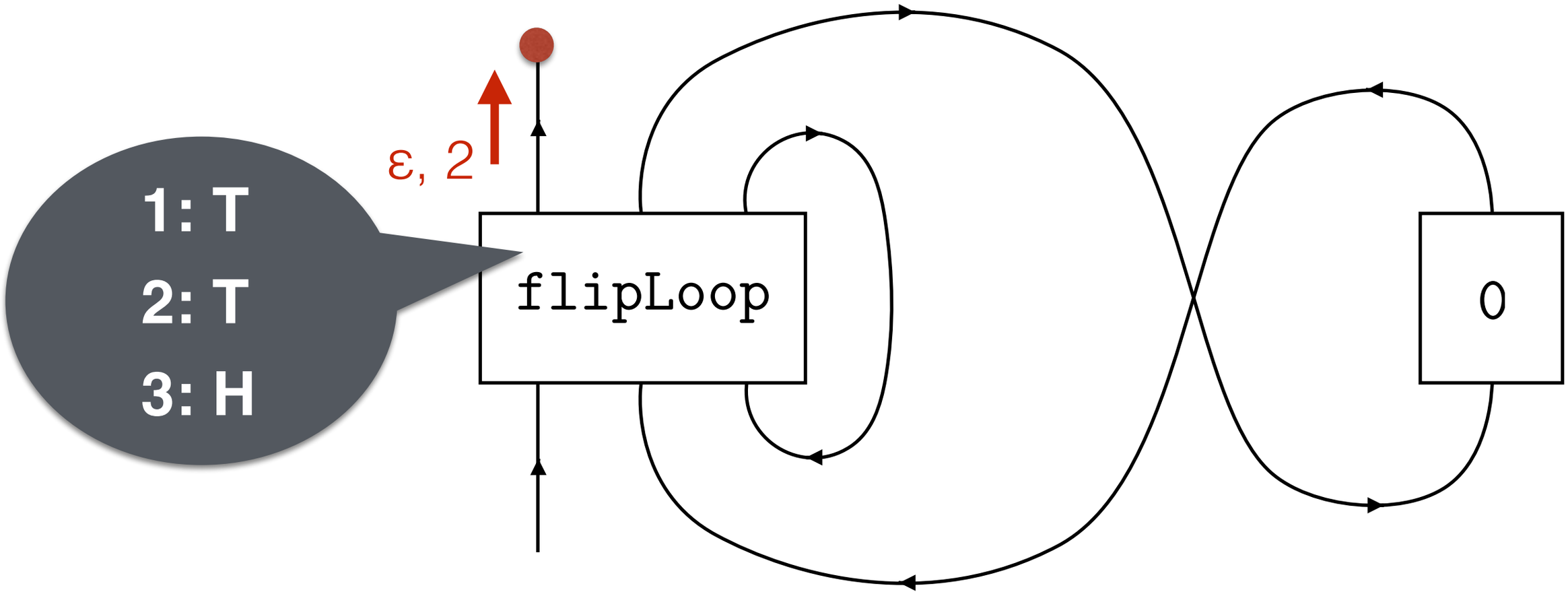
