Coalgebraic Trace Semantics
for
Büchi and Parity Automata

Natsuki Urabe, Shunsuke Shimizu and Ichiro Hasuo
The University of Tokyo

Overview

- Preliminary I:
  Behavioral Domain via Final Coalgebra
- Preliminary II:
  Coalgebraic Finite & Infinitary Trace Semantics
- Main Result:
  Coalgebraic Trace Semantics for Büchi & Parity Automata
- Related Work, Conclusions & Future Work

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Coalgebra

- Categorical model for state-based dynamics
- Numerous theoretical & practical results
- An arrow $c : X \to FX$ in a category $\mathbb{C}$
  - $F : \mathbb{C} \to \mathbb{C}$

Examples

• Deterministic Automaton	$c : X \to X^{\Sigma} \times \{0,1\}$
• Nondeterministic Automaton	$c : X \to \mathcal{P}(\{\checkmark\} + \Sigma \times X)$
• Mealy Machine	$c : X \to (B \times X)^{A}$

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- Bisimulation

Span

$$\begin{array}{ccccc} FX & \xleftarrow{F\pi_1} & FR & \xrightarrow{F\pi_2} & FY \\ \uparrow c & = & \uparrow e & = & \uparrow d \\ X & \xleftarrow{\pi_1} & R & \xrightarrow{\pi_2} & Y \end{array}$$

- Behavioral Equivalence

Cospan

$$\begin{array}{ccccc} FX & \xrightarrow{Ff} & FE & \xleftarrow{Fg} & FY \\ \uparrow c & = & \uparrow e & = & \uparrow d \\ X & \xrightarrow{f} & E & \xleftarrow{g} & Y \end{array}$$

Behavioral Domain via Final Coalgebra

(see e.g. [J. Rutten, TCS '00], [Jacobs,'12])

Example:

Deterministic Automaton

$$\mathcal{A} = (X, \Sigma, \delta, \mathsf{Acc})$$

where $\delta : X \times \Sigma \to X$

in $\mathbf{Sets}$

$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array}$$

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final

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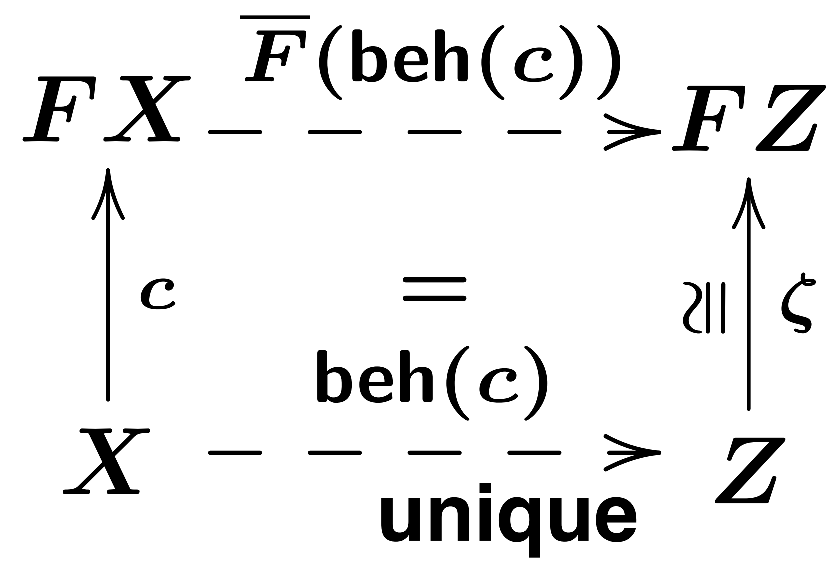

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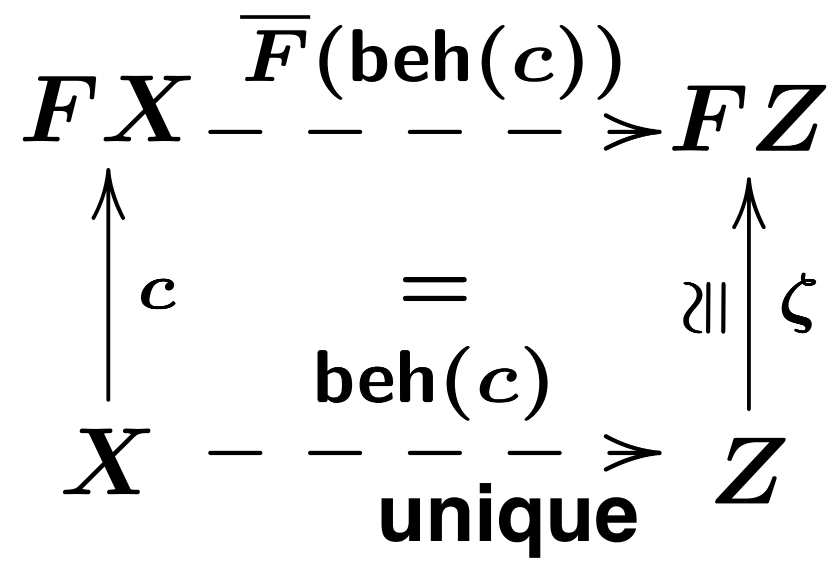

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- For many F, a ***final coalgebra*** exists

Behavioral Domain via Final Coalgebra

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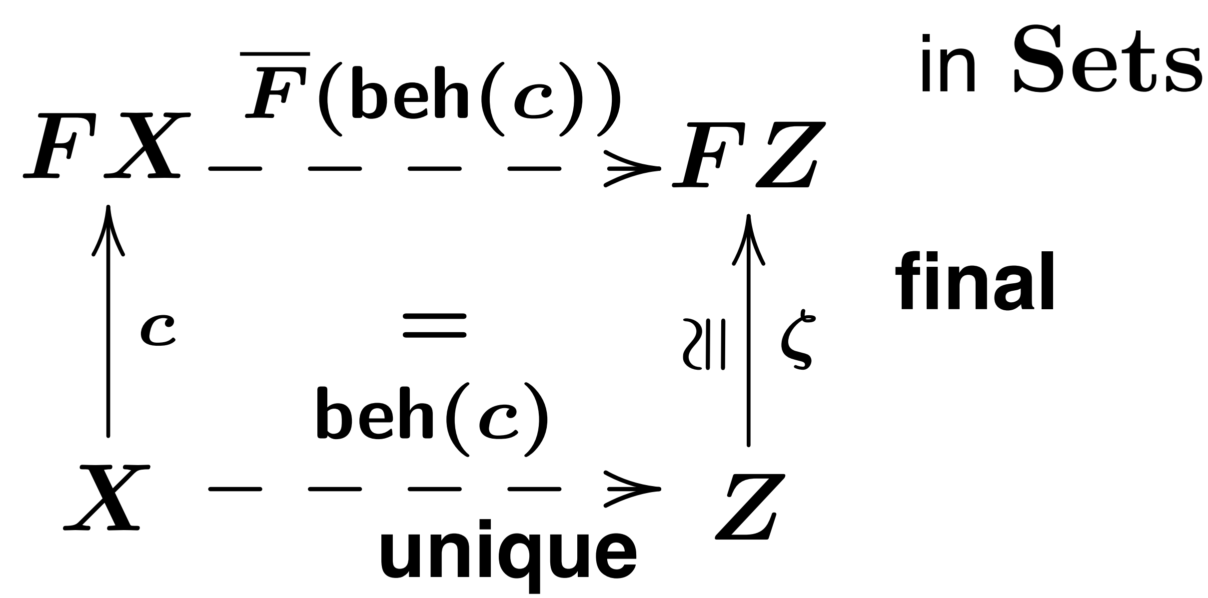

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- For many F, a ***final coalgebra*** exists
- For many systems, the unique $\mathsf{beh}(c)$ describes the behavior

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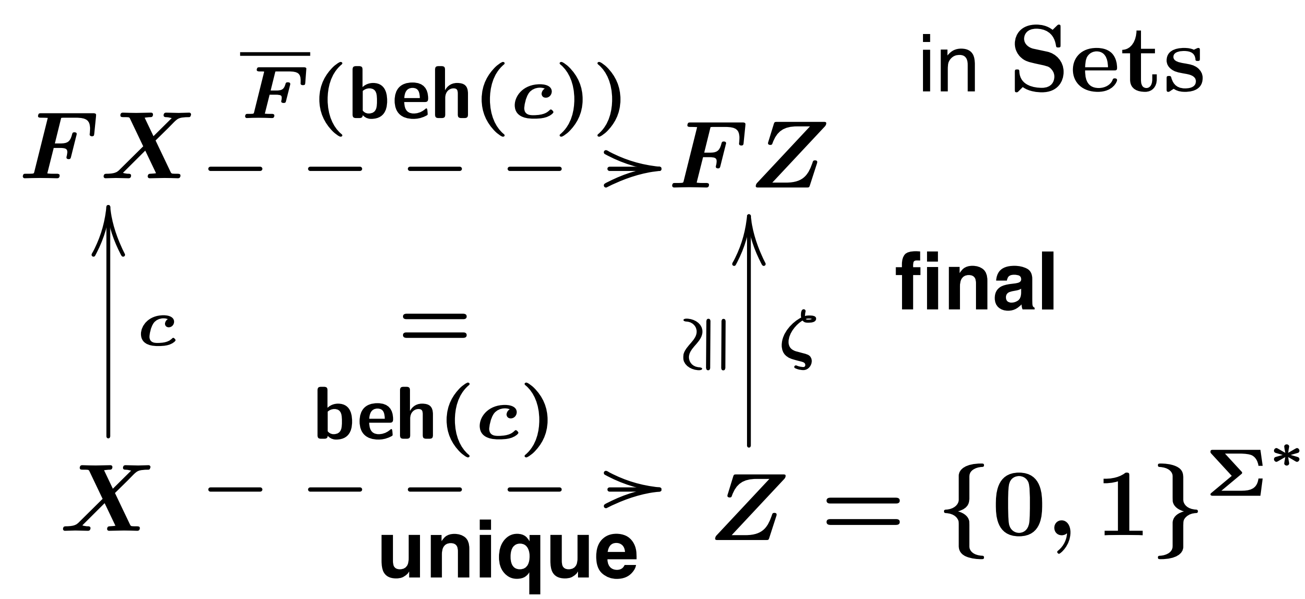

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Thm:

$\mathsf{beh}(c)(x)(w) = 1$ iff w is accepted by $\mathcal{A}$

- For many F, a ***final coalgebra*** exists
- For many systems, the unique $\mathsf{beh}(c)$ describes the behavior

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Final Coalgebra in Kleisli Category

[Power & Turi, CTCS ’99], [Hasuo et al., ’07]

in $\mathbf{Sets}$

Nondeterministic Automaton

$$\mathcal{A} = (X, \Sigma, \delta, \mathsf{Acc})$$

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$$\begin{array}{c} FX \\ \uparrow c \\ X \end{array}$$

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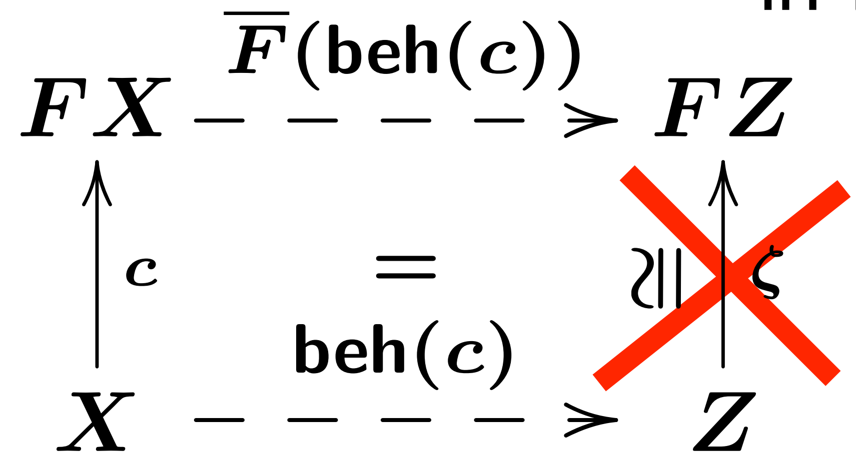

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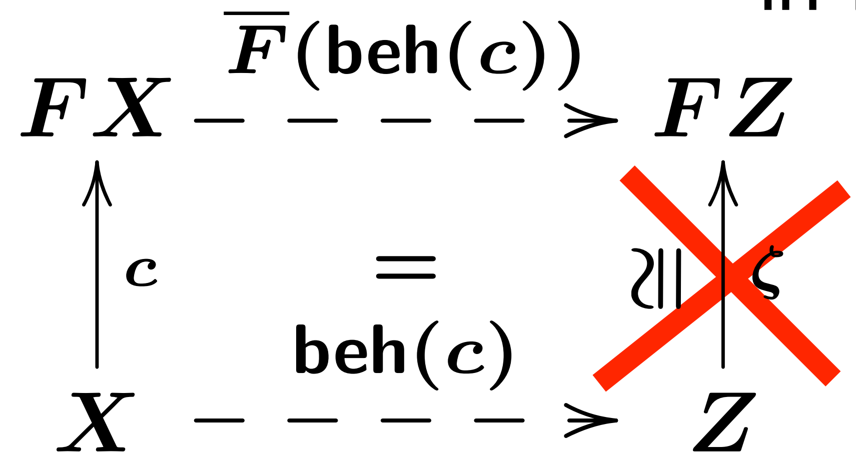

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- $\mathcal{P}$ is a **monad**

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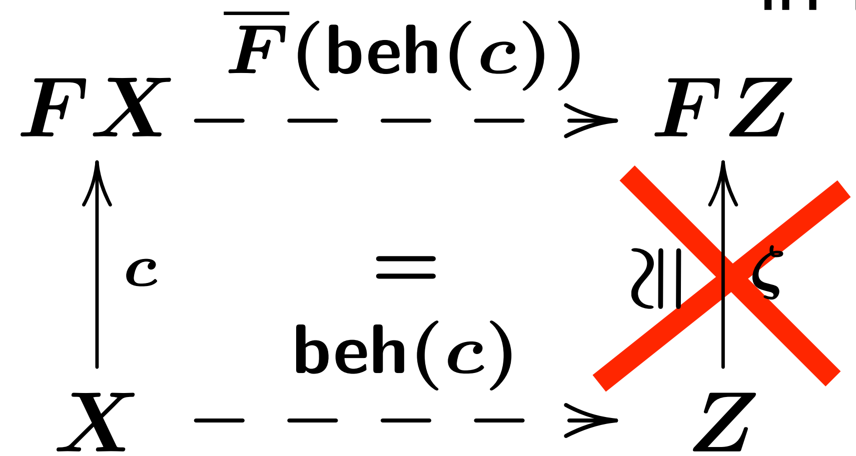

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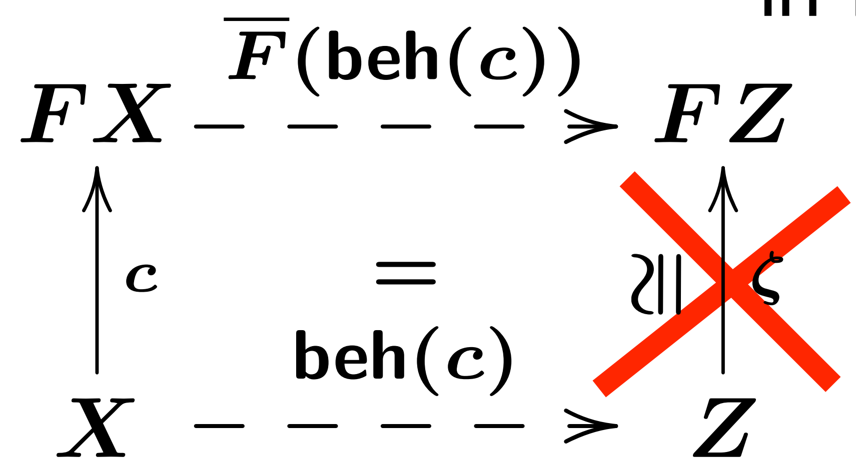

