Weighted Automata Extraction from Recurrent Neural Networks via Regression on State Spaces

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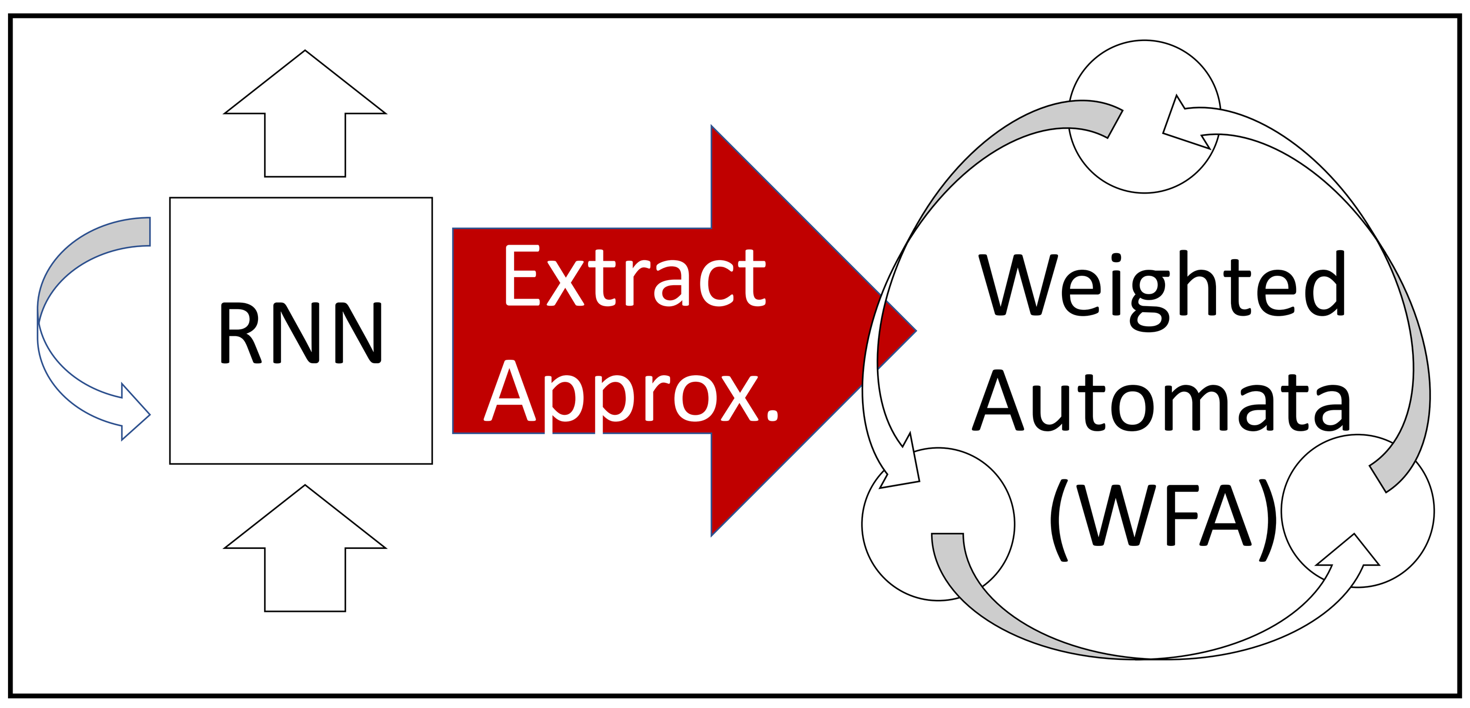
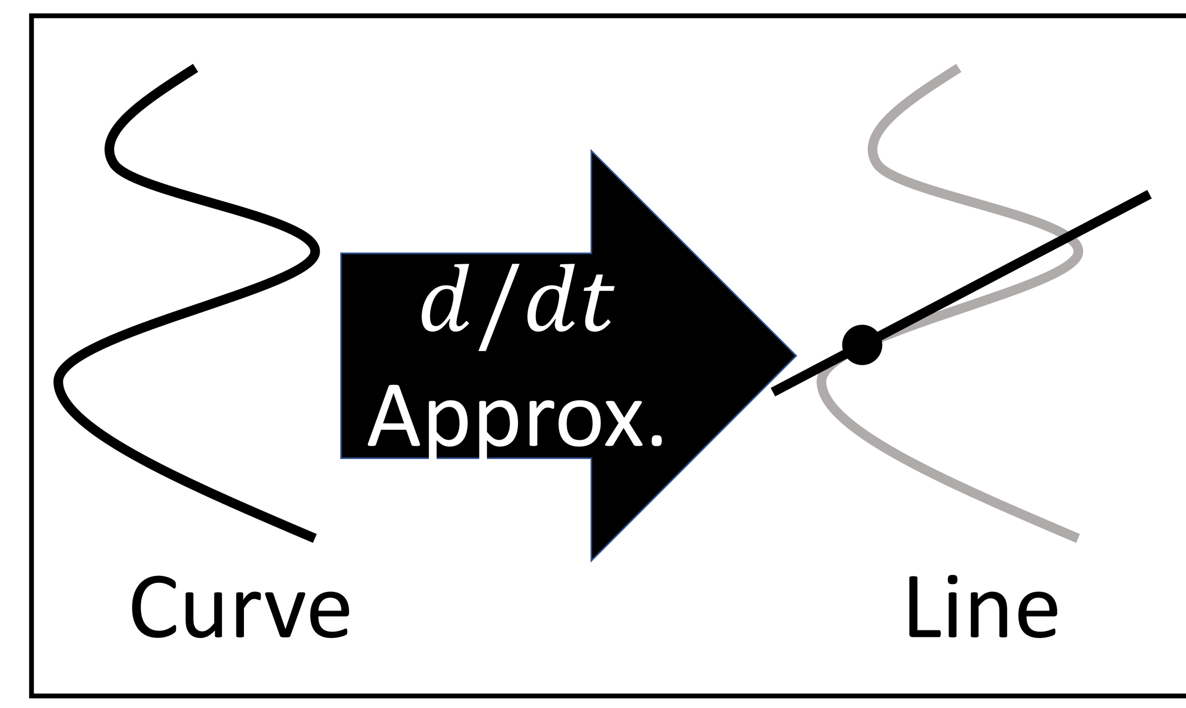
What's this?

