Constrained Optimization for Hybrid System Falsification and Application to Conjunctive Synthesis

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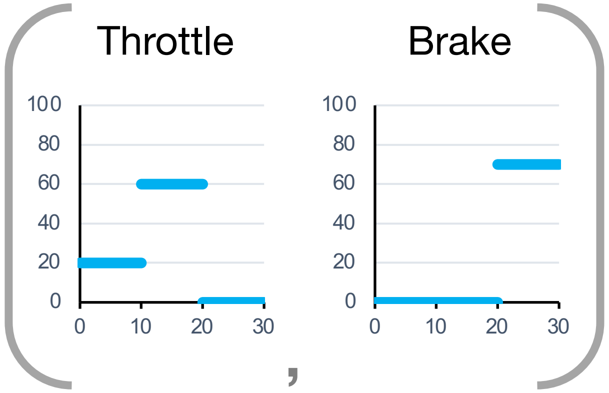
Hybrid system falsification of CPS

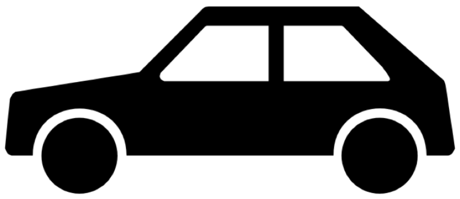
[Fainekos & Pappas, Theor. Comput. Sci. 2009]

Input u

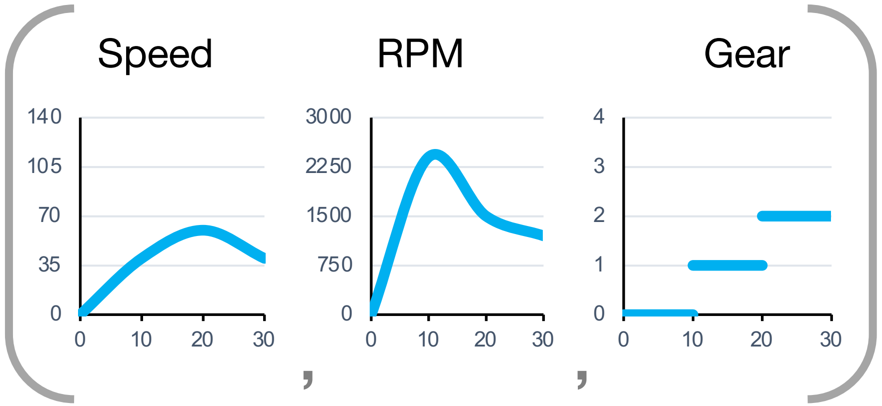


System model $\mathcal{M}$

Automatic transmission system (Simulink)



Output $\mathcal{M}(u)$



Specification φ

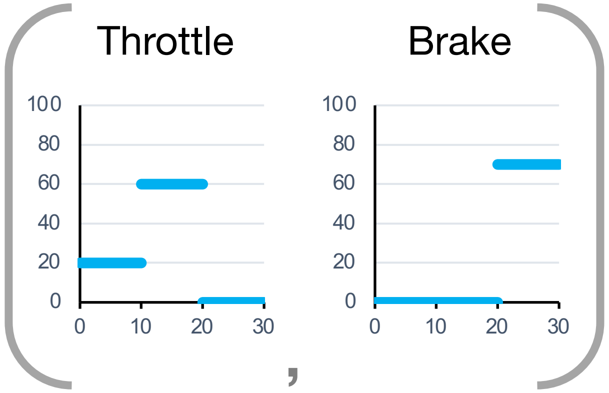
$$\varphi :\equiv \square_{[0,30]}(\text{speed} < 100)$$

Goal: to find an input signal u, s.t. $\mathcal{M}(u) \not\models \varphi$

Hybrid system falsification of CPS

[Fainekos & Pappas, Theor. Comput. Sci. 2009]

Input u



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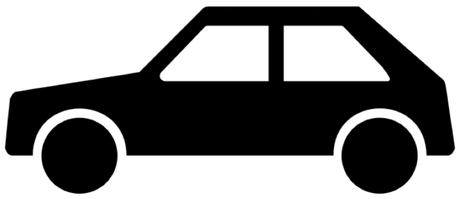