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- $\mathcal{P}$ is a **monad** → **Kleisli category** $\mathcal{K}\ell(\mathcal{P})$

$$\frac{f : X \nrightarrow Y \text{ in } \mathcal{K}\ell(\mathcal{P})}{f : X \to \mathcal{P}Y \text{ in } \mathbf{Sets}}$$

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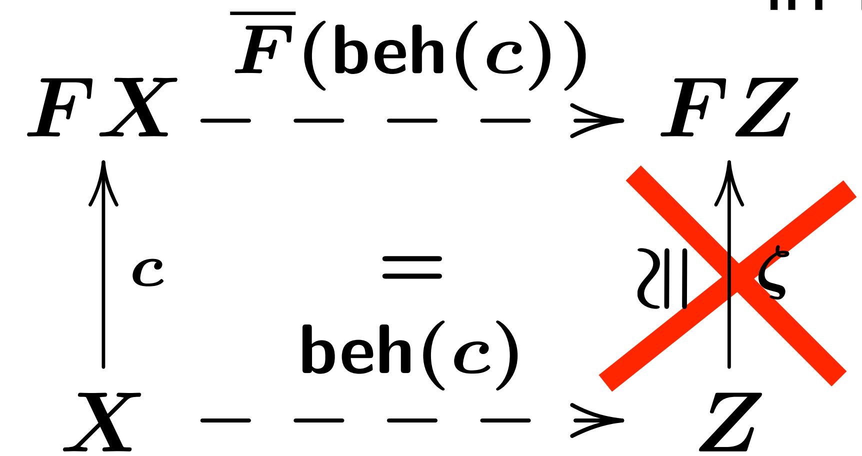

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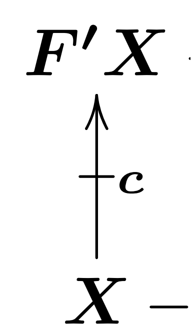

where $F' := \{\checkmark\} + \Sigma \times (\_)$

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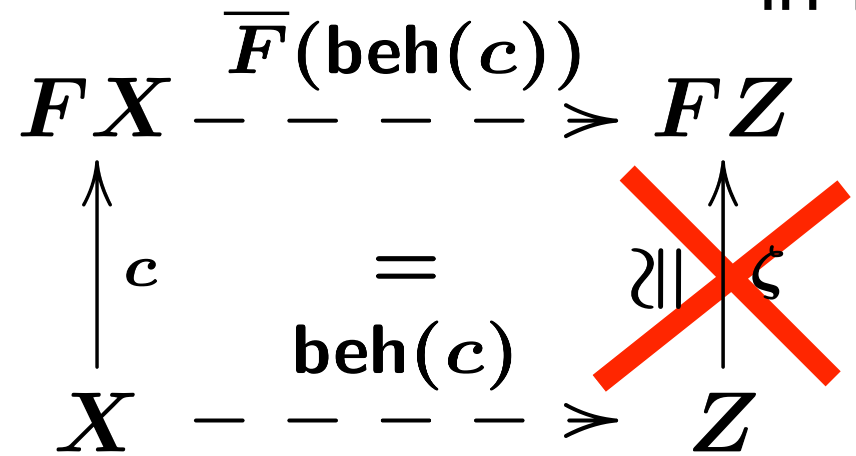

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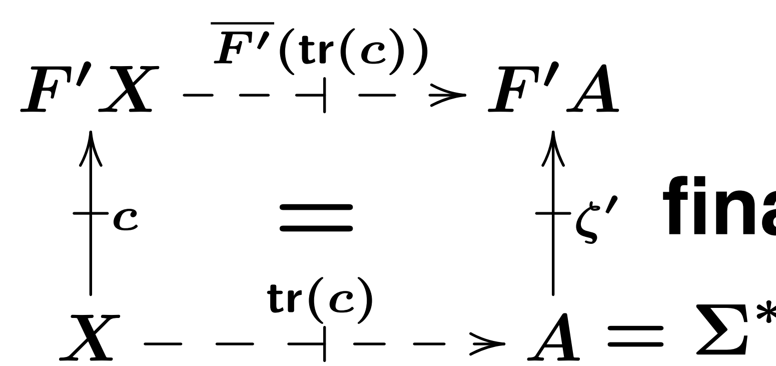

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Finite Trace Semantics

in $\mathcal{K}\ell(\mathcal{P})$

$$
\begin{array}{ccc}
F'X & \xrightarrow{\overline{F'}(\mathsf{tr}(c))} & F'A \\
\uparrow c & = & \cong \uparrow \zeta' \ \textbf{final} \\
X & \xrightarrow{\mathsf{tr}(c)} & A = \Sigma^*
\end{array}
$$

$$
\frac{\mathsf{tr}(c) : X \nrightarrow \Sigma^* \quad \text{in } \mathcal{K}\ell(\mathcal{P})}{\mathsf{tr}(c) : X \to \mathcal{P}\Sigma^* \quad \text{in } \mathbf{Sets}}
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Thm:

$\mathsf{tr}(c)$ characterizes **finite trace** $L(\mathcal{A})$

Def.

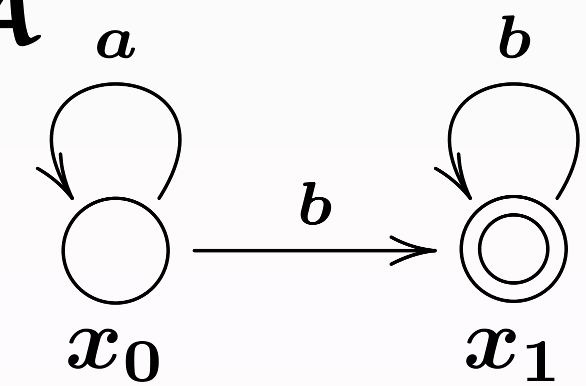
For $\mathcal{A} = (X, \Sigma, \delta, \mathsf{Acc})$,

finite trace semantics

$$L(\mathcal{A})(x) := \left\{ a_0 \ldots a_{n-1} \in \Sigma^* \;\middle|\; x = x_0 \xrightarrow{a_0} \cdots \xrightarrow{a_{n-1}} x_n \in \mathsf{Acc} \right\}$$

Example:

$\mathcal{A}$



$L(\mathcal{A})(x_0) = a^* b b^*$

Extension to Various Systems

in $\mathcal{K}\ell(\mathcal{P})$

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final where

$$F' := \{\checkmark\} + \Sigma \times (\_\,)$$

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- $F' = \{\checkmark\} + \Sigma \times (\_)$ $\quad$ $F' = \coprod_i \Sigma_i \times (\_)^i$ (polynomial functor)
  - **Words** to **Trees**
- $T = \mathcal{P}$

Extension to Various Systems

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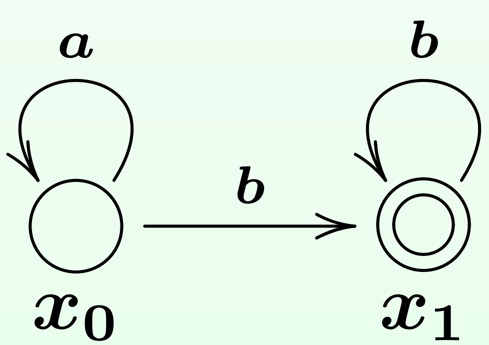
- $F' = \{\checkmark\} + \Sigma \times (\_) \Rightarrow F' = \coprod_i \Sigma_i \times (\_)^i$ (polynomial functor)
  - **Words** to **Trees**
- $T = \mathcal{P} \Rightarrow T = \mathcal{G}$ (the sub-Giry monad)
  - **Nondeterministic** to (generative) **Probabilistic**

Coalgebraic Finite Trace Semantics

Finite Trace

$$L(\mathcal{A})(x) := \left\{ a_0 \dots a_{n-1} \in \Sigma^* \;\middle|\; x = x_0 \xrightarrow{a_0} \cdots \xrightarrow{a_{n-1}} x_n \in \mathsf{Acc} \right\}$$

Example:



$$L(\mathcal{A}) = a^* b b^*$$

$$F = \{\checkmark\} + \Sigma \times (\_)$$

in $\mathcal{K}\ell(\mathcal{P})$

$$\begin{array}{ccc} FX & \xrightarrow{\overline{F}(\mathsf{tr}(c))} & F\Sigma^* \\ \uparrow c & = & \wr\!\| \uparrow \zeta' \\ X & \xrightarrow{\mathsf{tr}(c)} & \Sigma^* \end{array}$$

final

unique

Thm:

$\mathsf{tr}(c)$ characterizes **finite trace** $L(\mathcal{A})$

Coalgebraic Infinitary Trace Semantics

[Jacobs, '04]

Infinitary Trace

$$L^{\infty}(\mathcal{A})(x) := L(\mathcal{A})(x)$$

$$\cup \left\{ \begin{array}{r} a_0 a_1 \dots \\ \in \Sigma^{\omega} \end{array} \middle| \begin{array}{r} x = x_0 \xrightarrow{a_0} x_1 \\ \xrightarrow{a_1} \dots \end{array} \right\}$$

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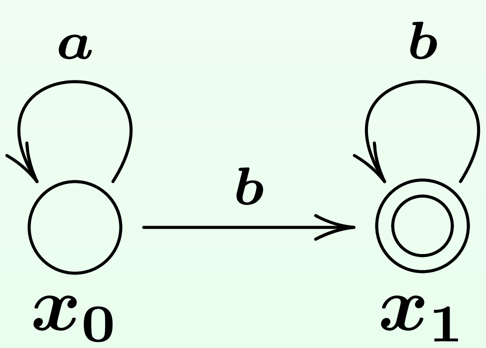

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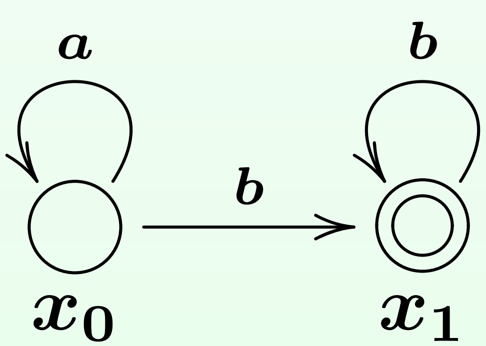

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$\mathcal{K}\ell(\mathcal{P})$

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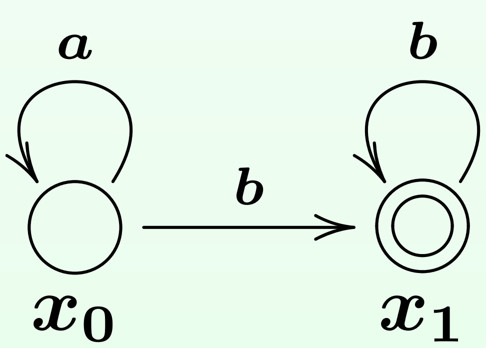

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$\mathcal{K}\ell(\mathcal{P})$

$$\begin{array}{ccc} FX & & F\Sigma^* \\ \uparrow c & & \cong \uparrow \zeta' \\ X & & \Sigma^* \end{array}$$

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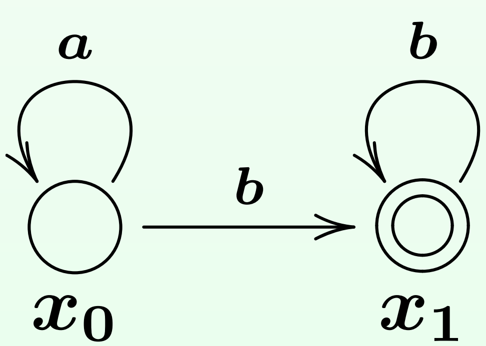

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$\mathcal{K}\ell(\mathcal{P})$ | in **Sets**

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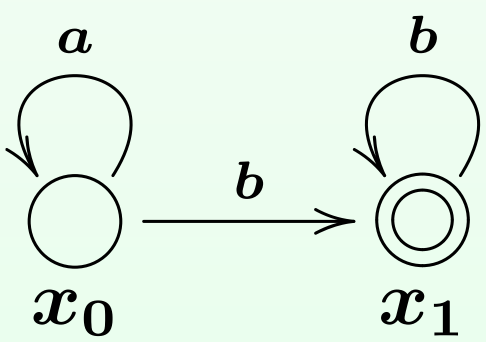

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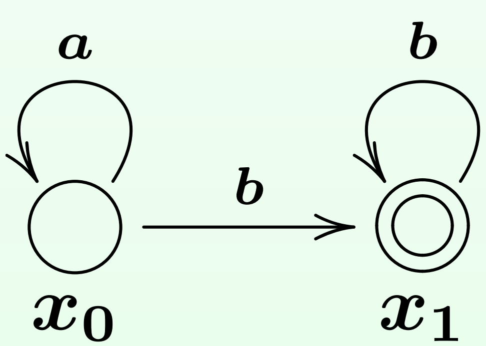

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$\mathcal{K}\ell(\mathcal{P})$

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weakly final

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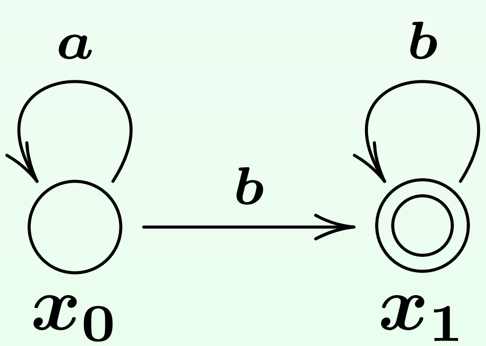

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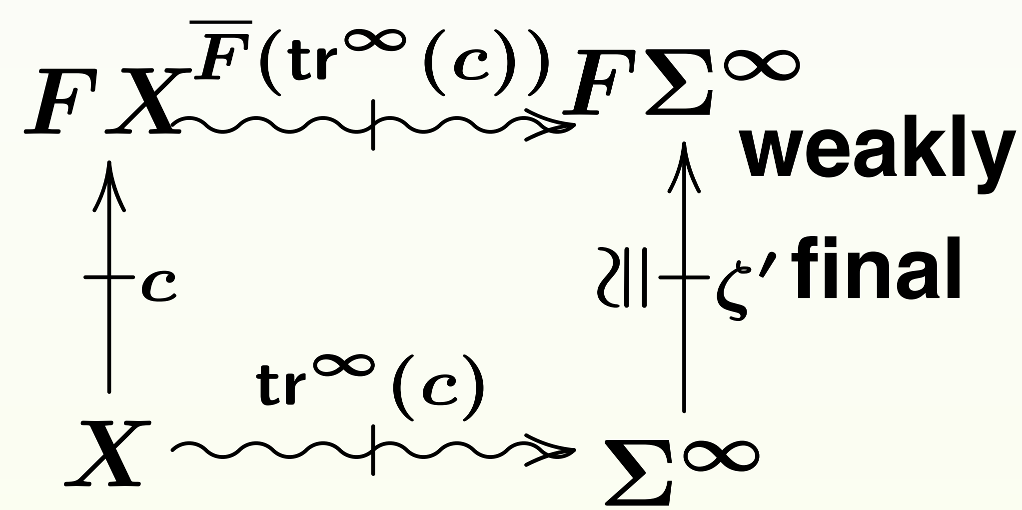

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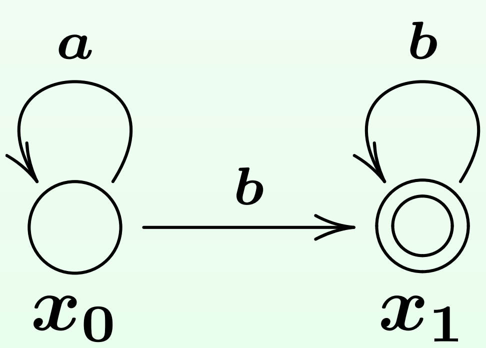