We did 

like we do 

!

Experimental results

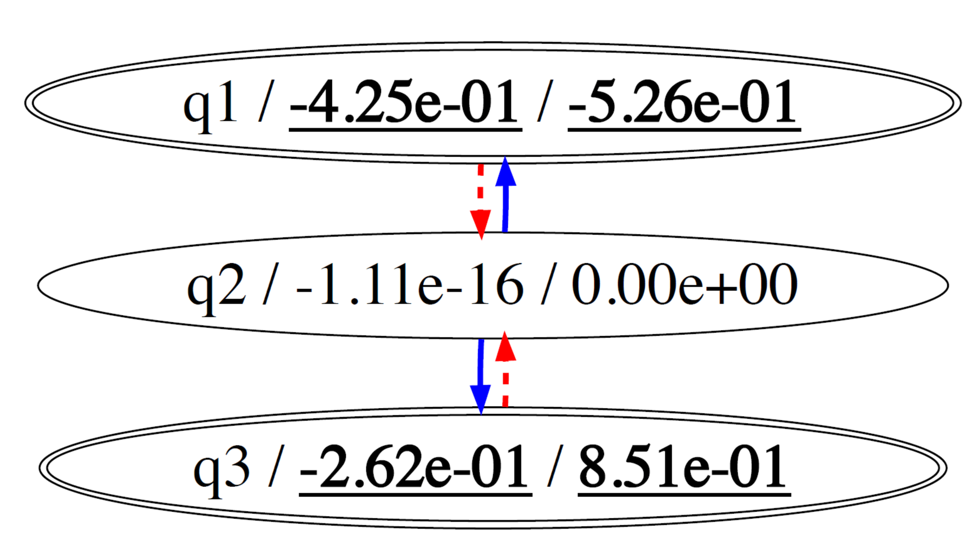
(1) Extraction is quick and accurate!