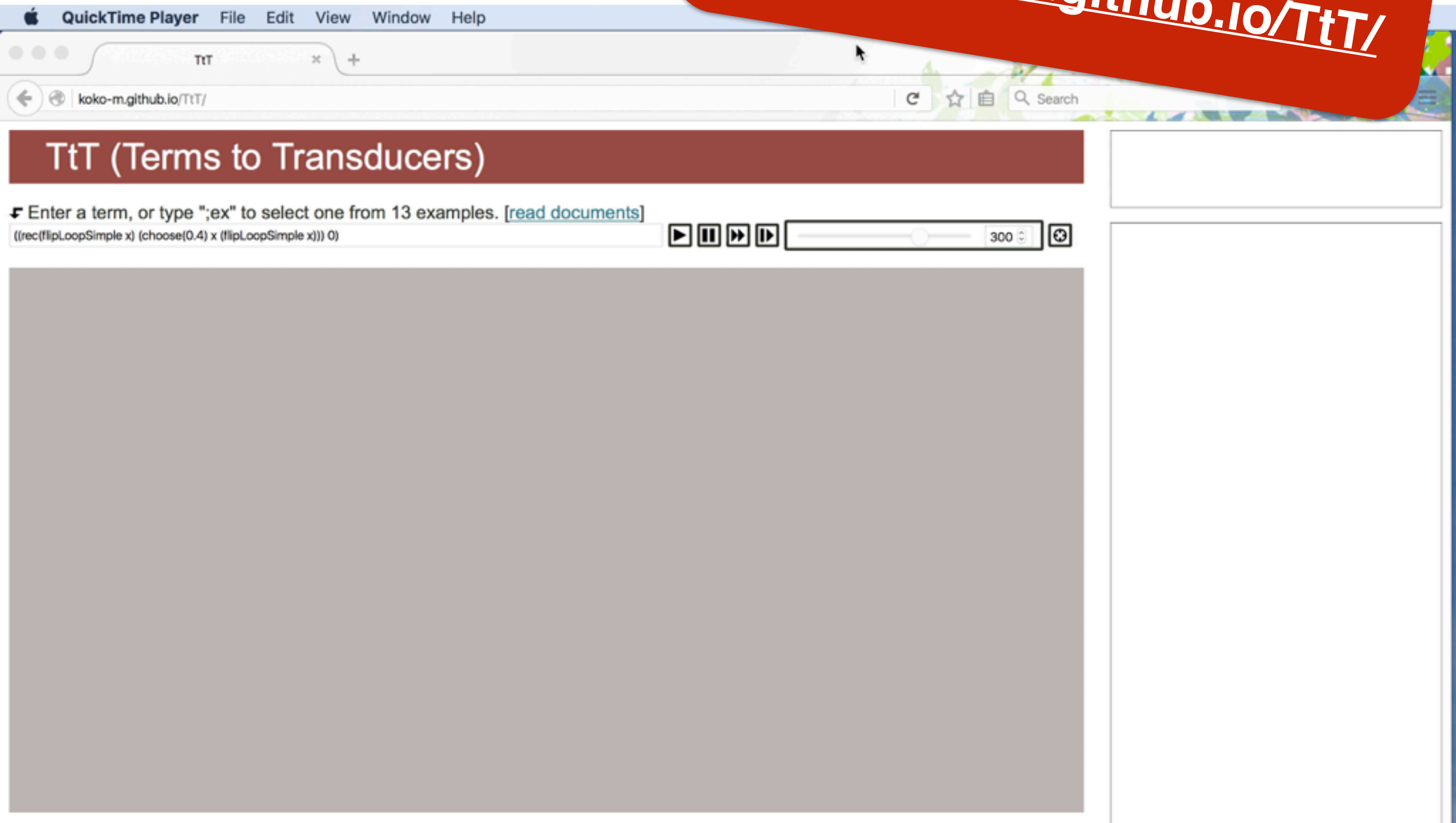