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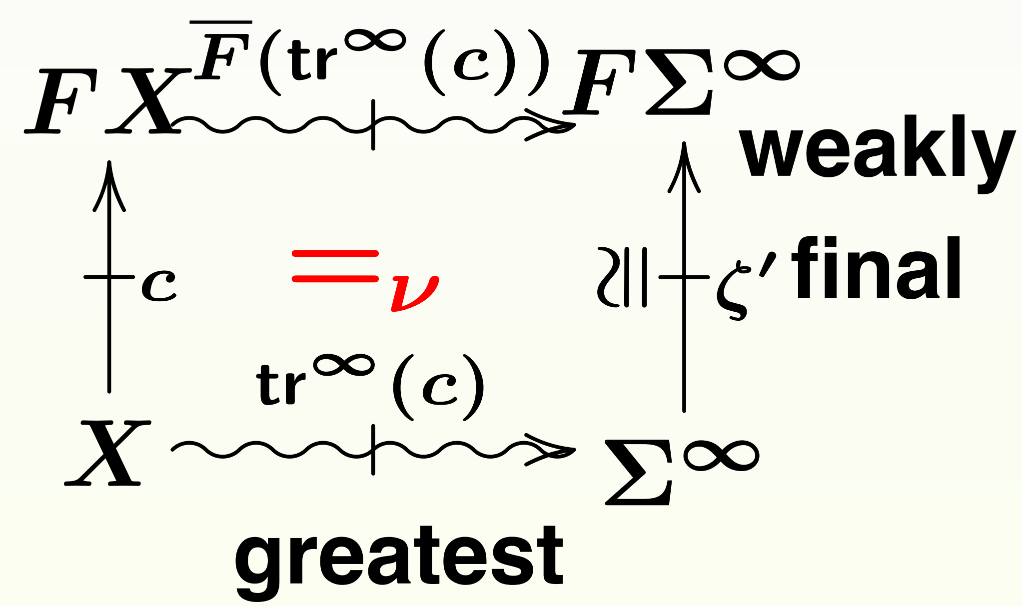


Coalgebraic Infinitary Trace Semantics

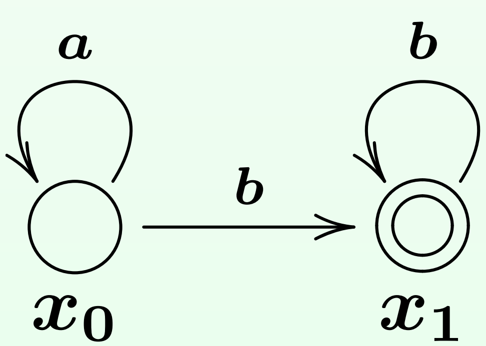
[Jacobs, '04]

Infinitary Trace

$$L^{\infty}(\mathcal{A})(x) := L(\mathcal{A})(x)$$

$$\cup \left\{ a_0 a_1 \ldots \in \Sigma^{\omega} \;\middle|\; x = x_0 \xrightarrow{a_0} x_1 \xrightarrow{a_1} \ldots \right\}$$

Example:



$$L^{\infty}(\mathcal{A})(x_0) = a^* b b^* + a^{\omega} + a^* b^{\omega}$$

$F = \{\checkmark\} + \Sigma \times (\_)$

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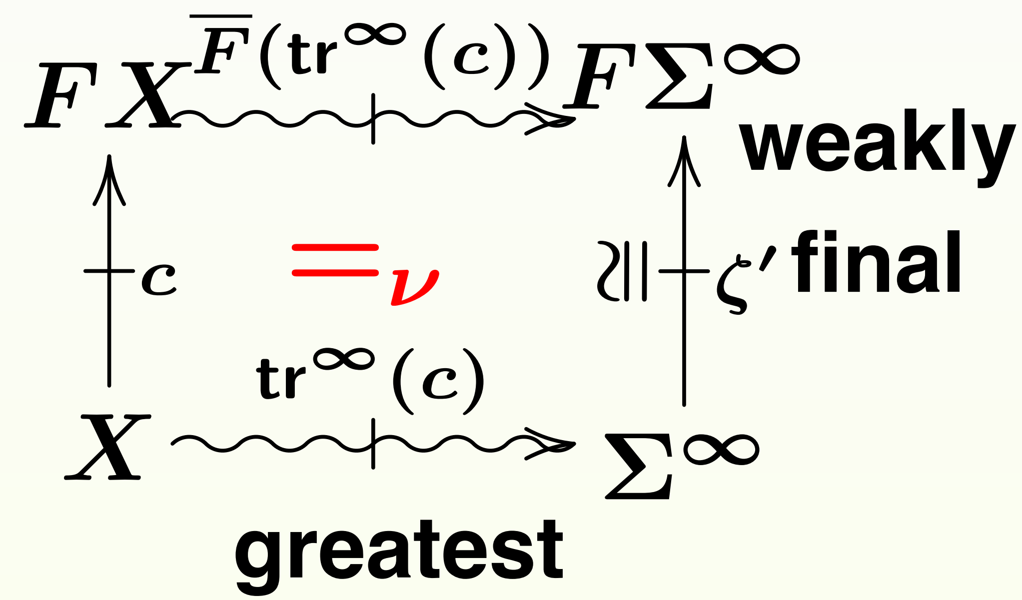

$$\frac{f : X \nrightarrow \Sigma^{\infty} \text{ in } \mathcal{K}\ell(\mathcal{P})}{f : X \to \mathcal{P}\Sigma^{\infty} \text{ in Sets}}$$

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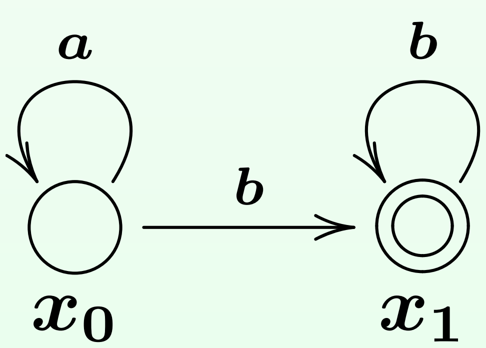

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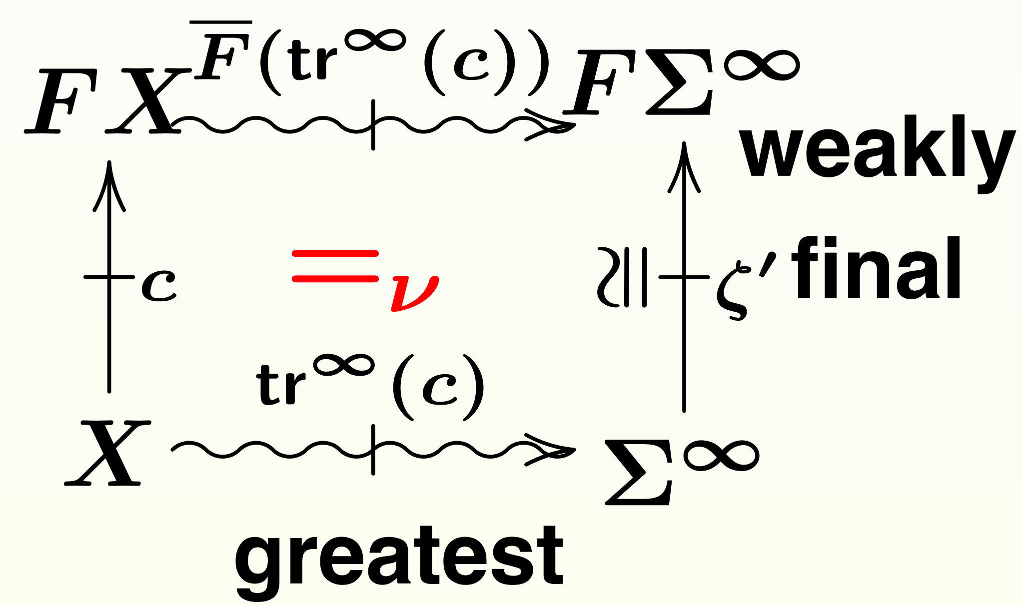

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Thm:

$\mathbf{tr}^\infty(c)$ characterizes **infinitary trace** $L^\infty(\mathcal{A})$

Coalgebraic Infinitary Trace Semantics

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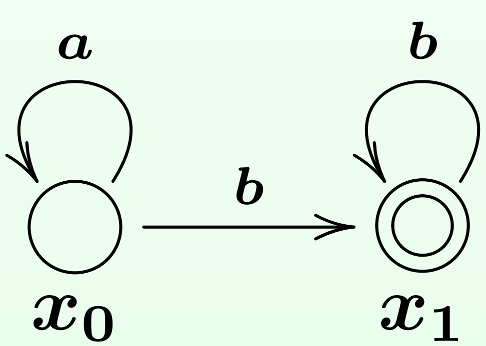

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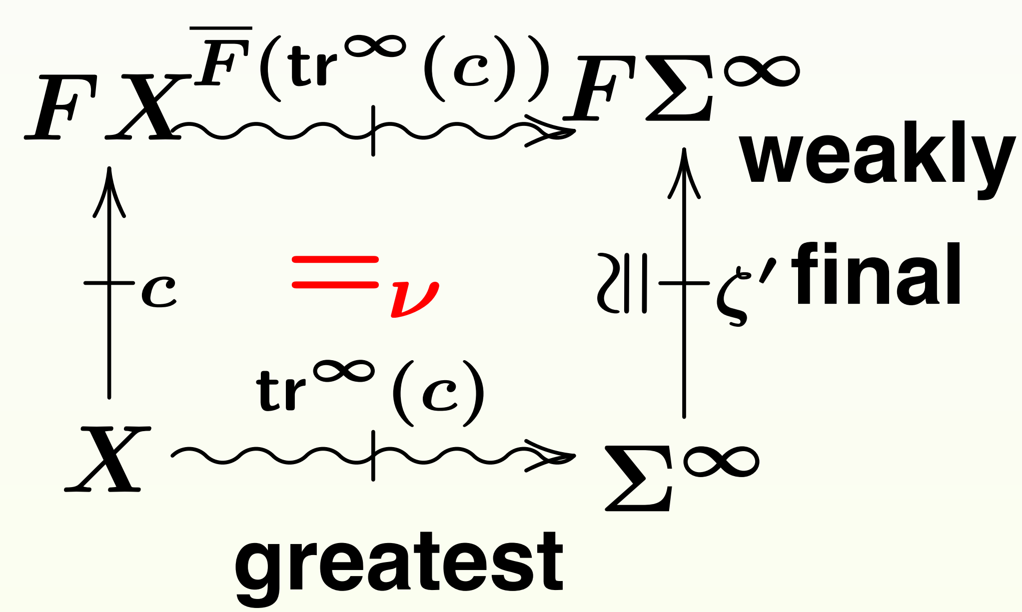
$\mathcal{K}\ell(\mathcal{P})$



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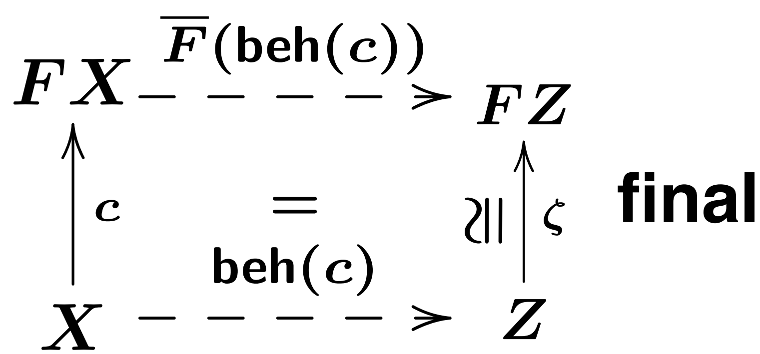
Thm:

$\mathrm{tr}^{\infty}(c)$ characterizes **infinitary trace** $L^{\infty}(\mathcal{A})$

➡ **Leave finality!**

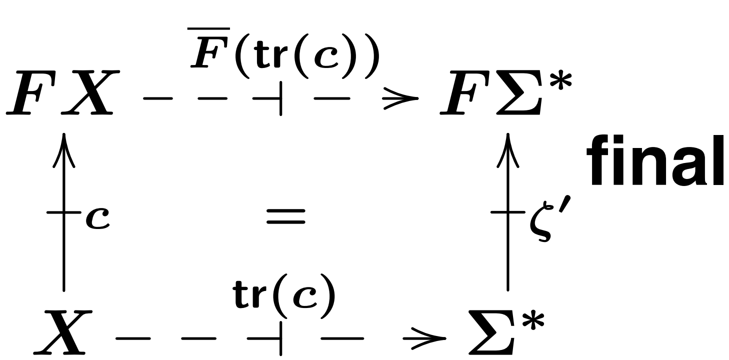
Summary

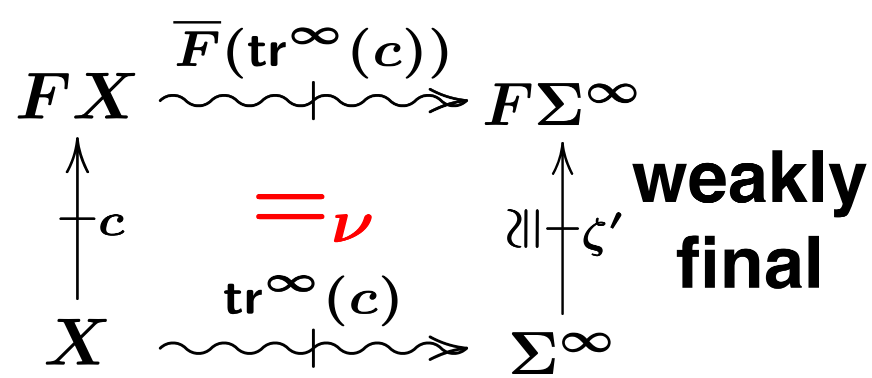
- Coalgebra is a model for **state-based dynamics**
- **Final coalgebra** captures the behavior



in $\mathbf{Sets}$

- For nondet. & prob. automata,
  - the final coalgebra in the **Kleisli category** captures the **finite** trace semantics
  - a weakly final coalgebra in the **Kleisli category** captures the **infinitary** trace semantics





in $\mathcal{K}\ell(\mathcal{P})$

Overview

- Preliminary I:
  Behavioral Domain via Final Coalgebra
- Preliminary II:
  Coalgebraic Finite & Infinitary Trace Semantics
- Main Result:
  Coalgebraic Trace Semantics for Büchi & Parity Automata
- Related Work, Conclusions & Future Work

Büchi Automaton $\mathcal{A} = (X, \Sigma, \delta, \mathbf{Acc})$

Def.

X : state space Σ : alphabet

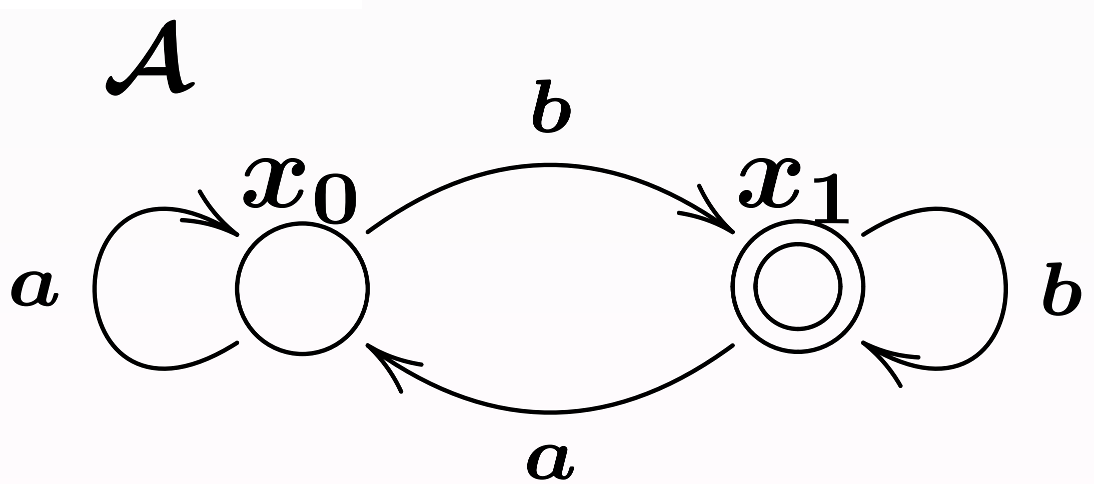
$\delta : X \to \mathcal{P}(\Sigma \times X)$: transition relation

$\mathbf{Acc} \subseteq X$: accepting states

Büchi language $L^B(\mathcal{A}) : X \to \mathcal{P}(\Sigma^\omega)$

$$L^B(\mathcal{A})(x) := \left\{ a_0 a_1 \ldots \in \Sigma^\omega \;\middle|\; \begin{array}{l} x = x_0 \xrightarrow{a_0} x_1 \xrightarrow{a_1} \cdots \\ \text{s.t. } x_k \in \mathbf{Acc} \text{ for inf. many } k\text{'s} \end{array} \right\}$$

Example:



$$L^B(\mathcal{A})(x_0) = \left\{ w \;\middle|\; \begin{array}{l} w \text{ contains} \\ \text{infinitely many } b\text{'s} \end{array} \right\}$$

Parity Automaton $\mathcal{A} = (X, \Sigma, \delta, p)$

Def.