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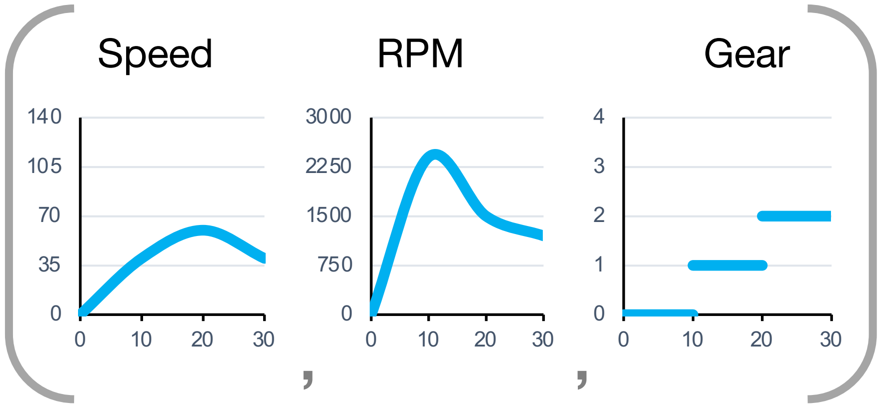


Specification φ

$$\varphi :\equiv \Box_{[0,30]}(\text{speed} < 100)$$

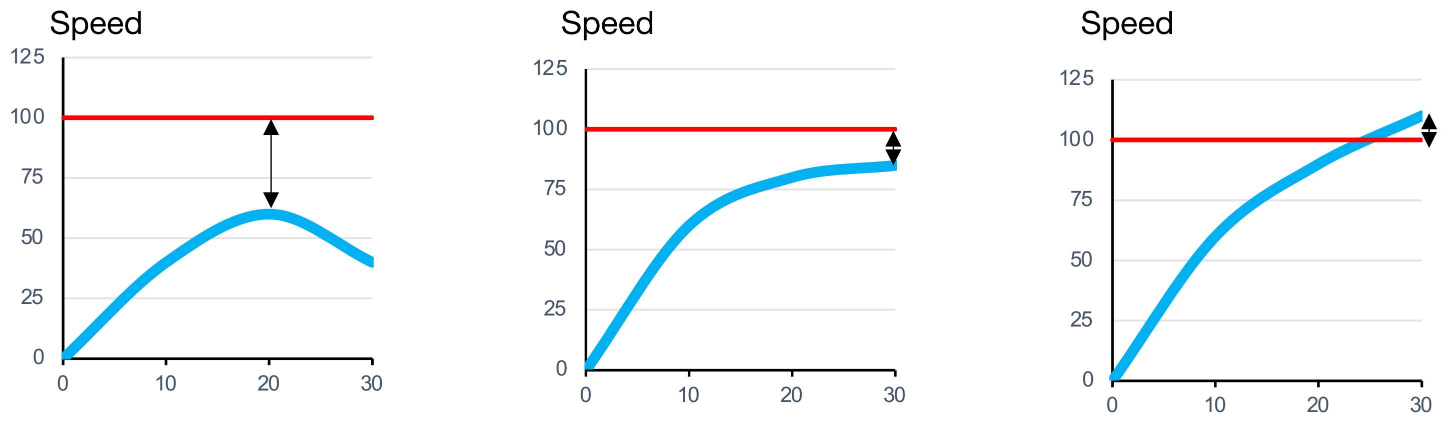
Always for $t \in [0,30]$

Speed is below 100

Goal: to find an input signal u, s.t. $\mathcal{M}(u) \not\models \varphi$

Robust semantics of STL [Donze & Maler, FORMATS'10]

Specification: $\varphi \equiv \Box_{[0,30]}(\mathrm{speed}(t) < 100)$



Violation degree	Far from violation	Almost violated	Violated
Robustness $[\![\mathcal{M}(\mathrm{u}), \varphi]\!]$	40	10	-10

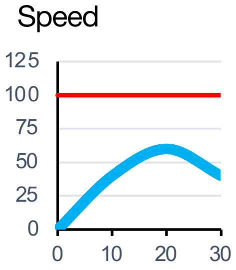
$$\mathcal{M}(u) \not\models \varphi \Leftrightarrow [\![\mathcal{M}(\mathrm{u}), \varphi]\!] < 0$$

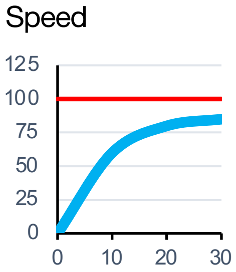
Optimization-based falsification

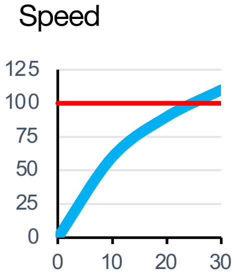
Falsification Problem is translated into:

Try minimizing $[\![\mathcal{M}(\mathrm{u}), \varphi]\!]$ and finish if $[\![\mathcal{M}(\mathrm{u}), \varphi]\!] < 0$