$(\|\Sigma\|, \|Q_{A\bullet}\|)$	**RGR**(2)	**RGR**(5)	**BFS**(500)	**BFS**(1000)	**BFS**(2000)	**BFS**(3000)	**BFS**(5000)
(4, 10)	2.17 / 286	2.39 / 338	26.8 / 165	9.77 / 279	4.36 / 545	4.07 / 716	2.33 / 1390
(6, 10)	2.45 / 1787	2.54 / 1302	6.99 / 386	4.48 / 641	4.08 / 1218	3.15 / 1410	2.28 / 2480
(10, 10)	4.68 / 7462	4.46 / 5311	22.5 / 928	11.9 / 1562	5.90 / 3521	4.55 / 3638	3.55 / 5571
(10, 15)	5.62 / 8941	5.78 / 8564	21.2 / 2155	10.6 / 4750	7.87 / 5692	5.71 / 7344	5.27 / 7612
(10, 20)	3.70 / 7610	3.79 / 7799	6.24 / 2465	10.1 / 2188	6.13 / 3106	3.70 / 5729	3.63 / 7473
(15, 10)	7.34 / 9569	5.52 / 10000	13.5 / 3227	8.01 / 6765	6.07 / 7916	5.98 / 8911	6.17 / 8979
(15, 15)	8.44 / 10000	5.58 / 9981	16.3 / 2675	9.24 / 4850	7.28 / 5135	9.88 / 7204	6.44 / 8425
(15, 20)	9.16 / 7344	5.15 / 7857	13.7 / 2224	7.26 / 3823	6.60 / 5744	4.96 / 5674	4.01 / 9464
Total	5.45 / 6625	4.40 / 6394	15.9 / 1778	8.92 / 3107	6.04 / 4110	5.25 / 5078	4.21 / 6549

$(\|\Sigma\|, \|Q_{A\bullet}\|)$	**RGR**(2)	**RGR**(5)	**BFS**(500)	**BFS**(1000)	**BFS**(2000)	**BFS**(3000)	**BFS**(5000)
(4, 10)	7.73 / 696	7.07 / 1135	15.0 / 199	7.96 / 424	6.62 / 650	6.61 / 762	9.06 / 1693
(6, 10)	4.92 / 1442	7.43 / 1247	1.46 / 552	6.95 / 660	5.90 / 1217	8.78 / 1557	3.54 / 2237
(10, 10)	5.02 / 5536	4.28 / 5951	7.70 / 1117	11.0 / 1738	4.77 / 2635	3.52 / 3926	4.52 / 4777
(10, 15)	7.15 / 6977	4.35 / 8315	19.4 / 1552	13.8 / 3271	16.8 / 3209	8.57 / 5293	5.08 / 6522
(10, 20)	6.98 / 4697	8.06 / 6704	18.6 / 1465	11.8 / 2046	12.7 / 2851	9.03 / 4259	8.01 / 4856
(15, 10)	5.97 / 8747	6.77 / 8882	23.3 / 2359	11.2 / 4668	9.88 / 6186	6.24 / 7557	6.02 / 8245
(15, 15)	5.78 / 8325	8.71 / 7546	16.6 / 2874	7.31 / 4380	9.92 / 6015	9.89 / 7110	6.40 / 8358
(15, 20)	4.60 / 7652	8.56 / 8334	36.9 / 1893	23.7 / 3069	12.8 / 3987	12.0 / 5262	8.38 / 6441
Total	6.02 / 5510	6.90 / 6015	19.0 / 1502	11.7 / 2532	9.92 / 3344	8.08 / 4466	6.38 / 5391

- We trained RNNs from randomly generated WFAs and applied the extraction to the RNNs. The cells show "{error}/{time}."
- Rows show various settings of the random WFAs / Columns show extraction methods: RGR(*) are ours, and BFS(*) are naïve baselines.
- Two tables use different ways to generate WFAs (see our paper!)

(2) Extraction yields interpretability!



- •We trained RNNs from a weighted language (see "Math stuff") that are so complex that any WFAs cannot express it and applied our extraction.
- The extracted WFAs represent a simplified version of the original language, and it can be graphically representable as above.

(3) Extraction makes the inference faster!

x1300 faster
by using WFA
Instead of RNN

- We used the same RNNs and extracted WFAs and compared inference time for random inputs.