X : state space Σ : alphabet

$\delta : X \to \mathcal{P}(\Sigma \times X)$: transition relation

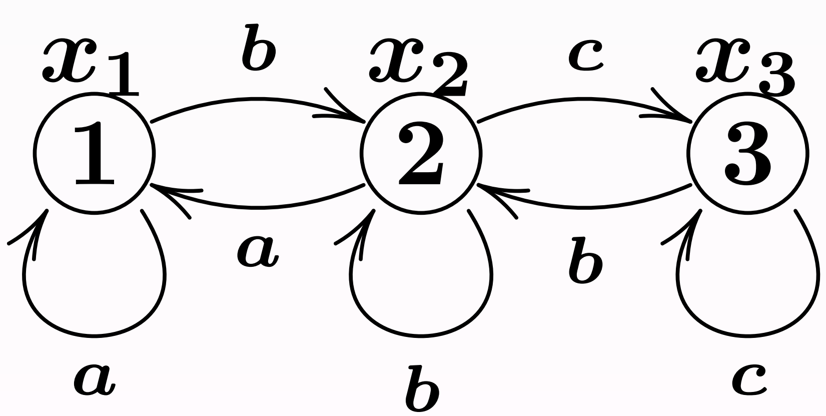
$p : X \to \{1, \ldots, 2n\}$: priority function

parity language $L^p(\mathcal{A}) : X \to \mathcal{P}(\Sigma^\omega)$

$$L^p(\mathcal{A})(x) := \left\{ a_0 a_1 \ldots \in \Sigma^\omega \;\middle|\; \begin{array}{l} x = x_0 \xrightarrow{a_0} x_1 \xrightarrow{a_1} \cdots \\ \text{s.t. } \limsup_{k\to\infty} p(x_k) \text{ is even} \end{array} \right\}$$

Example:

$\mathcal{A}$



$L^p(\mathcal{A})(x_1)$

$$= \left\{ w \;\middle|\; \begin{array}{l} w \text{ contains} \\ \quad \text{infinitely many } b\text{'s, but} \\ \quad \text{only finitely many } c\text{'s} \end{array} \right\}$$

Difficulty

- Büchi/parity acceptance condition considers **infinite** behaviors
  - "Visit **Acc** **infinitely** many times"
  - "Maximum **infinitely** visited priority is even"

➡ **Nonlocal**

Difficulty

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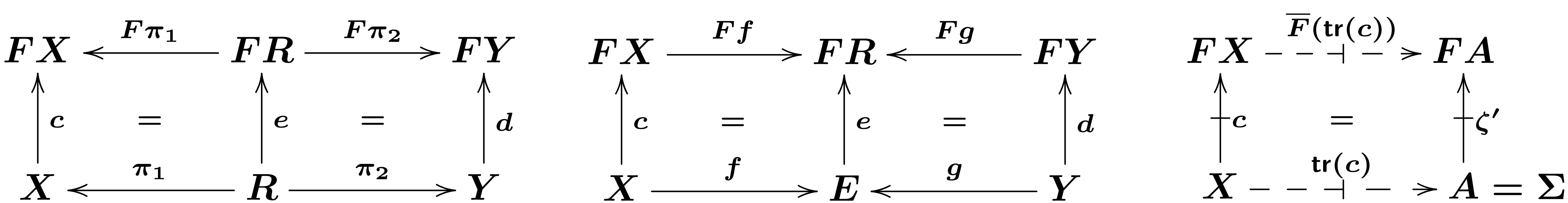
➡ **Nonlocal**

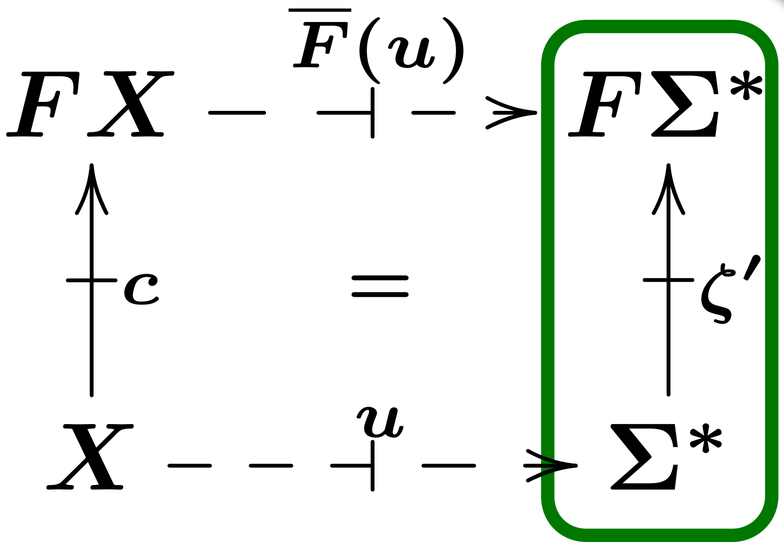
- Theory of coalgebra is centered around **homomorphisms**

$\approx$ **stepwise correspondence**



➡ **Local**

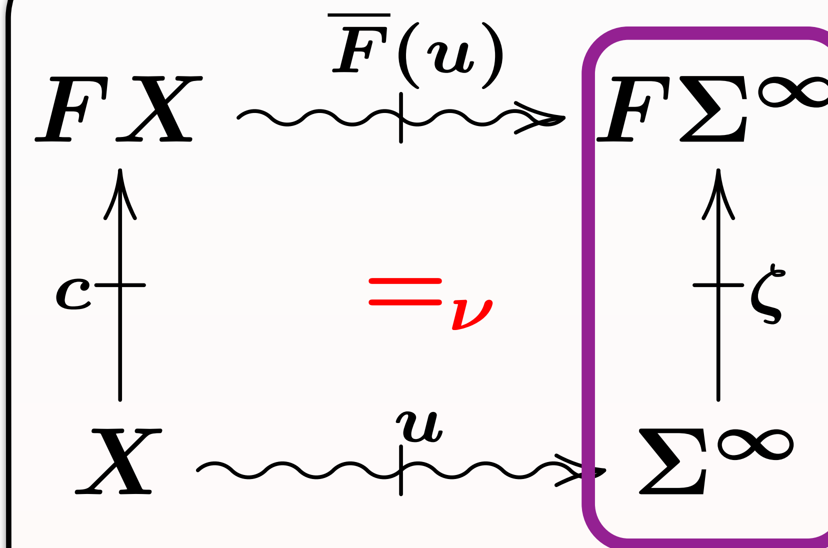
Least Homomorphism?



Unique Homomorphism

$$L(\mathcal{A}) \in \mathcal{P}(\Sigma^*)^X$$

Finite Trace



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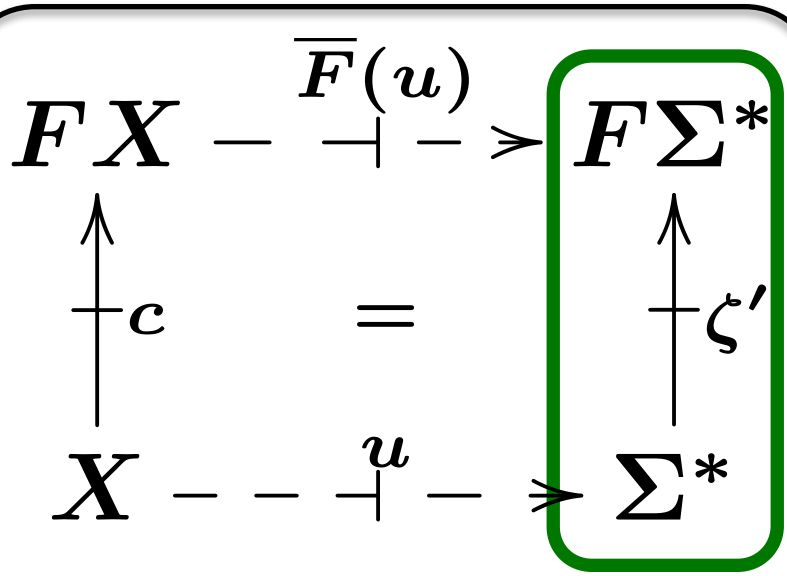

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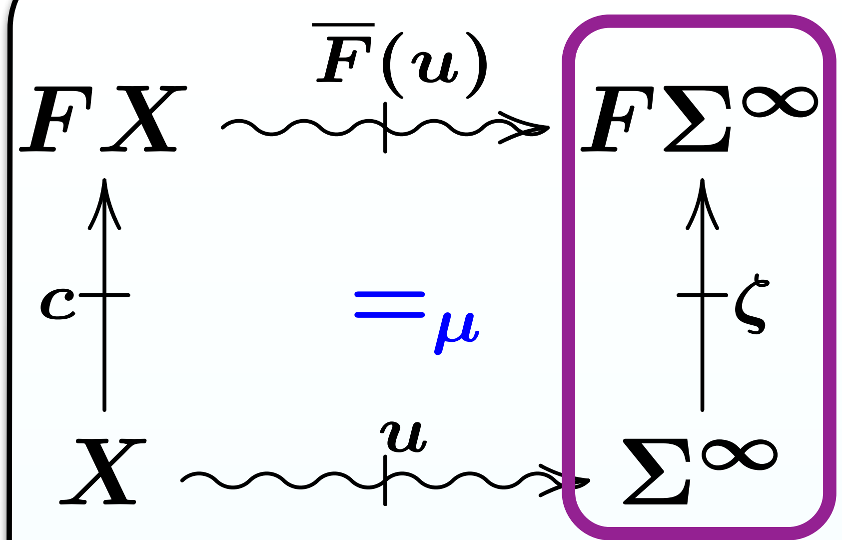

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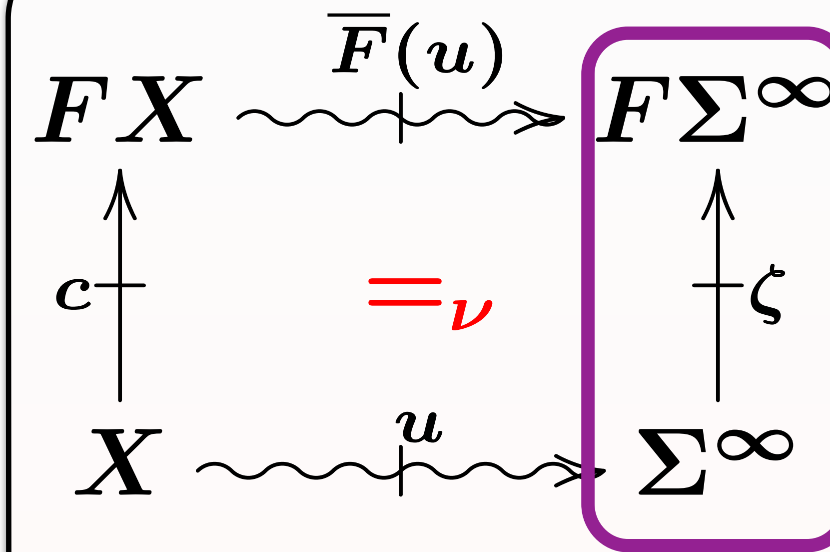

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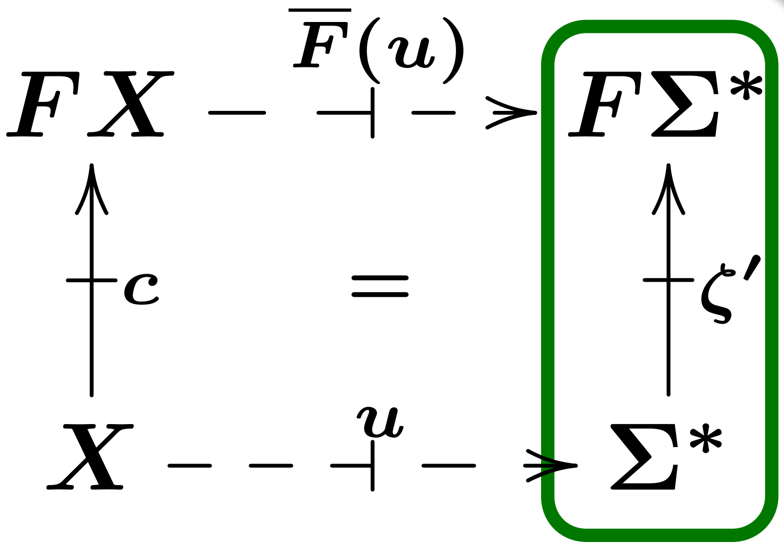

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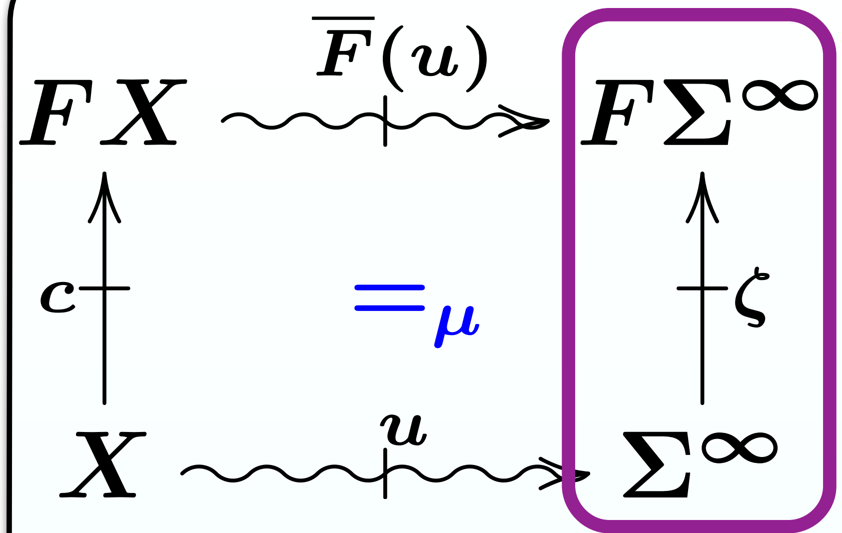

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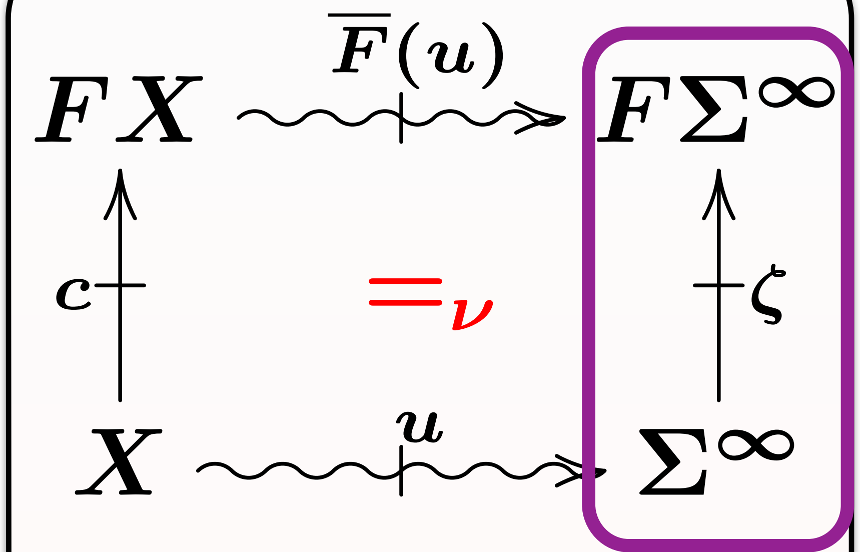