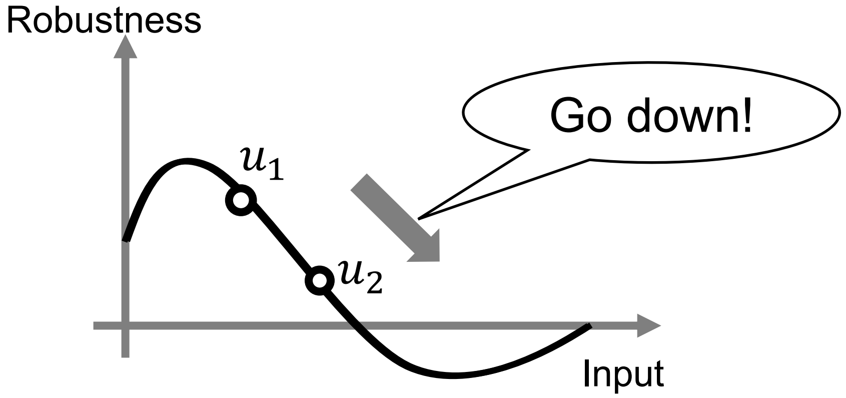
Input	u_1	Less robust →	u_2	Less robust →	u_3
Robustness $[\![\mathcal{M}(\mathrm{u}), \varphi]\!]$	40		10		**-10**







Optimized by
Hill-climbing style algorithms
(HC, SA, GNM, CMA-ES, …)



Synthesis is the dual of falsification

Falsification problem

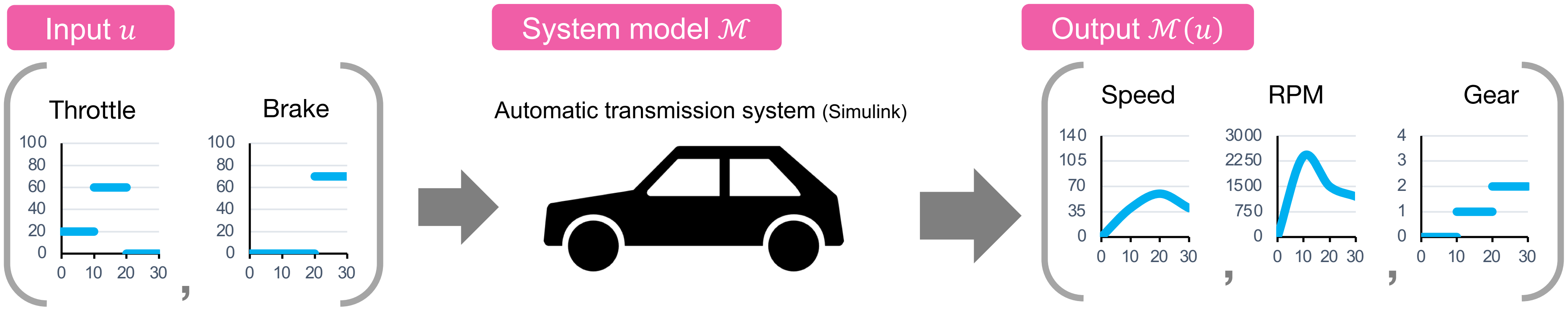
Try minimizing $[\![\mathcal{M}(\mathrm{u}), \neg\varphi]\!]$ **and finish if** $[\![\mathcal{M}(\mathrm{u}), \neg\varphi]\!] < 0$

$\mathcal{M}(u) \nvDash \neg\varphi \;\Leftrightarrow\; \mathcal{M}(u) \vDash \varphi$

Synthesis problem

Try maximizing $[\![\mathcal{M}(\mathrm{u}), \varphi]\!]$ **and finish if** $[\![\mathcal{M}(\mathrm{u}), \varphi]\!] > 0$

Conjunctive synthesis



Specification φ

and

$$\varphi :\equiv \varphi_1 \land \cdots \land \varphi_m$$

Goal: to find an input signal u, s.t. $\mathcal{M}(u) \vDash \varphi_1 \land \cdots \land \varphi_m$

$(\Leftrightarrow \mathcal{M}(u) \vDash \varphi_1, \ \ldots, \mathcal{M}(u) \vDash \varphi_m)$

S. Sato <sotasato@nii.ac.jp>

Example of Conjunctive synthesis

Conjunctive specification $\varphi^{\mathrm{AT}} :\equiv \varphi_1^{AT} \land \varphi_2^{AT} \land \varphi_3^{AT}$

$\varphi_1^{AT} :\equiv \Box_{[0,30]}(\mathrm{rpm} \leq 2400)$, $\varphi_2^{AT} :\equiv \Box_{[0,30]}(\mathrm{speed} \leq 60)$, $\varphi_3^{AT} :\equiv \Diamond_{[0,30]}(\mathrm{gear} \geq 3)$