Our Tool ***TtT***

http://koko-m.github.io/TtT/



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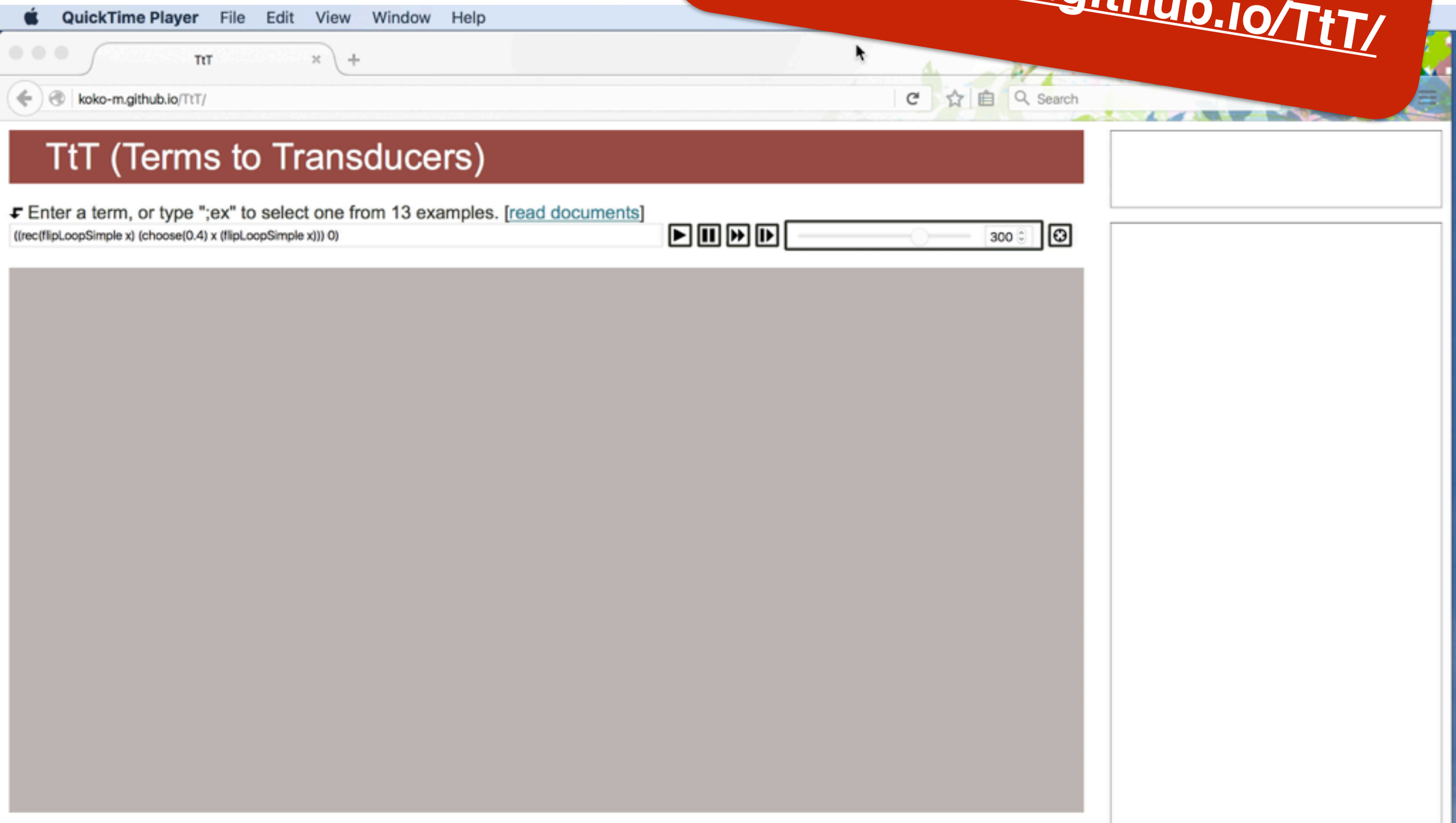

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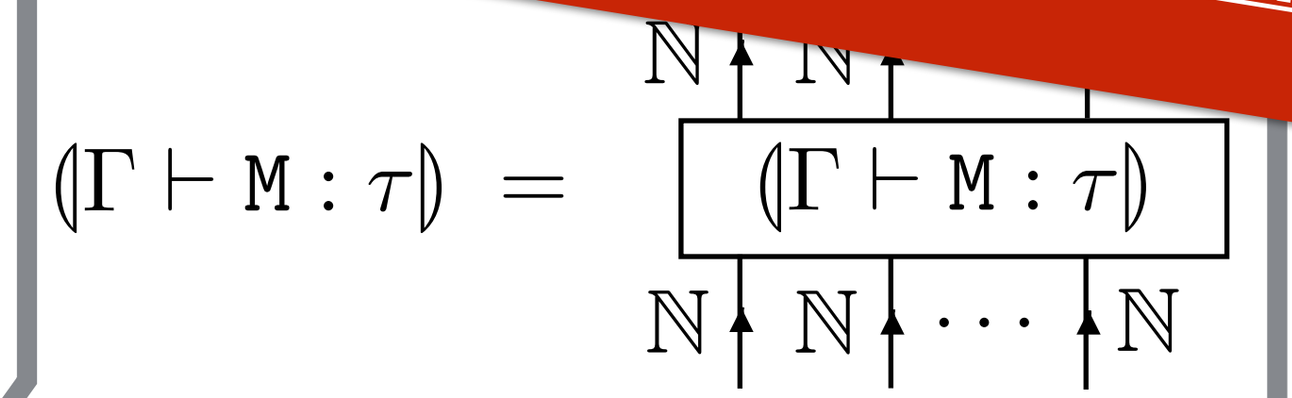

adequate translation

via coalgebraic component calculus