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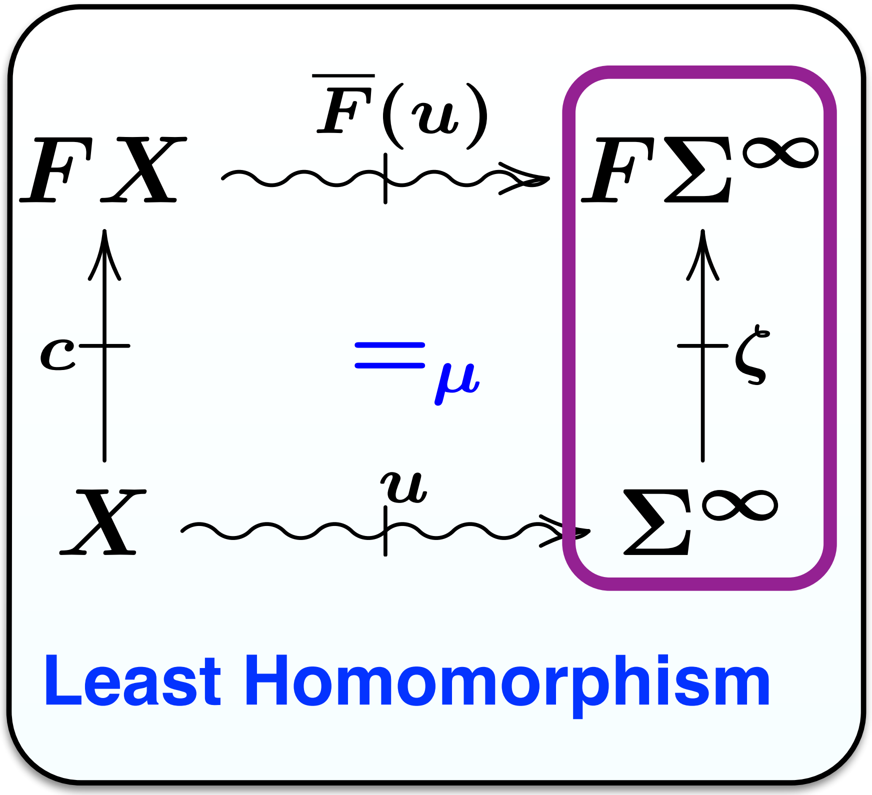
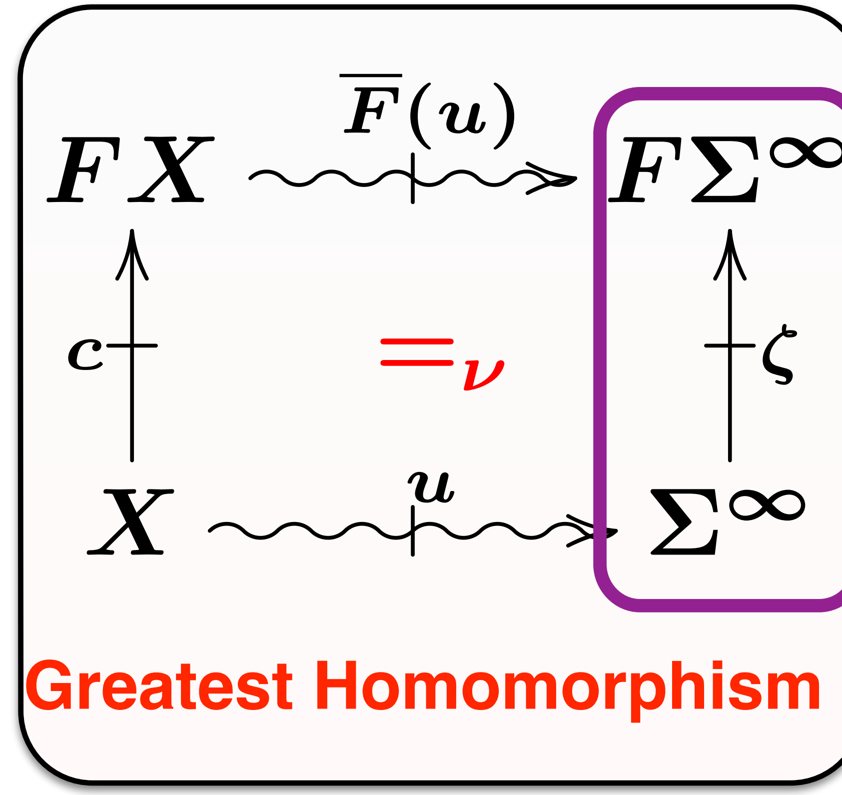
Between the Least and Greatest





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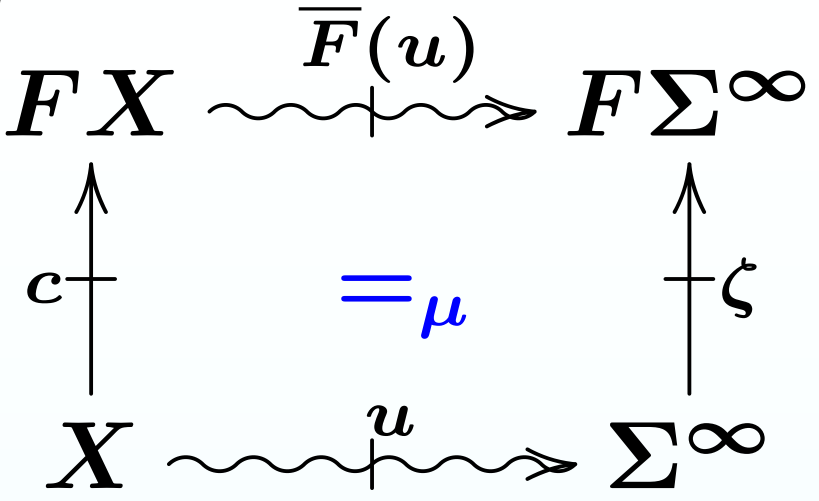

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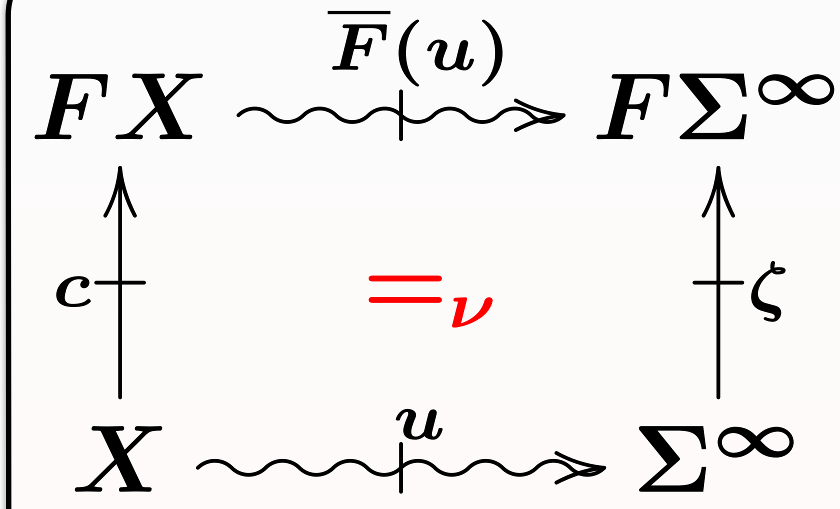

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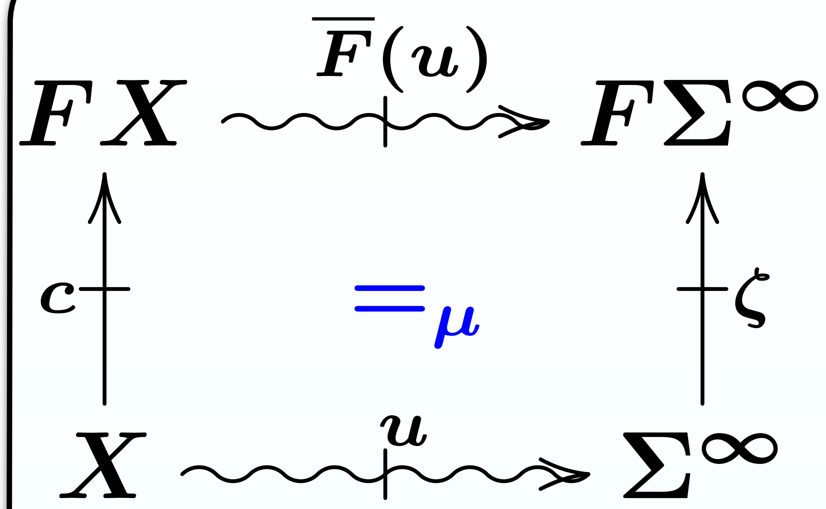

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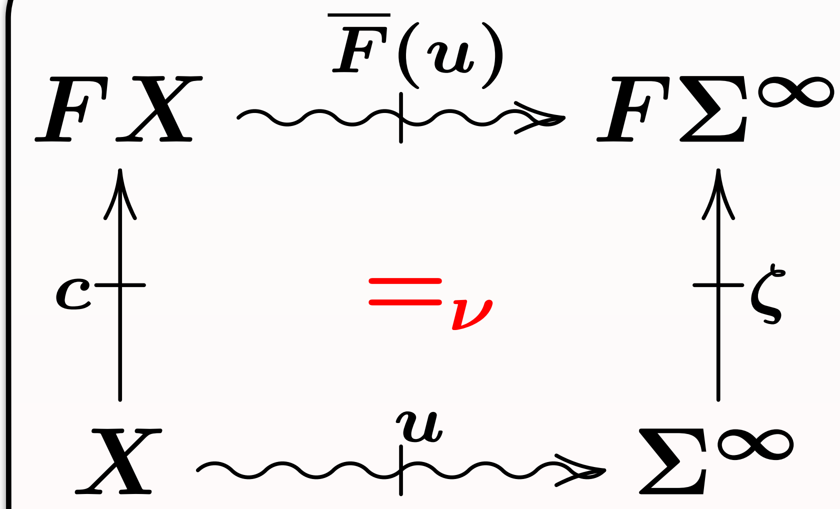
Finite Trace

(**No** infinite word)



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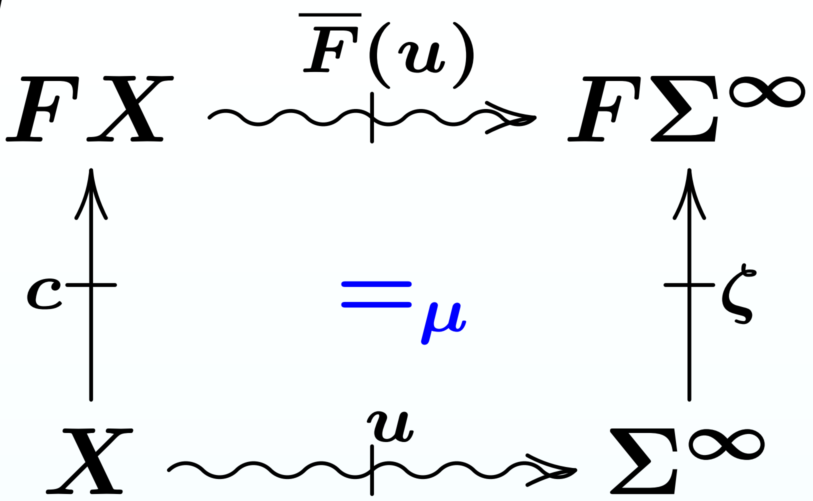

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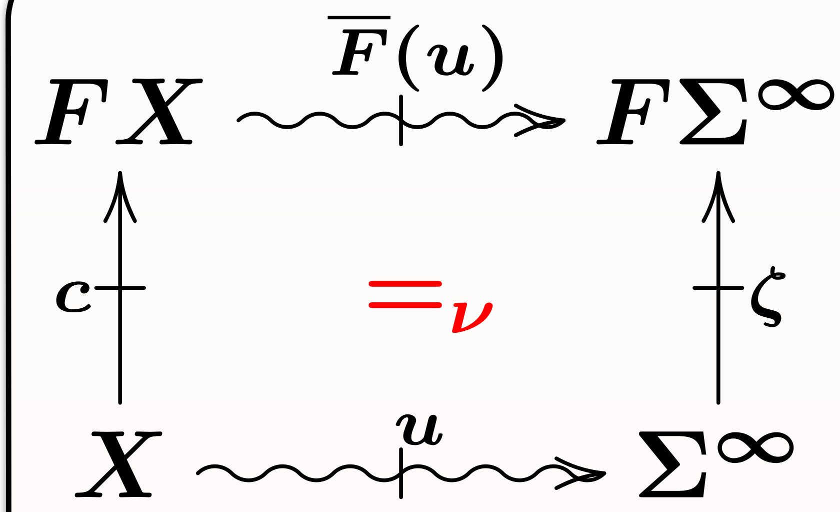

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Parity Language

(**Accepted** infinite words)

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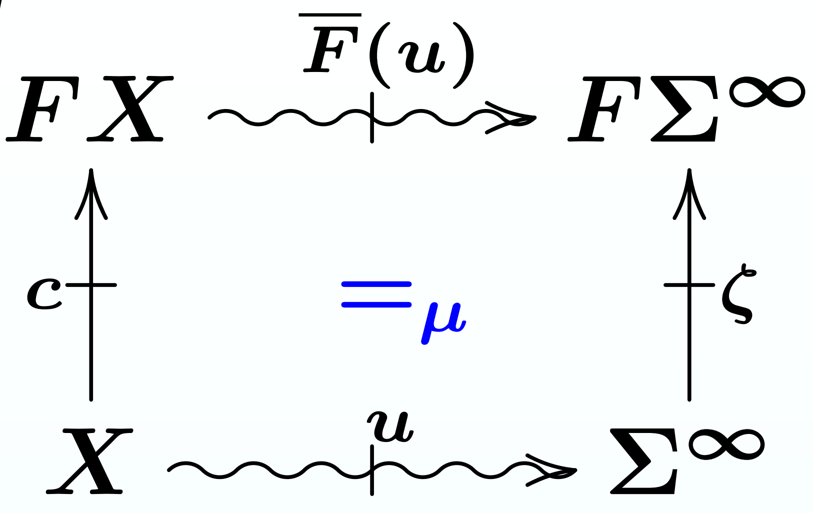
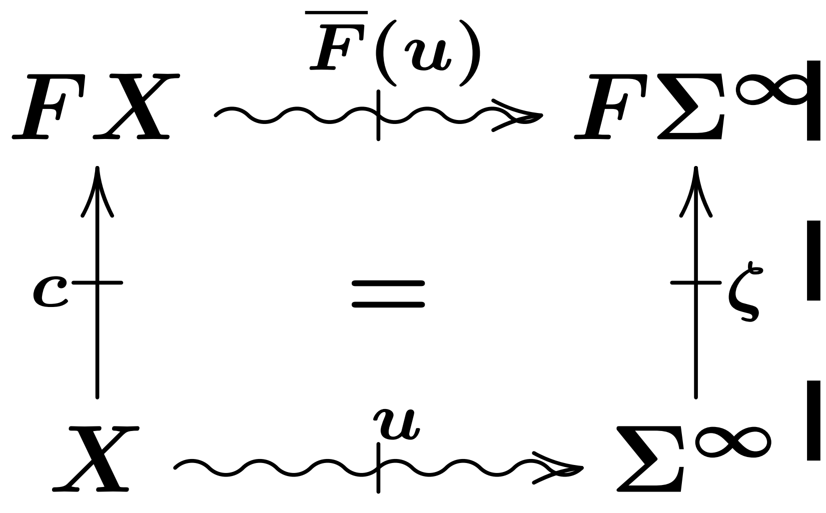
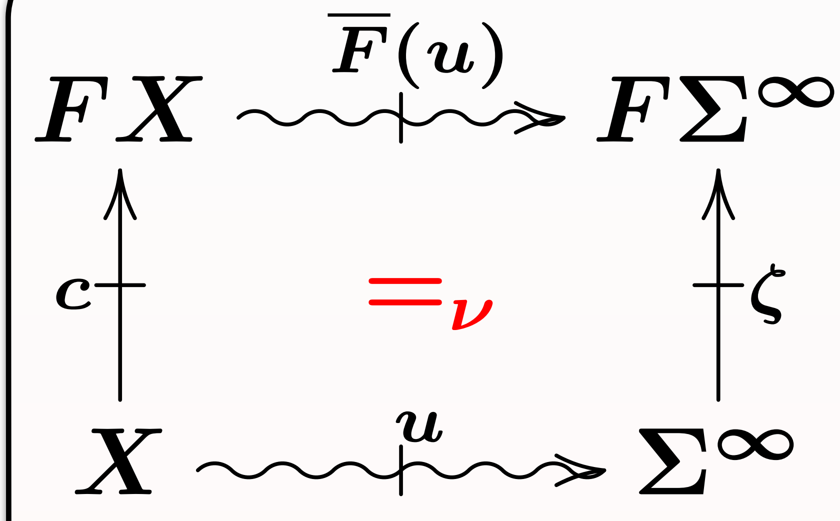