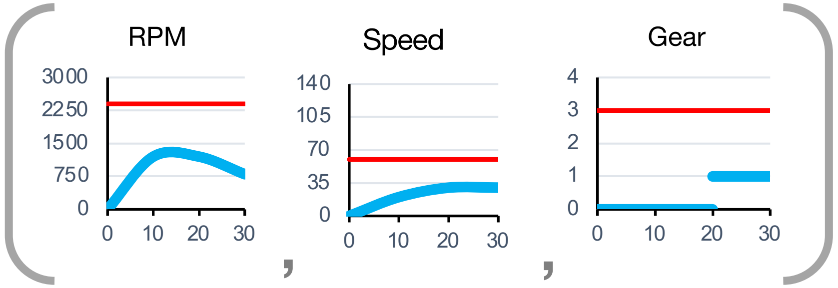
Eventually reach $\mathrm{gear} \geq 3$

Example of Conjunctive synthesis

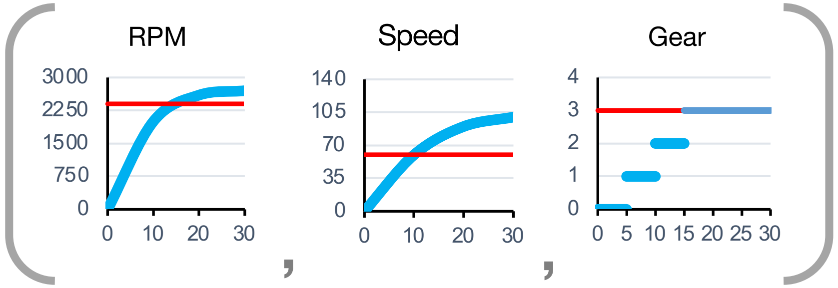
Conjunctive specification $\varphi^{\mathrm{AT}} :\equiv \varphi_1^{AT} \wedge \varphi_2^{AT} \wedge \varphi_3^{AT}$

$\varphi_1^{AT} :\equiv \Box_{[0,30]}(\mathrm{rpm} \leq 2400)$, $\varphi_2^{AT} :\equiv \Box_{[0,30]}(\mathrm{speed} \leq 60)$, $\varphi_3^{AT} :\equiv \Diamond_{[0,30]}(\mathrm{gear} \geq 3)$

Eventually reach $\mathrm{gear} \geq 3$



φ_1^{AT} : Satisfied φ_2^{AT} : Satisfied φ_3^{AT} : **Violated**



φ_1^{AT} : **Violated** φ_2^{AT} : **Violated** φ_3^{AT} : Satisfied

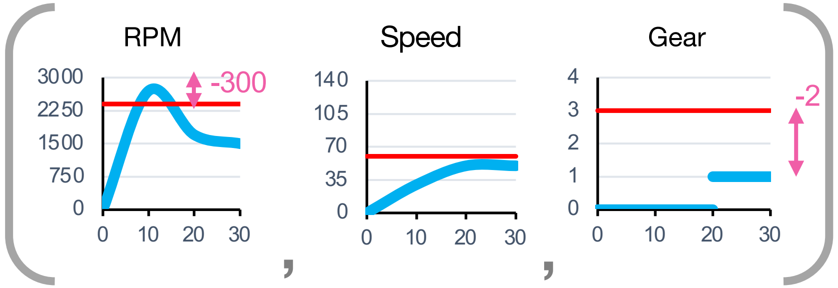
Satisfying all conjuncts are required

Challenge: Scale problem

- Usual robust semantics of conjunction [Fainekos & Pappas, FATES & RV'06]

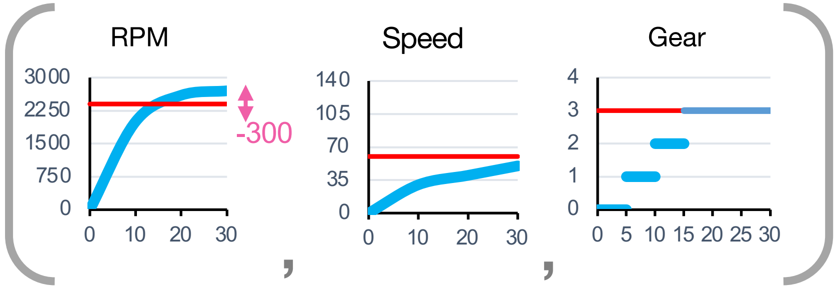
$$[\![\mathcal{M}(u), \varphi^{AT}]\!] = [\![\mathcal{M}(u), \varphi_1^{AT}]\!] \sqcap \cdots \sqcap [\![\mathcal{M}(u), \varphi_3^{AT}]\!]$$

infimum



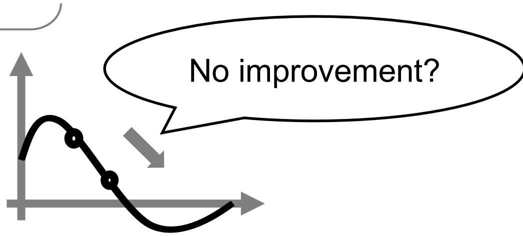
φ_1^{AT} : **Violated** φ_2^{AT} : Satisfied φ_3^{AT} : **Violated**

$[\![\mathcal{M}(u), \varphi^{AT}]\!]$ = **-300**



φ_1^{AT} : **Violated** φ_2^{AT} : Satisfied φ_3^{AT} : **Satisfied**

$[\![\mathcal{M}(u), \varphi^{AT}]\!]$ = **-300**



👎 **Contribution of small-scale conjunct is masked**

Conjunctive synthesis by constrained optim.