Method

Algorithm 1 Answering equivalence queries

1: **procedure** ANS-EQQ
Input: RNN $R = (\alpha_R, \beta_R, g_R)$, WFA $A = (Q_A, \alpha_A, \beta_A, (A_\sigma)_{\sigma\in\Sigma})$, error tolerance $e > 0$ and concentration threshold $M \in \mathbb{N}$
Output: a counterexample, or Equivalent
2: Initialize $p\colon \mathbb{R}^d \to \mathbb{R}^{Q_A}$ so that $p(\delta_R(\varepsilon)) = \delta_A(\varepsilon)$
3: queue $\leftarrow \langle\varepsilon\rangle$; visited $\leftarrow \emptyset$
4: **while** queue is non-empty **do**
5: $h \leftarrow$ **pop**(queue) ▷ Pop the element of the maximum priority
6: **if** $|f_R(h) - f_A(h)| \geq e$ **then**
7: **return** h ▷ return a counterexample
8: result $\leftarrow$ CONSISTENT?(h, visited, p)
9: **if** result = NG **then**
10: learn p by regression, so that $p(\delta_R(h')) = \delta_A(h')$ holds for all $h' \in$ visited $\cup \{h\}$
11: visited $\leftarrow$ visited $\cup \{ h \}$
12: visited′ $\leftarrow p(\delta_R($visited$))$
13: $\#vn \leftarrow |\{ x \in$ visited′ $\mid x \simeq_A p(\delta_R(h)) \}|$
14: **if** $\#vn \leq M$ **then**
15: pr $\leftarrow \min_{h' \in \text{visited}\setminus\{h\}} d(p(\delta_R(h)), p(\delta_R(h')))$ ▷ d is the Euclidean distance
16: **push** $h\sigma$ **to** queue with priority pr **for** $\sigma \in \Sigma$
17: **return** Equivalent

- Balle and Mohri's WFA learning algorithm reduces the problem of checking whether the target RNN and a given WFA are equivalent in respect of behavior.
- The algorithm is on the left. The main ideas are:
  - Run best-first search on Σ^* and find the difference between the RNN and the WFA (Line 4-7, 16)
  - Find the relation between the RNN and the WFA by making a map from the internal state of RNN to the internal state of WFA by GPR (Line 10)
  - Prioritize the visit where "the search is not enough." (Line 5, 14-15)
  - Prune the searching where "the search is enough." (Line 12-13)

Math stuff…

[Weighted language] Function $\Sigma^* \to \mathbb{R}$
[WFA] WFA A is a quadruple of
- Q_A ——— states
- $\alpha_A \in \mathbb{R}^{Q_A}$ ——— initial vector
- $\beta_A \in \mathbb{R}^{Q_A}$ ——— final vector
- $(A_\sigma)_{\sigma\in\Sigma} \in \mathbb{R}^{Q_A \times Q_A}$ ——— transition matrix

WFA induces its configuration δ_A and output f_A by:
- $\delta_A(w_1 \dots w_n) = \alpha_A^T A_{w_1} \dots A_{w_n}$
- $f_A(w_1 \dots w_n) = \delta_A(w_1 \dots w_n)\beta$

[RNN] RNN R is a triple of
- $\alpha_R \in \mathbb{R}^d$ ——— initial state
- $\beta_R \in \mathbb{R}^d \to \mathbb{R}$ ——— final state
- $g_R\colon \mathbb{R}^d \times \Sigma^* \to \mathbb{R}^d$ ——— transition func.

RNN induces its configuration δ_R and output f_A by:
- $\delta_R(w_1 \dots w_n) = \begin{cases} g_R(\delta_R(w_1 \dots w_{n-1}), w_n) & n > 0 \\ \alpha_A & n = 0 \end{cases}$
- $f_R(w_1 \dots w_n) = \beta_R(\delta_R(w_1 \dots w_n))$

Future work

- Improving scalability
  Currently, $|\Sigma| = 15$ is the maximum. Can we make it work for larger $|\Sigma|$ so that it is applicable for NLP?
- Giving a theoretical guarantee
  It is a heuristics now, and there is no guarantee for the superiority of our method nor termination.
- Applying for quality assurance of RNN
  Model-checking is a field to generate the proof of the (typically) safety of the system. Can we combine our technique with model checking of WFAs and make a technique for the quality assurance of RNN?
- Forcing the WFA to be probabilistic
  Even if the target system outputs the values in [0, 1], Balle and Mohri's algorithm does not necessarily output WFAs whose output is in it. Can we fix it and improve the applicability to the RNNs whose outputs are probabilistic?

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