Least Homomorphism



Intermediate



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Infinitary Trace

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Coalgebraic Modeling of Parity Automaton

parity automaton $\mathcal{A} = (\, X,\, \Sigma,\, \delta,\, p\,)$

$c : X \nrightarrow \Sigma \times X$ in $\mathcal{K}\ell(\mathcal{P})$ and

$$X = X_1 + \cdots + X_{2n}$$

$X_i := p^{-1}(i)$

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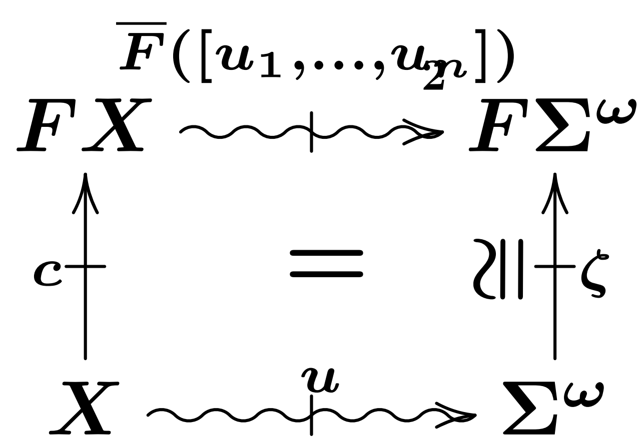

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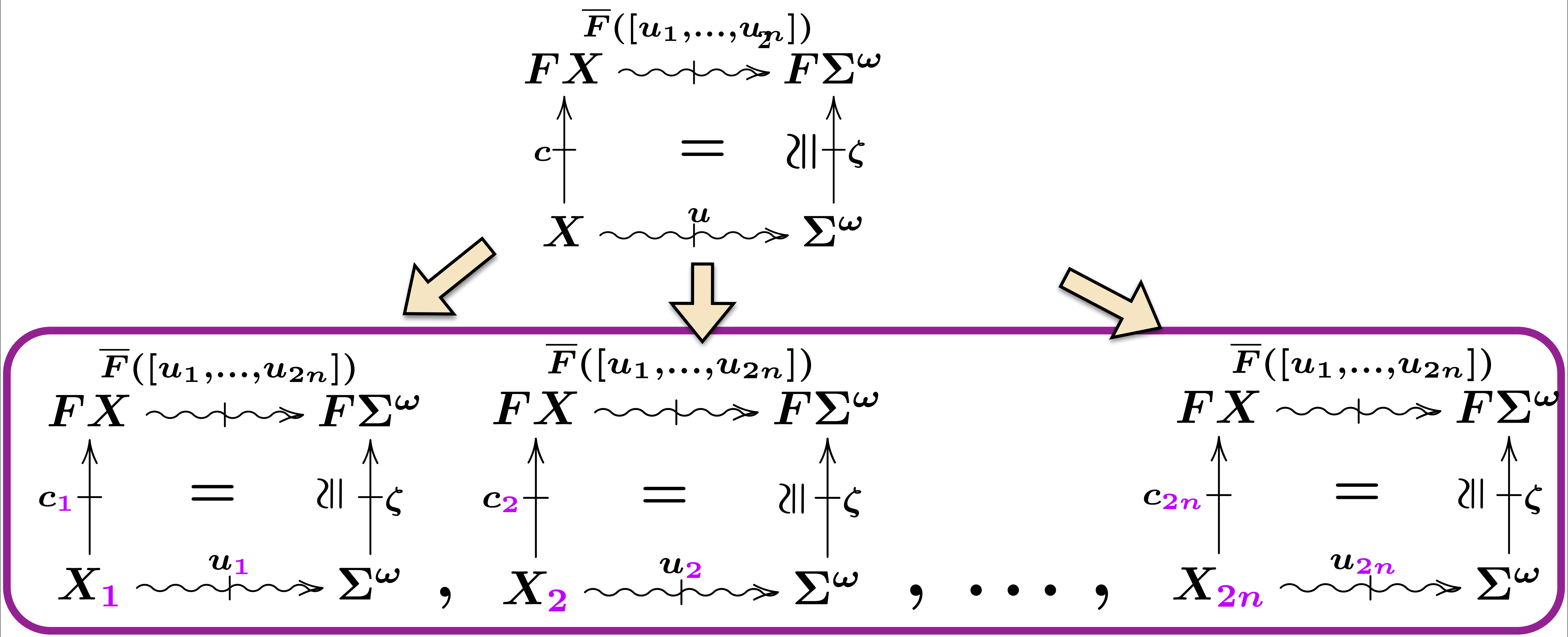

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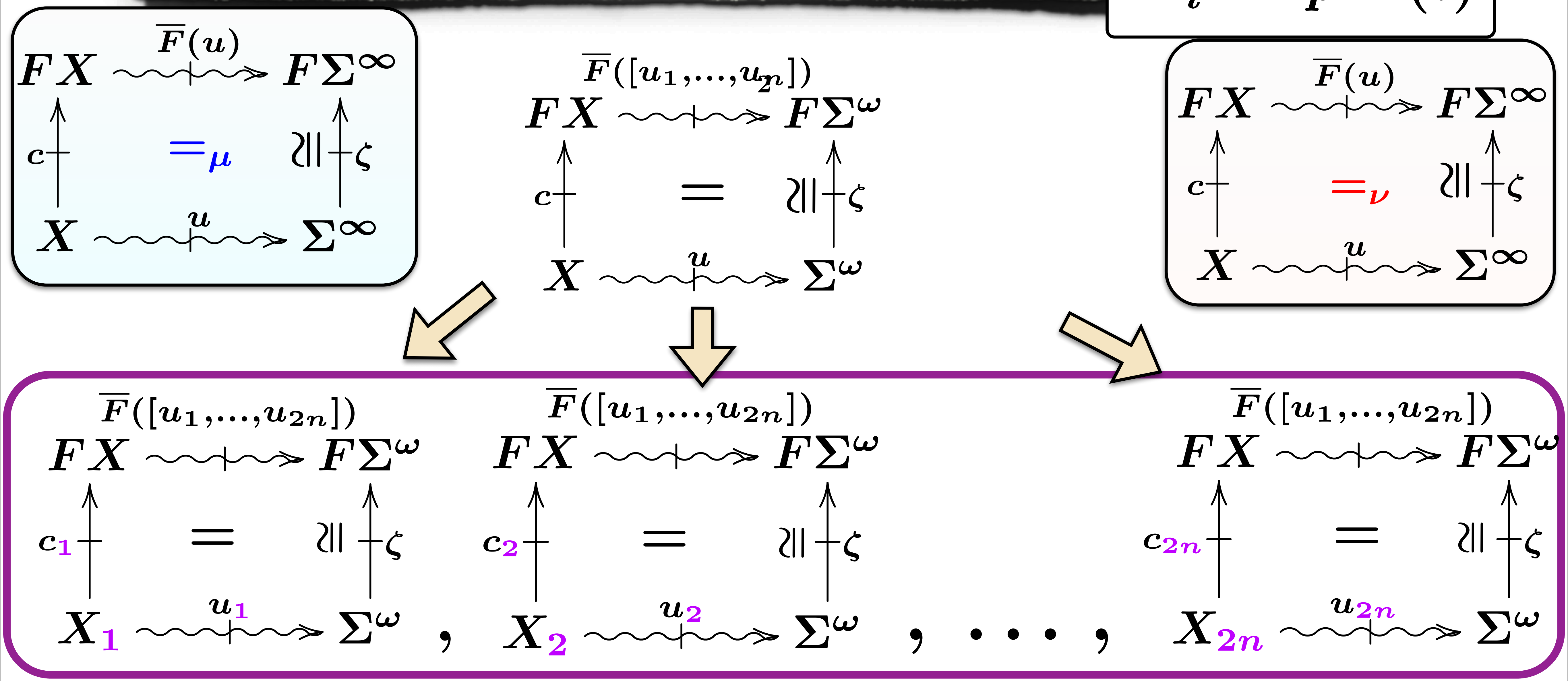

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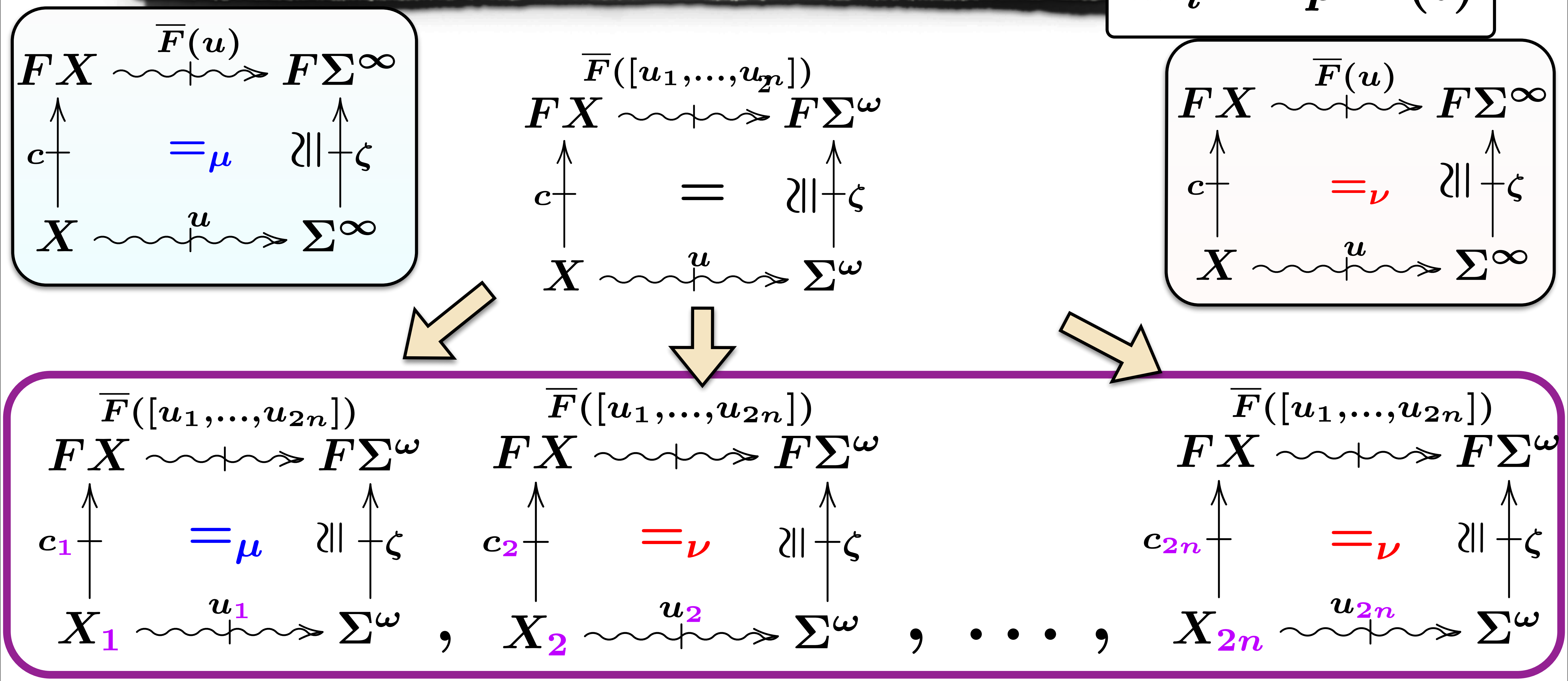

Solution of System of Diagrams

$$
\begin{array}{ccc}
FX & \xrightarrow{\overline{F}([u_1,\ldots,u_{2n}])} & F\Sigma^\omega \\
{\scriptstyle c_1}\uparrow & =_\mu & \cong\uparrow{\scriptstyle \zeta} \\
X_1 & \xrightarrow{u_1} & \Sigma^\omega
\end{array}
,\quad
\begin{array}{ccc}
FX & \xrightarrow{\overline{F}([u_1,\ldots,u_{2n}])} & F\Sigma^\omega \\
{\scriptstyle c_2}\uparrow & =_\nu & \cong\uparrow{\scriptstyle \zeta} \\
X_2 & \xrightarrow{u_2} & \Sigma^\omega
\end{array}
,\quad \ldots ,\quad
\begin{array}{ccc}
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$$

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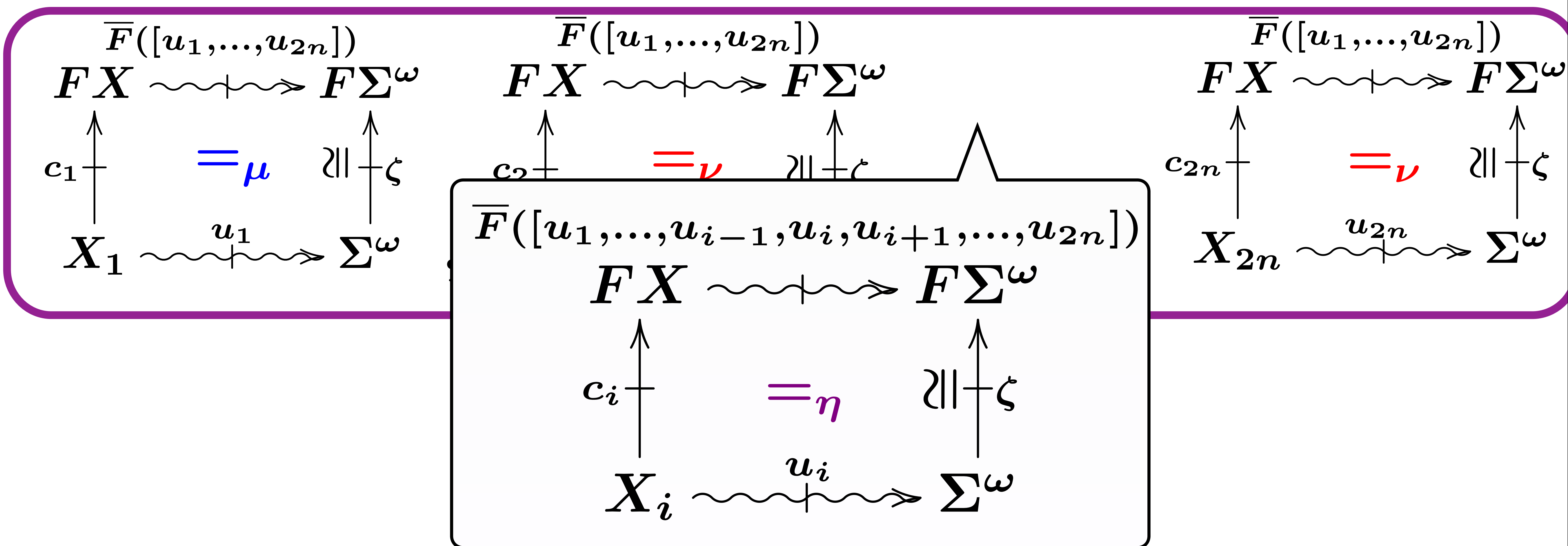