Maximize $[\![\mathcal{M}(u), \varphi_1^{AT}]\!] \sqcap \cdots \sqcap [\![\mathcal{M}(u), \varphi_3^{AT}]\!]$

Maximize $[\![\mathcal{M}(u), \varphi_1^{AT}]\!]$

Subject to $[\![\mathcal{M}(u), \varphi_2^{AT}]\!] > 0,\ [\![\mathcal{M}(u), \varphi_3^{AT}]\!] > 0$

Choose one conjunct as the optimization target

No masking

Multiple constraint ranking

How to effectively search the solution of constrained optimization?
➔ MCR [de Paula Garcia et al., Computers and Structures 2017]

- Balances multiple preferences of the solution of constrained optimization
  - Objective function, violation degrees, the number of violated constraints
- Scale-invariant
- No hyper-parameter

Preferred solutions of constrained optim.

Robustness for each conjuncts

	φ_1^{AT} (objective)	φ_2^{AT}	φ_3^{AT}
u_1	1400	-10	-3
$\underline{u_2}$	1400	59.9	0

Feasible is preferred

	φ_1^{AT} (objective)	φ_2^{AT}	φ_3^{AT}
u_1	1400	59.9	-2
$\underline{u_2}$	1400	59.9	-1

Small violation degree is preferred

	φ_1^{AT} (objective)	φ_2^{AT}	φ_3^{AT}
u_1	-1000	59.9	0
$\underline{u_2}$	-30	59.9	0

Large fitness is preferred

Formal definition of MCR

For a population X of candidate inputs, one prioritizes individuals $u \in X$ by

Feasibles are always prior to infeasibles

$$F_X(u) := \begin{cases} \mathrm{RVNum}_X(u) + \sum_{j=2}^{m} \mathrm{RCon}_X^j(u) & \text{(if no feasible solution)} \\ \mathrm{RObj}_X(u) + \mathrm{RVNum}_X(u) + \sum_{j=2}^{m} \mathrm{RCon}_X^j(u) & \text{(otherwise)} \end{cases}$$

Smaller is better

- **RObj** compares the value of objective function
- **RCon** compares the violation degree of each constraints
- **RVNum** compares the number of violated constraints

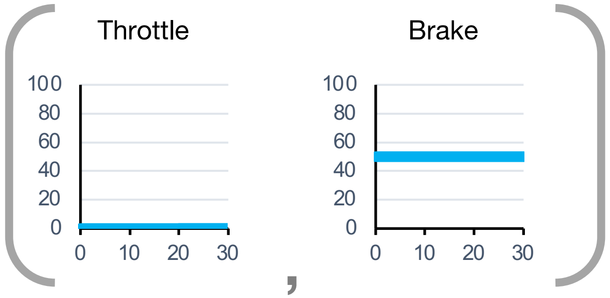
Example: Usual semantics

Robustness for each conjuncts

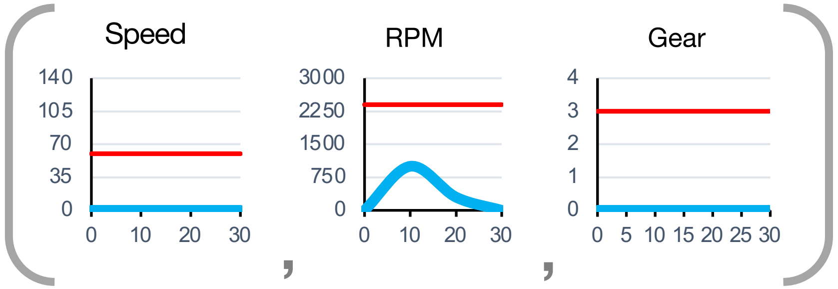
		φ_1^{AT} (objective)	φ_2^{AT}	φ_3^{AT}
Individual	u_1	1400	59.9	-3
	u_2	-9	2	0
	u_3	-180	2	-1

(Naive input)

