

1. The category  $\mathbb{C}$  has a final object  $\mathbf{1}$  and  $F$  preserves a limit  $(Z, (\gamma_i : Z \rightarrow F^i \mathbf{1})_{i \in \omega})$  of the final sequence  $\mathbf{1} \leftarrow F\mathbf{1} \leftarrow F^2\mathbf{1} \leftarrow \dots$ . (This limit yields a final  $F$ -coalgebra  $\zeta : Z \rightarrow FZ$ .)
2. The Kleisli category  $\mathcal{Kl}(T)$  is **Dcpo**-enriched. That is, for each pair of objects  $X$  and  $Y$  in  $\mathcal{Kl}(T)$ , the homset  $\mathcal{Kl}(T)(X, Y)$  carries an order  $\sqsubseteq$  and each directed subset of  $\mathcal{Kl}(T)(X, Y)$  has its infimum.
3. For each object  $X \in \mathbb{C}$ , the lifting  $J(!_X)$  of the unique arrow to the final object is the largest element in  $\mathcal{Kl}(T)(X, \mathbf{1})$ .
4. There exists a distributive law  $\lambda : FT \Rightarrow TF$ , and hence  $F$  can be lifted to an endofunctor  $\overline{F}$  on  $\mathcal{Kl}(T)$ .
5. The functor  $\overline{F}$  is monotone.
6. The functor  $J$  lifts the limit in 1. to a weak 2-limit. Namely, for any cone  $(X, (\pi_i : X \rightarrow F^i \mathbf{1})_{i \in \omega})$  over the sequence  $\mathbf{1} \leftarrow \overline{F}\mathbf{1} \leftarrow \overline{F}^2\mathbf{1} \leftarrow \dots$ , there exists the *largest mediating arrow*  $b_{\max} : X \rightarrow Z$  that satisfies the following conditions.
  - The arrow  $b_{\max}$  is a mediating arrow (i.e. for each  $i \in \omega$ ,  $\pi_i = J\gamma_i \odot b_{\max}$ ).
  - If  $b : X \rightarrow Z$  satisfies  $J\gamma_i \odot b \sqsubseteq J\gamma_i \odot b_{\max}$  for each  $i \in \omega$ , then  $b \sqsubseteq b_{\max}$  holds.