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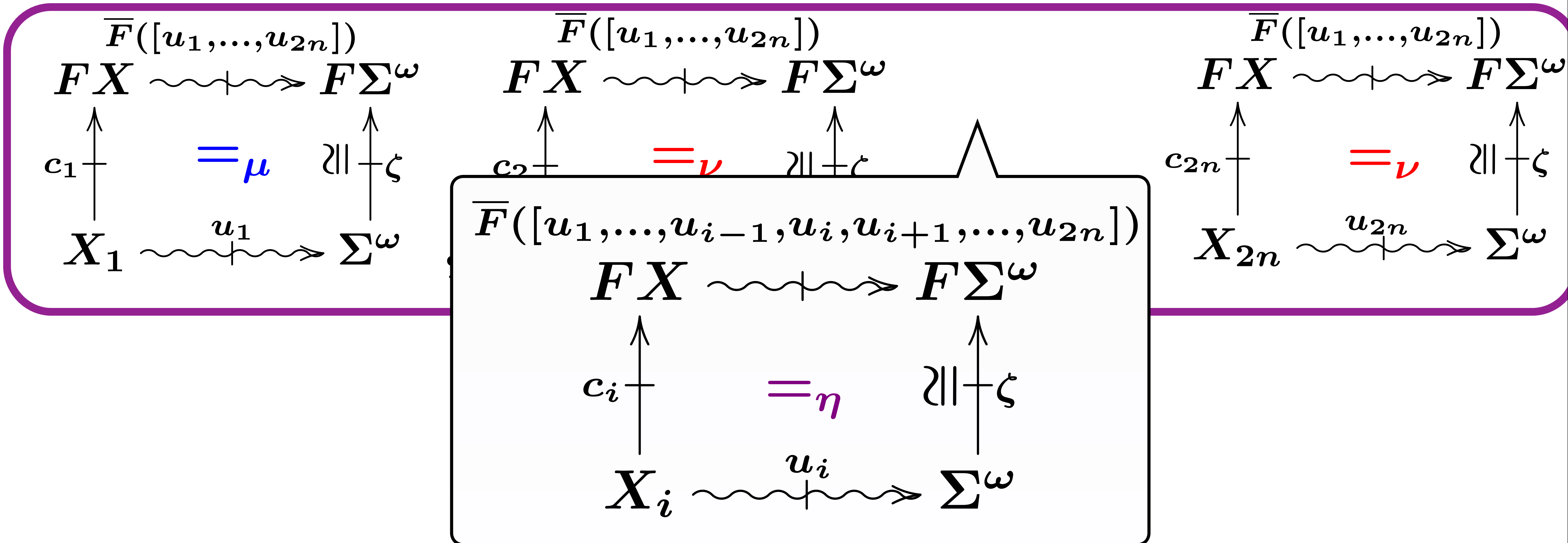
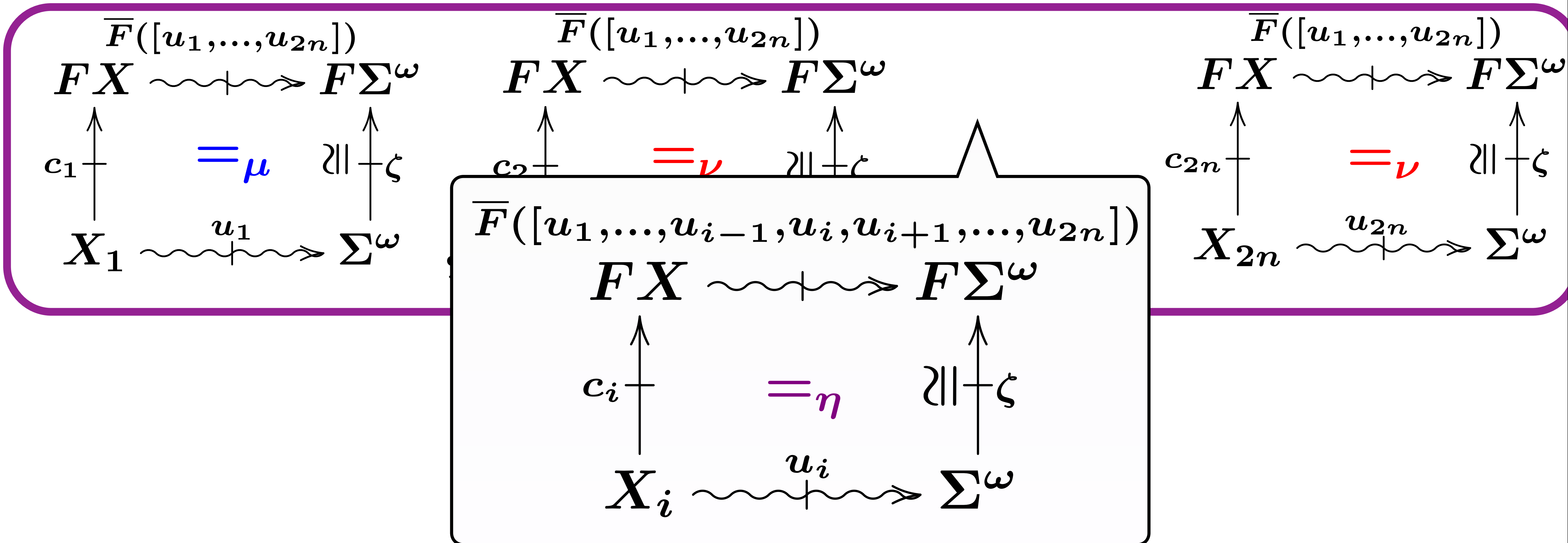


- We solve from the left to the right

Solution of System of Diagrams



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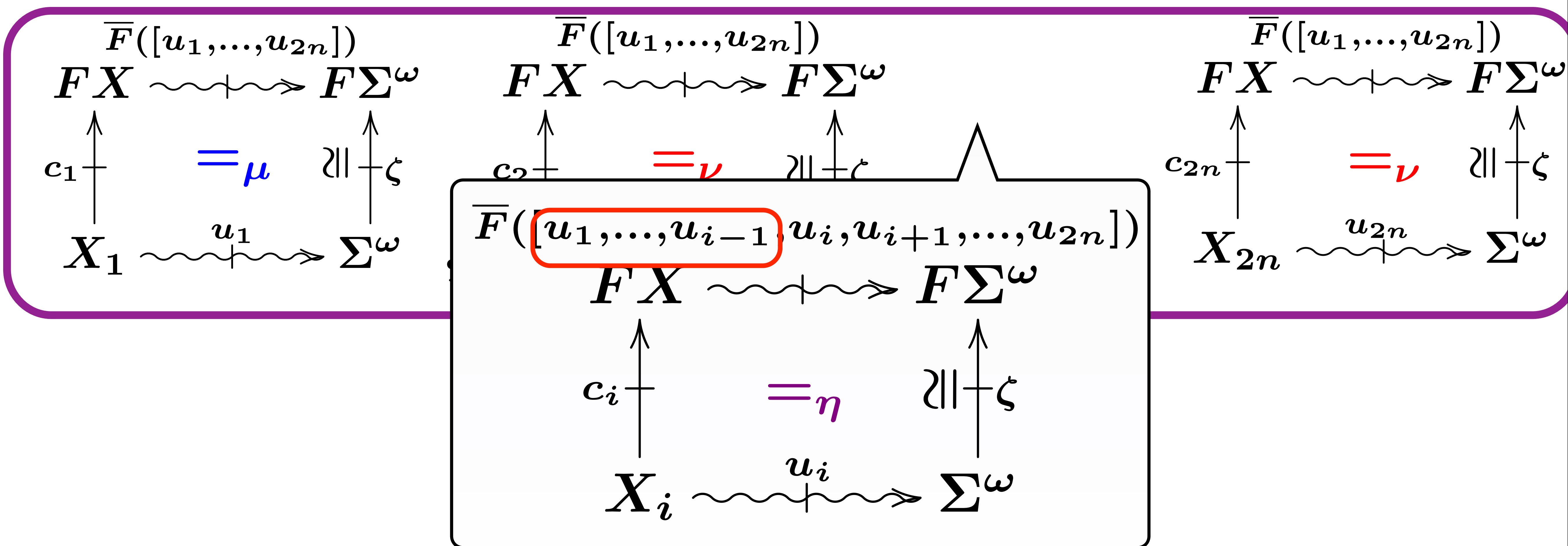

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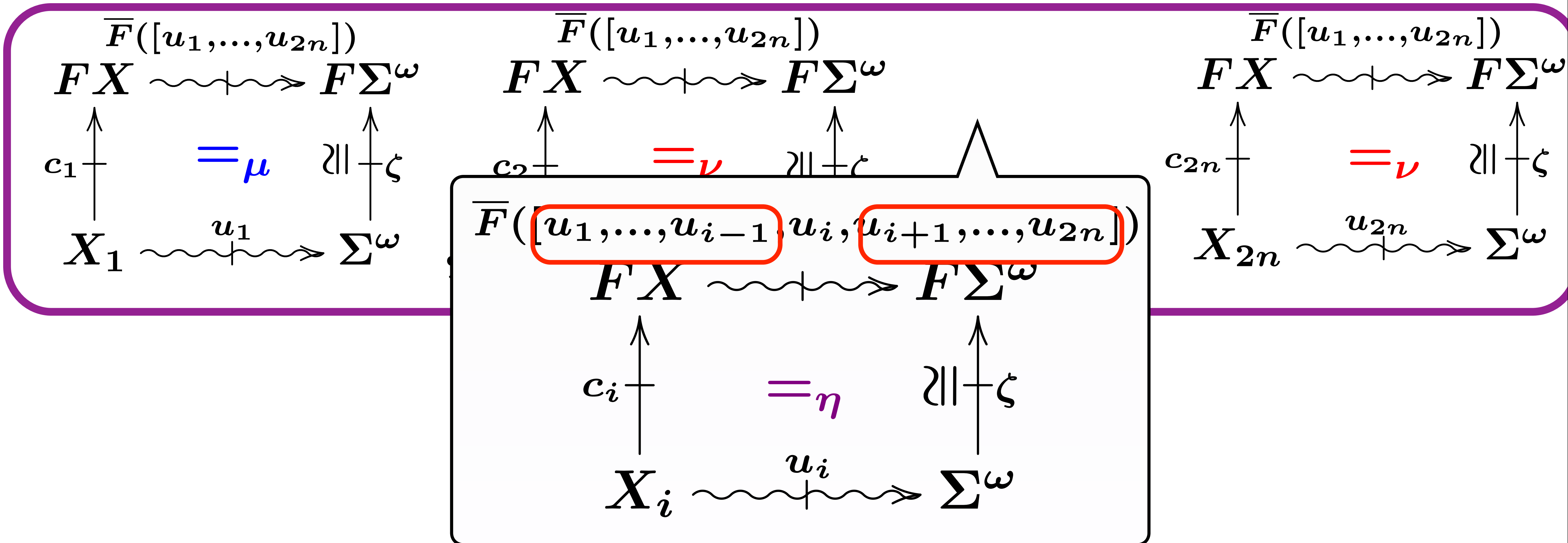

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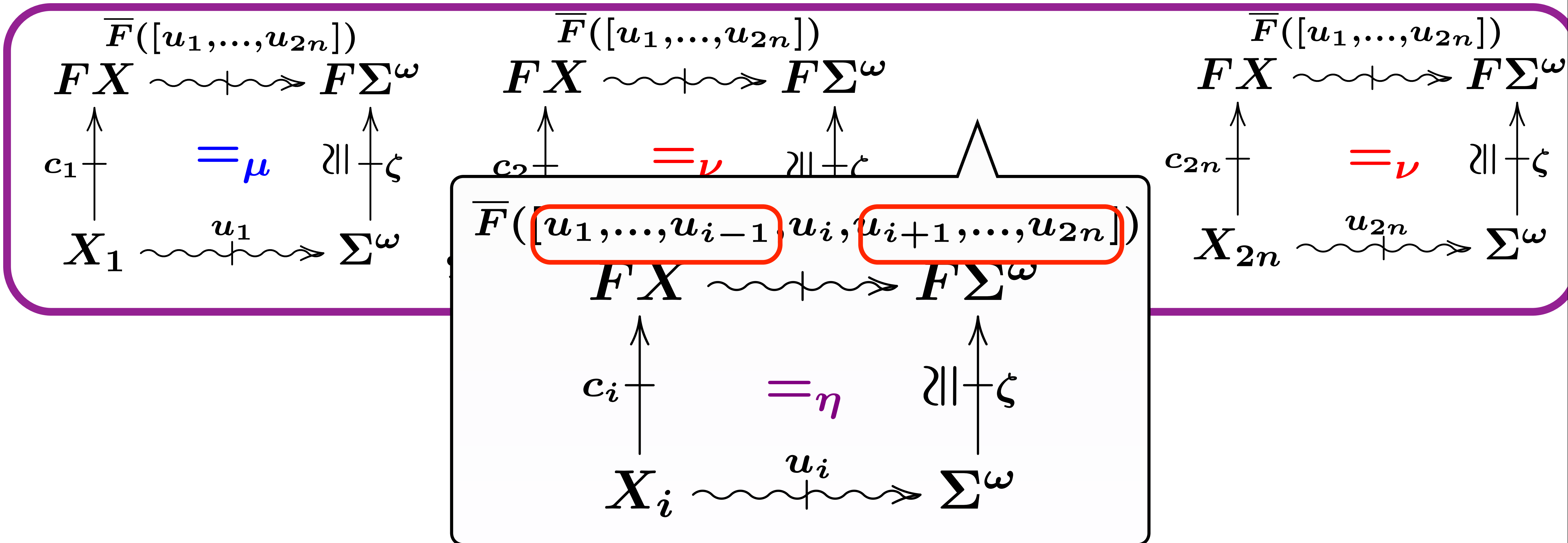

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- To solve the i'th diagram,
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    - regard $u_{i+1}, \dots, u_{2n}$ as parameters
- c.f. [Cleaveland et al., CAV '92], [Arnold & Niwinski, '01]

“Sanity-check Result”

Thm:

For a parity automaton $\mathcal{A} = (X, \Sigma, \delta, p)$, we define

$c : X \nrightarrow \Sigma \times X$ in $\mathcal{K}\ell(\mathcal{P})$ and $X_1 + \cdots + X_{2n} = X$ by

$$c = \delta \quad \text{and} \quad X_i := p^{-1}(i) \qquad .$$

Let $u_1^{\mathrm{sol}}, \ldots, u_n^{\mathrm{sol}}$ be the solution of the following system.

$$\begin{array}{ccc} FX & \xrightarrow{\overline{F}([u_1,\ldots,u_{2n}])} & F\Sigma^\omega \\ {\scriptstyle c_1}\uparrow & =_\mu & \uparrow{\scriptstyle \zeta}\ \cong \\ X_1 & \xrightarrow{u_1} & \Sigma^\omega \end{array} \quad , \quad \begin{array}{ccc} FX & \xrightarrow{\overline{F}([u_1,\ldots,u_{2n}])} & F\Sigma^\omega \\ {\scriptstyle c_2}\uparrow & =_\nu & \uparrow{\scriptstyle \zeta}\ \cong \\ X_2 & \xrightarrow{u_2} & \Sigma^\omega \end{array} \quad , \ \ldots , \quad \begin{array}{ccc} FX & \xrightarrow{\overline{F}([u_1,\ldots,u_{2n}])} & F\Sigma^\omega \\ {\scriptstyle c_{2n}}\uparrow & =_\nu & \uparrow{\scriptstyle \zeta}\ \cong \\ X_{2n} & \xrightarrow{u_{2n}} & \Sigma^\omega \end{array}$$

Then we have:

$$\left[\, u_1^{\mathrm{sol}}, \ldots, u_{2n}^{\mathrm{sol}} \,\right] = L^p(\mathcal{A})$$

Function Φ_c

$$\Phi_c : \quad \{ f : X \nrightarrow \Sigma^\omega \} \;\to\; \{ f : X \nrightarrow \Sigma^\omega \}$$

$$\left(X \xrightarrow{\;f\;} \Sigma^\omega \right) \;\mapsto\; \begin{array}{ccc} \overline{F}X & \xrightarrow{\;\overline{F}f\;} & \overline{F}\Sigma^\omega \\ \uparrow c & & \cong \downarrow \zeta^{-1} \\ X & & \Sigma^\omega \end{array}$$