$u_1 =$

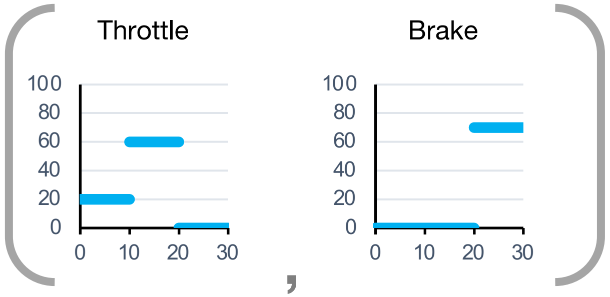


$\mathcal{M}(u_1) =$

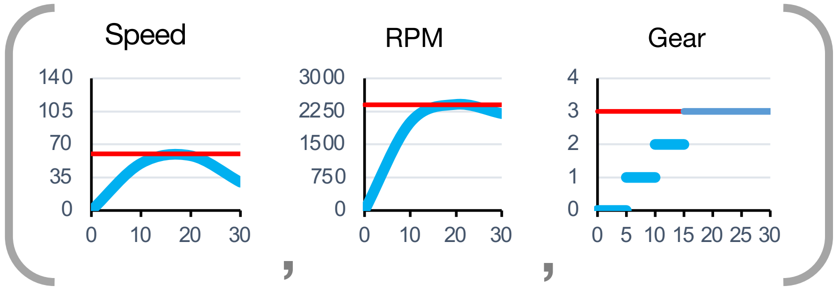


(Almost-satisfying input)

$u_2 =$



$\mathcal{M}(u_2) =$



Example: Usual semantics

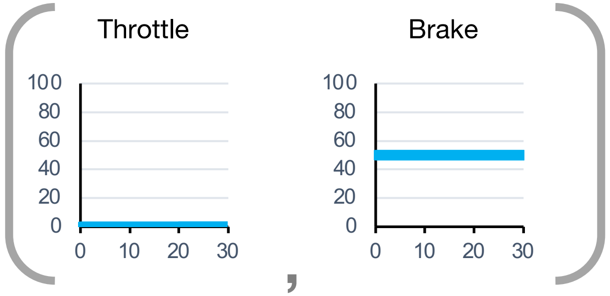
Robustness for each conjuncts

		φ_1^{AT} (objective)	φ_2^{AT}	φ_3^{AT}	infimum
Individual	u_1	1400	59.9	**-3**	**-3**
	u_2	**-9**	2	0	**-9**
	u_3	**-180**	2	-1	**-180**

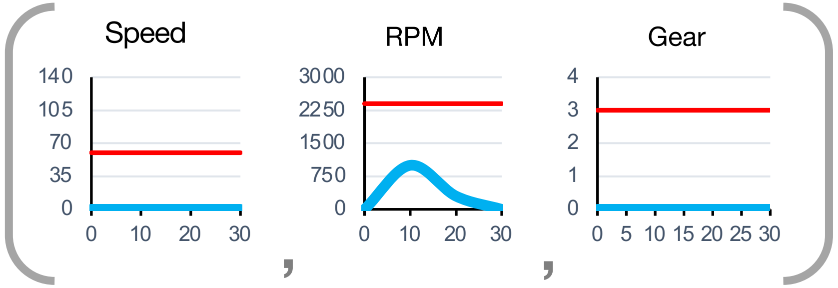
Usual (infimum) semantics says u_1 is the most preferred

(Naive input)

$u_1 =$

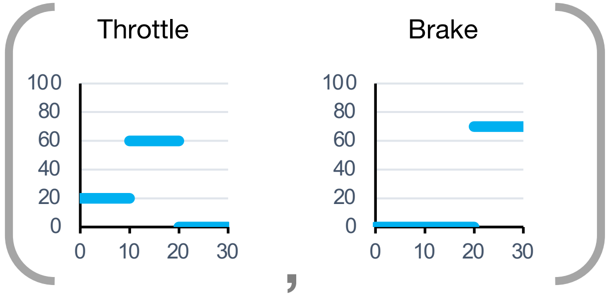


$\mathcal{M}(u_1) =$

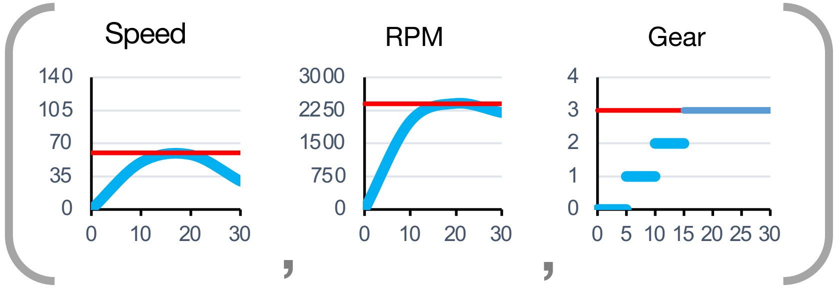


(Almost-satisfying input)