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- f is a homomorphism $\Leftrightarrow$ f is a fixed point of Φ_c

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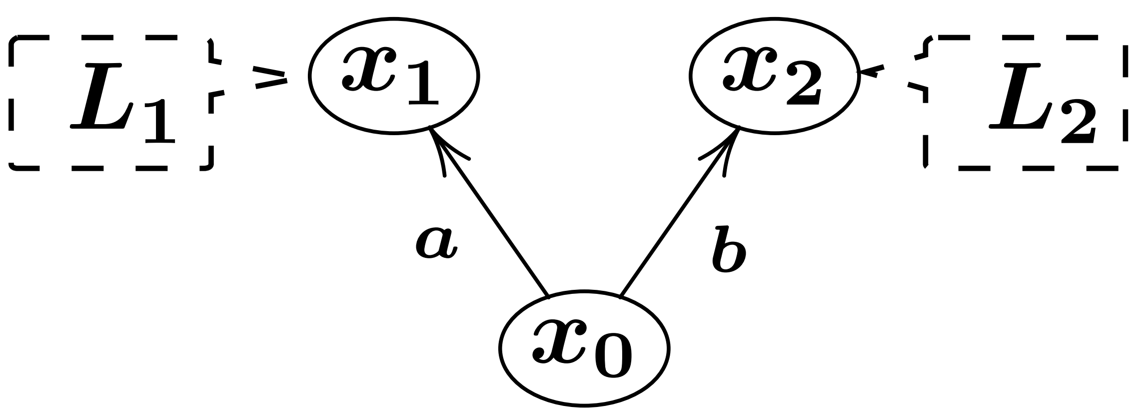

f

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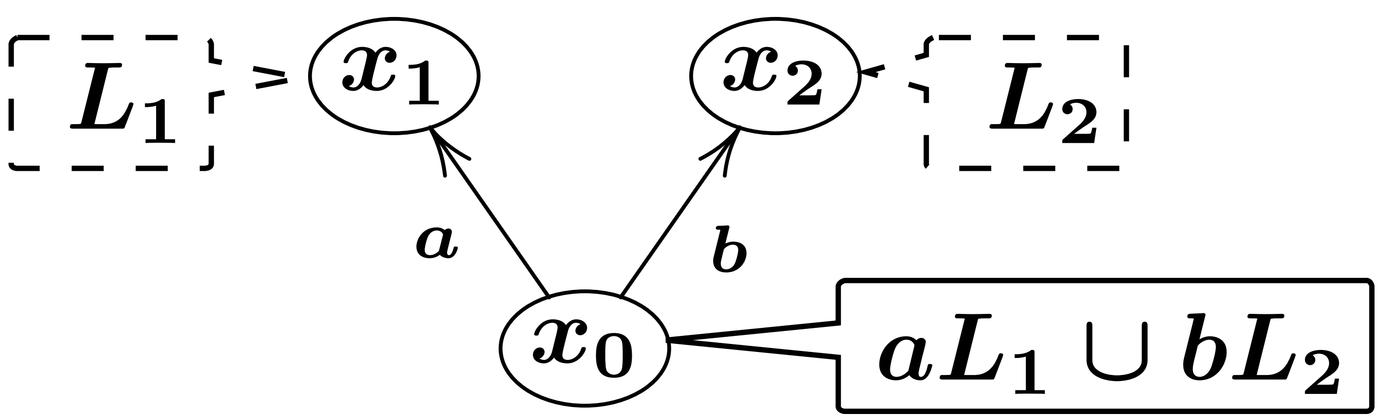

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$\Phi_c(f)$

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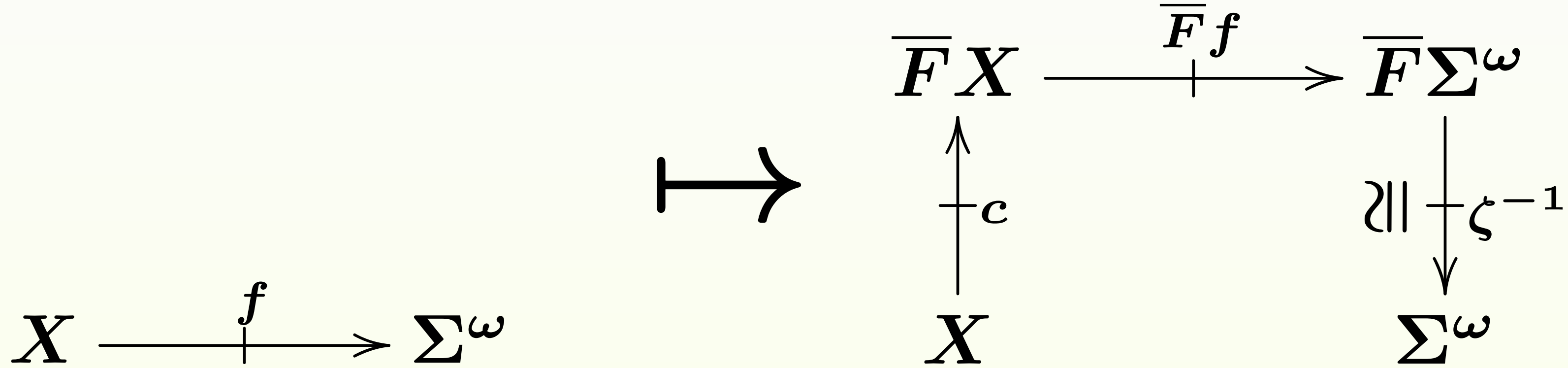

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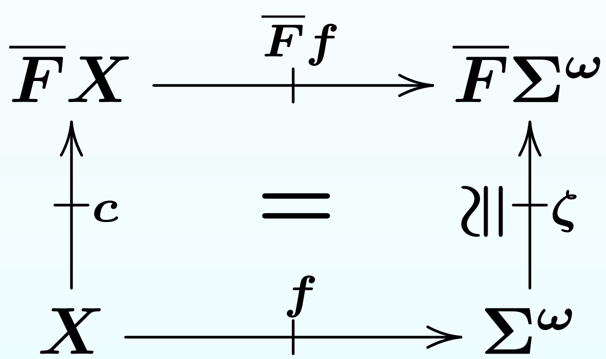
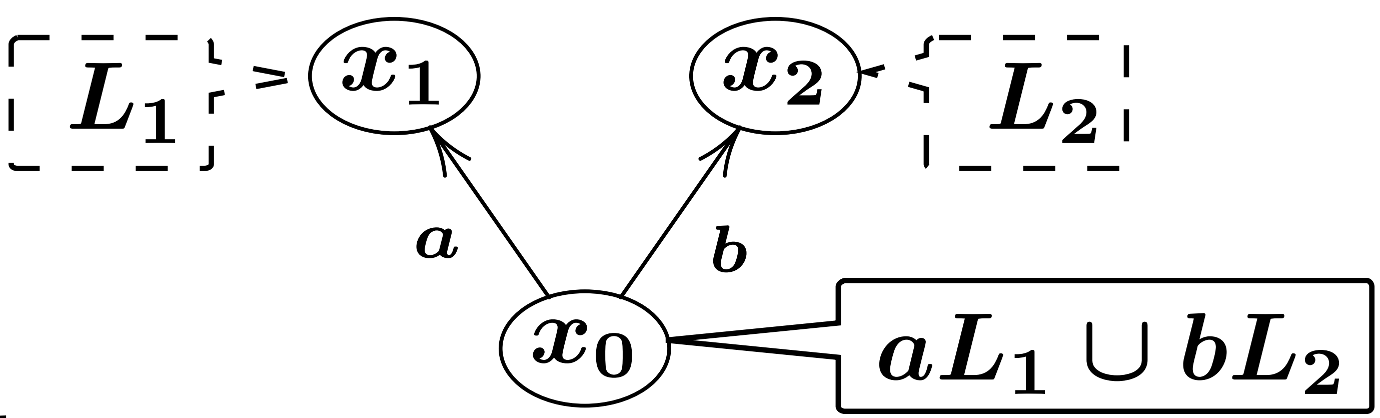


$$f = \Phi_c(f)$$



f

$\Phi_c(f)$

- Φ_c is the one often denoted by $\Diamond_\delta : \mathcal{P}(\Sigma^\omega)^X \to \mathcal{P}(\Sigma^\omega)^X$

Fixed Point Semantics for Parity Automaton

$$\Phi_c \quad \begin{array}{ccc} FX & \xrightarrow{\overline{F}([u_1,\ldots,u_{2n}])} & F\Sigma^\omega \\ {\scriptstyle c_1}\uparrow & =_\mu & \cong\uparrow{\scriptstyle \zeta} \\ X_1 & \xrightarrow{u_1} & \Sigma^\omega \end{array} \;,\; \begin{array}{ccc} FX & \xrightarrow{\overline{F}([u_1,\ldots,u_{2n}])} & F\Sigma^\omega \\ {\scriptstyle c_2}\uparrow & =_\nu & \cong\uparrow{\scriptstyle \zeta} \\ X_2 & \xrightarrow{u_2} & \Sigma^\omega \end{array} \;,\; \ldots \;,\; \begin{array}{ccc} FX & \xrightarrow{\overline{F}([u_1,\ldots,u_{2n}])} & F\Sigma^\omega \\ {\scriptstyle c_{2n}}\uparrow & =_\nu & \cong\uparrow{\scriptstyle \zeta} \\ X_{2n} & \xrightarrow{u_{2n}} & \Sigma^\omega \end{array}$$

$$\downarrow \qquad \overline{\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}}$$

$$\Diamond_\delta$$

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$$\downarrow$$

$$\Diamond_\delta \; \begin{cases} u_1 =_\mu \Diamond_\delta\big([\,u_1, u_2, \dots, u_{2n}\,]\big) \restriction_{X_1} & \in \mathcal{P}(\Sigma^\omega)^{X_1} \\ u_2 =_\nu \Diamond_\delta\big([\,u_1, u_2, \dots, u_{2n}\,]\big) \restriction_{X_2} & \in \mathcal{P}(\Sigma^\omega)^{X_2} \\ \vdots & \\ u_{2n} =_\nu \Diamond_\delta\big([\,u_1, u_2, \dots, u_{2n}\,]\big) \restriction_{X_{2n}} & \in \mathcal{P}(\Sigma^\omega)^{X_{2n}} \end{cases}$$

Equational System for Parity Automaton

Thm:

The solution of the following equational system characterizes **parity** language

$$\begin{cases} u_1 =_{\mu} \Diamond_\delta([u_1, u_2, \ldots, u_{2n}]) \restriction_{X_1} & \in \mathcal{P}(\Sigma^\omega)^{X_1} \\ u_2 =_{\nu} \Diamond_\delta([u_1, u_2, \ldots, u_{2n}]) \restriction_{X_2} & \in \mathcal{P}(\Sigma^\omega)^{X_2} \\ \vdots & \\ u_{2n} =_{\nu} \Diamond_\delta([u_1, u_2, \ldots, u_{2n}]) \restriction_{X_{2n}} & \in \mathcal{P}(\Sigma^\omega)^{X_{2n}} \end{cases}$$

c.f.

$$\nu u_2. \Big(\mu u_1. \big(\Diamond_\delta u_1 \lor (F \land \Diamond_\delta u_2)\big)\Big)$$ for Büchi

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$\nu u_2. \Big(\mu u_1. \big(\Diamond_\delta u_1 \vee (F \wedge \Diamond_\delta u_2)\big)\Big)$ for Büchi

$$\left[\, u_1^{\mathrm{sol}}, \ldots, u_{2n}^{\mathrm{sol}} \,\right] = L^p(\mathcal{A})$$

Extension to Various Systems

$$
\begin{array}{ccc}
FX & \xrightarrow{\overline{F}([u_1,\dots,u_{2n}])} & F\Sigma^\omega \\
{\scriptstyle c_1}\uparrow & =_{\mu} & \cong\uparrow{\scriptstyle \zeta} \\
X_1 & \xrightarrow{u_1} & \Sigma^\omega
\end{array}
\;,\quad
\begin{array}{ccc}
FX & \xrightarrow{\overline{F}([u_1,\dots,u_{2n}])} & F\Sigma^\omega \\
{\scriptstyle c_2}\uparrow & =_{\nu} & \cong\uparrow{\scriptstyle \zeta} \\
X_2 & \xrightarrow{u_2} & \Sigma^\omega
\end{array}
\;,\;\dots\;,\quad
\begin{array}{ccc}
FX & \xrightarrow{\overline{F}([u_1,\dots,u_{2n}])} & F\Sigma^\omega \\
{\scriptstyle c_{2n}}\uparrow & =_{\nu} & \cong\uparrow{\scriptstyle \zeta} \\
X_{2n} & \xrightarrow{u_{2n}} & \Sigma^\omega
\end{array}
$$

- $F = \Sigma \times (\_)$ ➡ $F = \coprod_i \Sigma_i \times (\_)^i$ (polynomial functor)
  - **Words** to **Trees**
- $T = \mathcal{P}$ ➡ $T = \mathcal{G}$ (the sub-Giry monad)
  - **Nondeterministic** to (generative) **Probabilistic**

Overview

- Preliminary I:
  Behavioral Domain via Final Coalgebra
- Preliminary II:
  Coalgebraic Finite & Infinitary Trace Semantics
- Main Result:
  Coalgebraic Trace Semantics for Büchi & Parity Automata
- Related Work, Conclusions & Future Work

Related Work

- Deterministic Muller automaton as a coalgebra

  [Ciancia & Venema, CMCS ’12]

  - Trick with **lasso characterization**

    ➡ Coalgebra on $\mathbf{Sets}^{\mathbf{2}}$

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Thm: **Bisimilarity** and **language equivalence** coincide

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  - **Decision procedures** for complementation, union, intersection, equivalence check

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Thm: **Bisimilarity** and **language equivalence** coincide

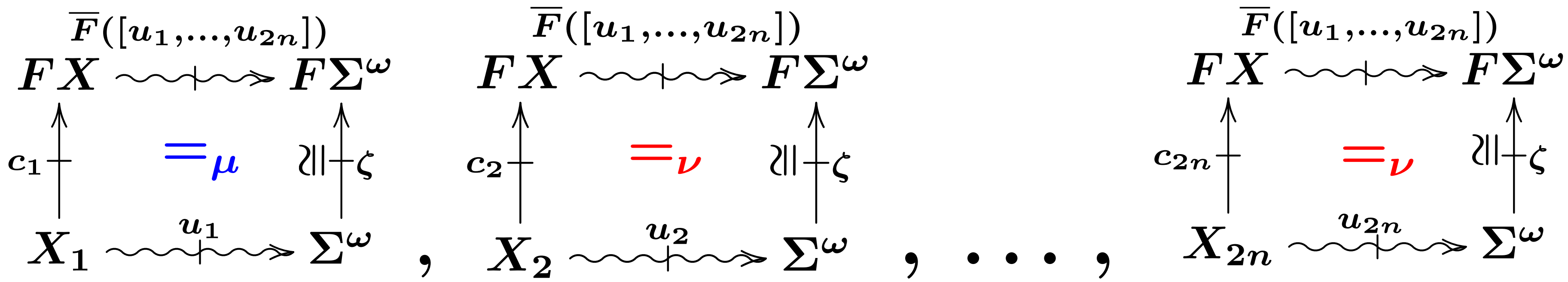
  - **Decision procedures** for complementation, union, intersection, equivalence check
  - Compared to our characterization:
    - Final coalgebra exists -> well-behaved
    - Extension to probabilistic systems seems difficult
    - Finite-state restriction

Conclusions

- Coalgebraic modeling of Büchi / parity automata

$$X = X_1 + \cdots + X_{2n}$$ $$X_i := p^{-1}(i)$$

- Coalgebraic characterization of their languages



- "sanity-check" proofs
- Applications
  - e.g. **coalgebraic fair simulation** [U., Shimizu, Hasuo, arXiv preprint, '16]

Future Work

- Extension to **2-player** setting
- Extension to **reactive** probabilistic system