$u_2 =$



$\mathcal{M}(u_2) =$



Example: Calculating MCR

Robustness for each conjuncts

Individual		φ_1^{AT} (objective)	φ_2^{AT}	φ_3^{AT}
	u_1	1400	59.9	-3
	u_2	-9	2	0
	u_3	-180	2	-1

$$F_X(u) := \begin{cases} \mathrm{RVNum}_X(u) + \sum_{j=2}^{m} \mathrm{RCon}_X^j(u) & \text{(if no feasible solution)} \\ \mathrm{RObj}_X(u) + \mathrm{RVNum}_X(u) + \sum_{j=2}^{m} \mathrm{RCon}_X^j(u) & \text{(otherwise)} \end{cases}$$

	Robj$_X$ / robustness of φ_1^{AT}	**RCon$_X^2$** / violation deg. of φ_2^{AT}	**RCon$_X^3$** / violation deg. of φ_3^{AT}	**RVNum$_X$** / # of violated constraints	**F$_X$**
u_1	**1st** / 1400	**1st** / 0	**3rd** / -3	**2nd** / 1	7 (= 1 + 1 + 3 + 2)
u_2	**2nd** / -9	**1st** / 0	**1st** / 0	**1st** / 0	5 (= 2 + 1 + 1 + 1)
u_3	**3rd** / -180	**1st** / 0	**2nd** / -1	**2nd** / 1	8 (= 3 + 1 + 2 + 1)

MCR says u_2 is the most preferred

Example: Usual semantics vs. MCR

Usual semantics

	φ_1^{AT} (objective)	φ_2^{AT}	φ_3^{AT}	infimum
u_1	1400	59.9	-3	**-3**
u_2	-9	2	0	-9
u_3	-180	2	-1	-180

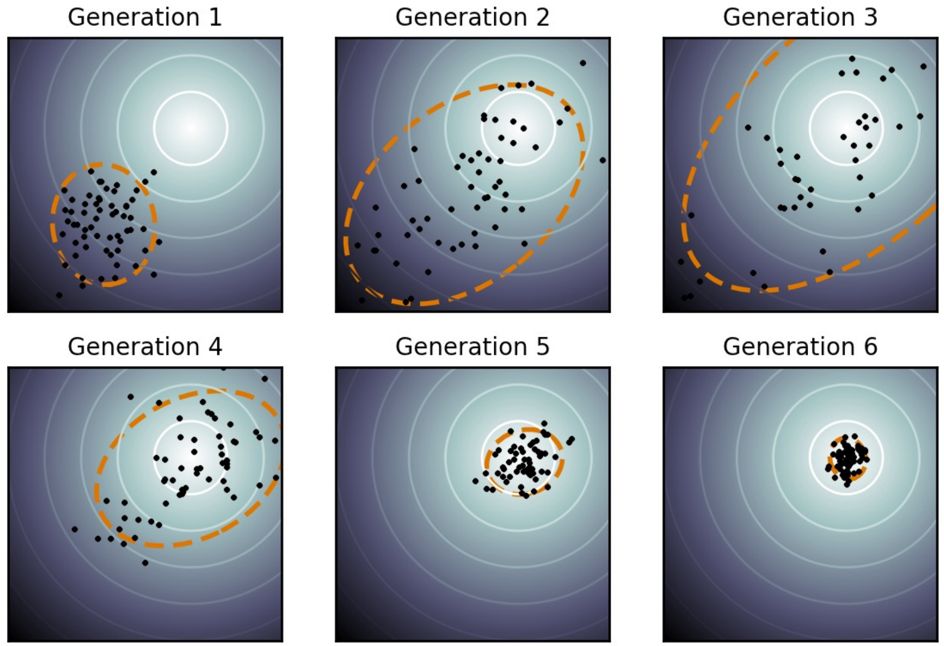
MCR

	φ_1^{AT} (objective)	φ_2^{AT}	φ_3^{AT}	F_X
u_1	1400	59.9	-3	7
u_2	-9	2	0	**5**
u_3	-180	2	-1	8

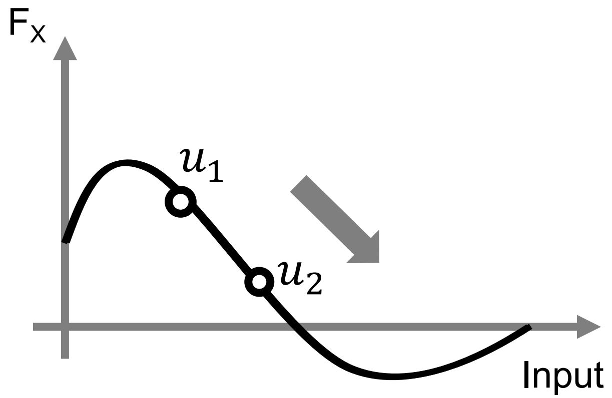
MCR fits our intuition

MCR for falsification

- Optimization algorithm should be population-based
- We adopt **CMA-ES** [Hansen & Ostermeier, International Conference on Evolutionary Computation 1996]
- CMA-ES is commonly used in optimization-based falsification



https://en.wikipedia.org/wiki/CMA-ES



Experimental setting

Model

- Automatic transmission [Hoxha et al., ARCH'15]
- Abstract fuel Control [Jin et al., HSCC'14]
- Wind Turbine [Schuler et al., ARCH'16]

Solver

- Breach (state-of-the-art falsification solver) [Donze, CAV'10]
- MCR (Breach + MCR calculator implemented in Python)
  - Choose each conjunct as **the optimization target** and report **best and worst results**

Metrics

- Success rate (per 60 trials)
- Average elapsed time of successful trials

Particularly exhibits the scale problem

Spec. ID	φ_1	φ_2	φ_3	φ_4
$\mathrm{AT1}_p$	$\Box_{[0,30]}(\mathsf{rpm} \leq p)$	$\Box_{[0,30]}(\mathsf{speed} \leq 60)$	$\Diamond_{[0,30]}(\mathsf{gear} \geq 3)$	
AT2	$\Diamond_{[0,29]}(\mathsf{speed} \geq 100)$	$\Diamond_{[29,30]}(\mathsf{speed} \leq 65)$		
$\mathrm{AT3}_{p1,p2}$	$\Diamond_{[0,10]}(\mathsf{speed} \geq p_1)$	$\Box_{[0,30]}(\mathsf{rpm} \leq p_2)$		
AFC	$\Box_{[31,50]}(\mathsf{mode} = 0)$	$\Diamond_{[11,20]}(\mathsf{mode} = 1)$	$\Box_{[0,30]}(\mathsf{throttle} > 40 \Rightarrow \mathsf{engine} < 1000)$	$\Diamond_{[0,50]}\Box_{[0,25]}(\mathsf{engine} > 1000)$
WT	$\Diamond_{[0,90]}\Box_{[0,5]}(\theta < 12 \land 15.5 \leq v \leq 15.95)$	$\Diamond_{[0,90]}(M_{g,d} \geq 47000)$	$\Diamond_{[0,90]}(\Omega < 9)$	

Experimental results

Our approach

The choice of optimization target

Model	Spec. φ	Breach		MCR (best)		MCR (worst)	
		SR	time [s]	SR	time	SR	time
AT	$AT1_{2500}$	58	34.3	**60**	33.9	**60**	38.9
	$AT1_{2400}$	18	72.7	**55**	147.0	28	87.3
	$AT1_{2300}$	0	—	**37**	326.4	0	—
	AT2	51	245.1	**54**	307.9	43	233.1
	$AT3_{80,4500}$	**60**	31.3	**60**	24.4	**60**	31.0
	$AT3_{50,2700}$	**60**	108.8	59	127.7	56	157.0
AFC	AFC	43	272.4	**54**	288.2	48	248.3
WT	WT	**60**	175.7	**60**	174.0	59	180.1

- Largerst success rates are show in blue
- Best combination of (SR, time) is highlighted

RQ1: Does MCR address the scale problem?

Model	Spec. φ	Breach		MCR (best)		MCR (worst)	
		SR	time [s]	SR	time	SR	time
AT	$AT1_{2500}$	58	34.3	**60**	33.9	**60**	[illegible]
	$AT1_{2400}$	18	72.7	**55**	147.0	28	[illegible]
	$AT1_{2300}$	0	—	**37**	326.4	0	—
	$AT2$	51	245.1	**54**	307.9	43	233.1
	$AT3_{80,4500}$	**60**	31.3	**60**	24.4	**60**	31.0
	$AT3_{50,2700}$	**60**	108.8	59	127.7	56	157.0
AFC	AFC	43	272.4	**54**	288.2	48	248.3
WT	WT	**60**	175.7	**60**	174.0	59	180.1

Breach always failed but MCR **succeeded 37 times** (/60)

- Yes. Our approach resulted higher SR in most cases (blue)
- Specifically, the advantage is obvious where the scale problem is more eminent

RQ2: How important is the choice of the objective conjunct in MCR?

The performance of MCR (worst) is **comparable or better** compared to Breach in every benchmarks.

Model	Spec. φ	Breach		MCR (best)			
		SR	time [s]	SR	time	SR	
AT	$AT1_{2500}$	58	34.3	**60**	33.9	**60**	38.9
	$AT1_{2400}$	18	72.7	**55**	147.0	28	87.3
	$AT1_{2300}$	0	—	**37**	326.4	0	—
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- A **bad choice** had **a negative effect** on the performance of MCR
- The effect is not so critical

Future work and Conclusion

Future work

- A method to choose the good objective conjunct
- Extension to more general form of specifications

Conclusion

- A method solving conjunctive synthesis via constrained optimization
- MCR and CMA-ES for effective optimization
- Our approach addresses the scale problem